Work ,Energy And Power

Assignment

1. Can any object have mechanical energy even if its momentum is zero? Explain.
Ans: Yes. An object at rest has zero momentum but can still possess potential energy. For example, a book kept on a shelf has zero momentum but has gravitational potential energy.
2. What is power? How do you differentiate kilowatt from kilowatt hour?
Ans: Power: The rate of doing work or transferring energy.

$$\text{Power} = \frac{\text{Work}}{\text{Time}}$$

- Kilowatt (kW): Unit of power.
  1 kW = 1000 W.
- Kilowatt-hour (kWh): Unit of energy.
  1 kWh = Energy consumed by a 1 kW appliance in 1 hour = 3.6×10^6 J.

3. Give one example each of potential energy:
Ans: (i) Due to position: Water stored in a dam.
(ii) Due to shape: A stretched rubber band or a compressed spring.
4. What kind of energy transformation takes place when a body is dropped from a certain height?
Ans: Potential energy is converted into kinetic energy as the body falls.
5. Can kinetic energy of a body be negative?
Ans: No. Kinetic energy cannot be negative because

$$KE = \frac{1}{2}mv^2$$

and v^2 is always positive or zero.
6. A freely falling object eventually stops on reaching the ground. What happens to its kinetic energy?
Ans: Its kinetic energy is converted into heat, sound, and deformation energy when it hits the ground.
7. What is the work done by the force of gravity on a satellite moving round the Earth? Justify your answer.
Ans: Zero. Gravity acts towards the Earth's center, while the satellite moves perpendicular to this force. Hence,

$$W = Fs\cos 90^\circ = 0.$$

8. The potential energy of a freely falling object decreases progressively. Does this violate the law of conservation of energy? Why?
Ans: No. The decrease in potential energy is exactly equal to the increase in kinetic energy, so the total mechanical energy remains constant.
9. Define 1 J of work.
Ans: One joule is the work done when a force of 1 newton moves an object through 1 metre in the direction of the force.
10. Derive the formula for potential energy.
Ans: Let a body of mass m be lifted through height h.
Force required = Weight = mg
Work done = Force × Distance

$$W = mg \times h = mgh$$

Since work done is stored as potential energy,

$$\boxed{PE = mgh}$$

where
m = mass,
g = acceleration due to gravity,
h = height.

11. Explain the Law of Conservation of Energy with a pendulum.
Ans: When a pendulum is at its highest position, its potential energy is maximum and kinetic energy is zero. As it swings downward, potential energy changes into kinetic energy. At the lowest point, kinetic energy is maximum and potential energy is minimum. While moving upward, kinetic energy again changes into potential energy. Thus, the total mechanical energy remains constant, proving the law of conservation of energy.
12. If energy is neither created nor destroyed, then from where do we get energy?
Ans: We do not create energy. We obtain usable energy by converting it from one form to another, such as chemical energy in food or fuels, solar energy, wind energy, and hydro energy.
13. State and explain one example where kinetic energy is present in a body and is used.
Ans: A moving bicycle has kinetic energy. This energy helps it move forward and can be used to do work, such as transporting a person.
14. Define power and give its unit.
Ans: Power is the rate at which work is done or energy is transferred.

$$P = \frac{W}{t}$$

SI unit: Watt (W).
15. How are work and energy related to each other?
Ans: Energy is the capacity to do work. When work is done on or by an object, energy is transferred. The amount of work done equals the energy transferred.
16. Give one example where work done on an object is negative.
Ans: When friction acts on a moving object, the force is opposite to the direction of motion, so the work done by friction is negative.
17. Give one example where work done on an object is zero.
Ans: A person carrying a bag while walking on a level road does no work on the bag in the physics sense because the force is vertical while the displacement is horizontal, making the angle 90°. Hence, work done is zero.

1. A spring which in initially in un-stretched condition, is first stretched by a length x and again by a further length x. The work done in the first case W_1 is one third of the work done in second case W_2. True or false?

Ans :
The work done in stretching a spring from extension x_1 to x_2 is:

$$W = \frac{1}{2}k(x_2^2 - x_1^2)$$

where k is the spring constant.
First stretch (0 to x)

$$W_1 = \frac{1}{2}kx^2$$

Second stretch (x to $2x$)

$$W_2 = \frac{1}{2}k[(2x)^2 - x^2]$$

$$= \frac{1}{2}k(4x^2 - x^2) = \frac{1}{2}(3kx^2) = \frac{3}{2}kx^2$$

Now,

$$\frac{W_1}{W_2} = \frac{\frac{1}{2}kx^2}{\frac{3}{2}kx^2} = \frac{1}{3}$$

Hence,

$$W_1 = \frac{1}{3}W_2$$

True.

2. A particle moves along X- axis from x=0 to x=8 under the influence of a force given by F= $3x^2 - 4x + 5$. Find the work done in the process

Ans: The work done by a variable force is given by

$$W = \int_{x_1}^{x_2} F(x)\, dx$$

Here,

$$F(x) = 3x^2 - 4x + 5, x_1 = 0, x_2 = 8$$

So,

= $W = \int_0^8 (3\,x^2 - 4x + 5)\, dx$

Integrating,

=$W = [x^3 - 2x^2 + 5x]_0^8$

Substitute the limits:

At $x = 8$,

= $8^3 - 2(8^2) + 5(8) = 512 - 128 + 40 = 424$

At $x = 0$,

$$0 - 0 + 0 = 0$$

Therefore,

$$W = 424 - 0 = 424 \text{ J}$$

$$\boxed{424 \text{ J}}$$

3. A body of mass 10 kg at rest is subjected to a force of 16N.Find the kinetic energy at the end of 10 s.

Ans: **Given:**

Mass, $m = 10$ kg

Force, $F = 16$ N

Time, $t = 10$ s

Initial velocity, $u = 0$

Using Newton's second law,

$$F = ma$$

= $a = \frac{F}{m} = \frac{16}{10} = 1.6 \text{ m/s}^2$

Using the equation of motion,

$$v = u + at$$

= $v = 0 + (1.6)(10) = 16$ m/s

= $KE = \frac{1}{2}mv^2$

= $KE = \frac{1}{2} \times 10 \times 16^2$

= $5 \times 256 = 1280$ J

4. A body of mass 5kg is thrown up vertically with a kinetic energy of 1000 J. If acceleration due to gravity is $10ms^{-2}$, find the height at which the kinetic energy becomes half of the original value.

Ans: **Given:**

Mass, $m = 5$ kg

Initial kinetic energy, $KE_1 = 1000$ J

Final kinetic energy, $KE_2 = \frac{1000}{2} = 500$ J

$$g = 10 \text{ m/s}^2$$

As the body moves upward, its kinetic energy decreases while its potential energy increases.

By the **law of conservation of energy**:

$$\text{Loss in KE} = \text{Gain in PE}$$

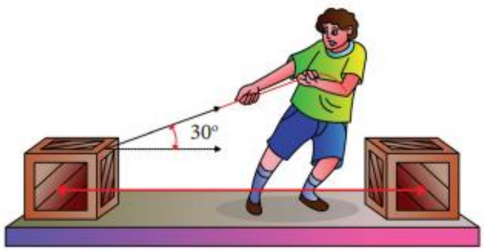
Loss in kinetic energy:

$$1000 - 500 = 500 \text{ J}$$

Gain in potential energy:

$$mgh = 500$$
$$= 5 \times 10 \times h = 500$$
$$= 50h = 500$$
$$= h = \frac{500}{50} = 10 \text{ m}$$

5. A box is pulled with a force of 25 N to produce a displacement of 15 m. If the angle between the force and displacement is 30°, find the work done by the force.



Ans: Given:

Force, $F = 25$ N

Displacement, $d = 15$ m

Angle, $\theta = 30^\circ$

Work done is calculated using:

$$W = Fd\cos\theta$$

Substitute the values:

$$W = 25 \times 15 \times \cos 30^\circ$$

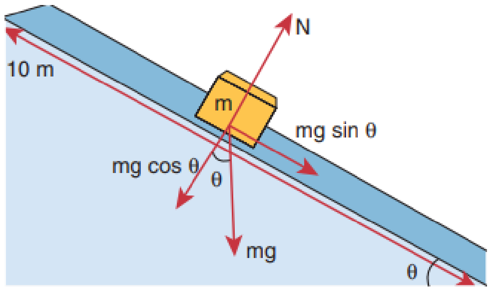
Since:

= $\cos 30^\circ = \frac{\sqrt{3}}{2} \approx 0.866$

= $W = 25 \times 15 \times 0.866$

$$= \boldsymbol{W = 375 \times 0.866 = 324.75} \textbf{ J}$$

6. An object of mass m=1 kg is sliding from top to bottom in the frictionless inclined plane of inclination angle θ = 30° and the length of inclined plane is 10 m as shown in the figure. Calculate the work done by gravitational force and normal force on the object. Assume acceleration due to gravity, g = 10 m s^{-2}



Ans: Given:

Mass, $m = 1$ kg

Length of incline, $s = 10$ m

Inclination, $\theta = 30^\circ$

Gravity, $g = 10$ m/s^2

Frictionless surface

Work done by gravitational force

The component of gravity along the incline is:

$$F = mg\sin\theta$$

$$F = 1 \times 10 \times \sin 30^\circ = 10 \times \frac{1}{2} = 5 \text{ N}$$

Work done by gravity:

$$W = Fs = 5 \times 10 = 50 \text{ J}$$

So,

$$\boxed{W_{\text{gravity}} = 50 \text{ J}}$$

Work done by the normal force

The normal force is perpendicular to the displacement along the incline. Therefore,

$$W = Fd\cos 90^\circ$$

Since $\cos 90^\circ = 0$,

$$W_{\text{normal}} = 0$$

So,

$$\boxed{W_{\text{normal}} = 0 \text{ J}}$$

7. If an object of mass 2 kg is thrown up from the ground reaches a height of 5 m and falls back to the Earth (neglect the air resistance). Calculate a) The work done by gravity when the object reaches 5 m height b) The

work done by gravity when the object comes back to Earth c) Total work done by gravity both in upward and downward motion and mention the physical significance of the result.

Ans: Given:

Mass, $m = 2$ kg

Height, $h = 5$ m

Acceleration due to gravity, $g = 10$ m/s^2

Weight of the object: $mg = 2 \times 10 = 20$ N

(a) Work done by gravity when the object reaches 5 m height

During upward motion, gravity acts downward while displacement is upward, so the angle is $180°$.

$$= W = Fd\cos 180°$$
$$= W = 20 \times 5 \times (-1)$$
$$= \boxed{W = -100\text{ J}}$$

(b) Work done by gravity when the object comes back to Earth

During downward motion, gravity and displacement are in the same direction, so the angle is $0°$.

$$W = Fd\cos 0°$$
$$W = 20 \times 5 \times 1$$
$$\boxed{W = +100\text{ J}}$$

(c) Total work done by gravity

$$W_{\text{total}} = (-100) + (+100)$$
$$\boxed{W_{\text{total}} = 0\text{ J}}$$

8. Consider an object of mass 2 kg moved by an external force 20 N in a surface having coefficient of kinetic friction 0.9 to a distance 10 m. What is the work done by the external force and kinetic friction ? Comment on the result. (Assume g = 10 ms^{-2})

Ans: Given:

Mass of object, $m = 2$ kg

External force, $F = 20$ N

Coefficient of kinetic friction, $\mu_k = 0.9$

Distance moved, $d = 10$ m

Acceleration due to gravity, $g = 10$ m/s^2

Work done by the external force

The external force and displacement are in the same direction, so:

$$W_{\text{external}} = Fd\cos 0°$$

$$W_{\text{external}} = 20 \times 10 \times 1$$

$$\boxed{W_{\text{external}} = 200\text{ J}}$$

Work done by kinetic friction

Normal force:

$$N = mg$$
$$N = 2 \times 10 = 20\text{ N}$$

Kinetic frictional force:

$$f_k = \mu_k N$$
$$f_k = 0.9 \times 20$$
$$f_k = 18\text{ N}$$

Friction acts opposite to the direction of displacement, so the angle is $180°$.

$$W_{\text{friction}} = f_k d\cos 180°$$
$$W_{\text{friction}} = 18 \times 10 \times (-1)$$

$$\boxed{W_{\text{friction}} = -180\text{ J}}$$

Step 3: Comment on the result

Net work done:

$$W_{\text{net}} = W_{\text{external}} + W_{\text{friction}}$$
$$W_{\text{net}} = 200 + (-180)$$

$$\boxed{W_{\text{net}} = 20 \text{ J}}$$

9. An object of mass 1 kg is falling from the height h = 10 m. Calculate a. The total energy of an object at h =10 m b. Potential energy of the object when it is at h = 4 m c. Kinetic energy of the object when it is at h = 4 m d. What will be the speed of the object when it hits the ground? (Assume g =10 m s^{-2})

Ans: **Given:**

Mass, $m = 1$ kg

Initial height, $h = 10$ m

Acceleration due to gravity, $g = 10$ m/s^2

Object is falling freely (air resistance neglected)

Total mechanical energy remains constant:

$$E_{\text{total}} = PE + KE$$

(a) Total energy of the object at $h = 10$ m

At the starting point, the object is at rest, so:

$$KE = 0$$

Potential energy:

$$PE = mgh$$
$$PE = 1 \times 10 \times 10$$
$$PE = 100 \text{ J}$$

Therefore,

$$E_{\text{total}} = PE + KE = 100 + 0$$
$$\boxed{E_{\text{total}} = 100 \text{ J}}$$

(b) Potential energy at $h = 4$ m

$$PE = mgh$$
$$PE = 1 \times 10 \times 4$$
$$\boxed{PE = 40 \text{ J}}$$

(c) Kinetic energy at $h = 4$ m

Total energy is conserved:

$$E_{\text{total}} = PE + KE$$

=$100 = 40 + KE$

=$KE = 100 - 40$

$$\boxed{KE = 60 \text{ J}}$$

(d) Speed of the object when it hits the ground

At the ground:

$$h = 0$$

So,

$$PE = 0$$

All the potential energy converts into kinetic energy:

$$KE = 100 \text{ J}$$

Using:

$$KE = \frac{1}{2}mv^2$$
$$100 = \frac{1}{2}(1)v^2 \qquad = v^2 = 200$$
$$v = \sqrt{200}$$

$v = 14.14$ m/s