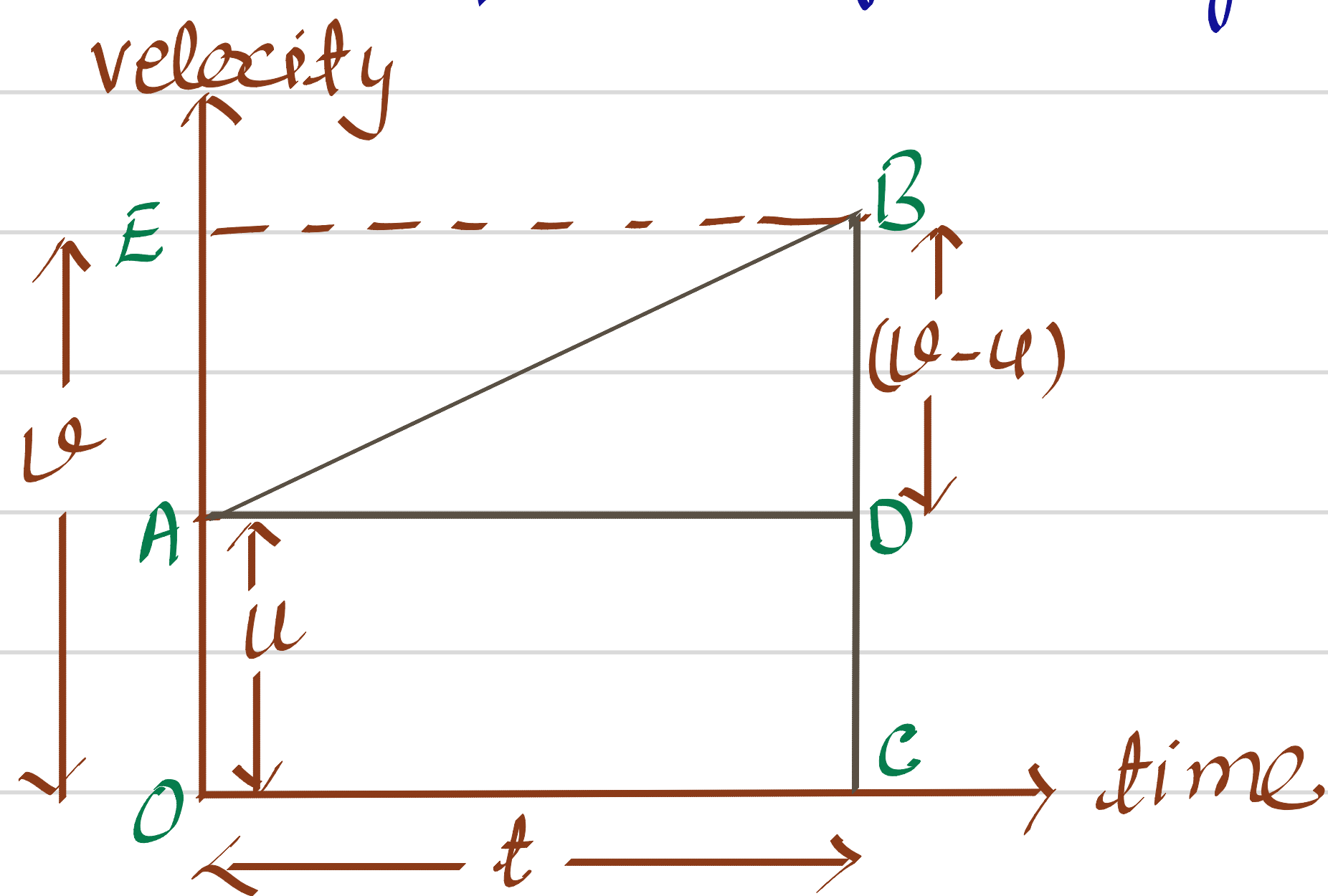


PHYSICS HALF-YEARLY EXAMSCLASS XI MOST IMPORTANT DERIVATIONSCHAPTER - MOTION IN A STRAIGHT LINE

Q.1 Derive three equations of motion (Kinematics equations) by graphical method.

A. Consider $v-t$ graph of a body shown in fig



$u \rightarrow$ initial velocity
 $v \rightarrow$ final velocity
 $BD = BC - CD = v - u$

1st Equation ($v = u + at$)

We know that the slope of a velocity-time graph is equal to acceleration (a), thus

$$a = \frac{BD}{AD} = \frac{v-u}{t} \quad [\because BD = v-u]$$

$$\Rightarrow at = v - u$$

$$\Rightarrow \boxed{v = u + at}$$

2nd Equation ($s = ut + \frac{1}{2}at^2$)

Suppose the body travels distance (s) in time (t)

Distance travelled $s =$ Area under the $v-t$ graph

$=$ Area of trapezium $OABC$

$$s = \frac{\text{sum of parallel sides} \times \text{height}}{2}$$

$$= \frac{(OA + CB) \times OC}{2}$$

$$= \frac{u + v}{2} \times t$$

$$\text{or } S = \frac{[u + (u + at)] \times t}{2} \quad [\because v = u + at]$$

$$\begin{aligned} \text{or } S &= \frac{(2u + at) \times t}{2} \\ &= \frac{(2ut + at^2)}{2} \end{aligned}$$

$$S = ut + \frac{1}{2}at^2$$

3rd Equation ($v^2 = u^2 + 2aS$)

$$\begin{aligned} S &= \text{Area under } v-t \text{ graph} \\ &= \text{Area of trapezium } OABC \\ &= \frac{\text{Sum of parallel side} \times \text{height}}{2} \end{aligned}$$

$$= \frac{(OA + BC) \times AD}{2}$$

$$= \frac{(u + v) \times t}{2}$$

$$= \left(\frac{u + v}{2} \right) \left(\frac{v - u}{a} \right) \quad \left[\begin{array}{l} \because \text{ by } v = u + at \\ t = \frac{v - u}{a} \end{array} \right]$$

$$S = \frac{v^2 - u^2}{2a}$$

$$\text{or } v^2 - u^2 = 2aS$$

$$\text{or } \boxed{v^2 = u^2 + 2aS}$$

Q.2 * (If it is in your syllabus)
Derive expression for distance travelled by an object in n th second. (Expression for S_{nth})

Expression for S_{nth}

$$\text{We know } S = ut + \frac{1}{2}at^2$$

here $s \rightarrow$ distance, $u \rightarrow$ initial velocity, $a \rightarrow$ acceleration
 $t \rightarrow$ time

Distance travelled in $t = n$ seconds -

$$S_n = un + \frac{1}{2}an^2 \quad (1)$$

distance travelled in $t = (n+1)$ seconds

$$S_{n+1} = u(n+1) + \frac{1}{2}a(n+1)^2 \quad (2)$$

Now distance travelled in n th second -

$$S_{nth} = S_n - S_{n-1}$$

put values of S_n and S_{n-1} from eqⁿ (1) and (2)
we get

$$S_{nth} = un + \frac{1}{2}an^2 - \left[u(n-1) + \frac{1}{2}a(n-1)^2 \right]$$

$$= un + \frac{1}{2}an^2 - u(n-1) - \frac{1}{2}a(n-1)^2$$

$$= \cancel{un} + \frac{1}{2}an^2 - \cancel{un} + u - \frac{1}{2}a(n^2 - 2n + 1)$$

$$= \cancel{\frac{1}{2}an^2} + u - \cancel{\frac{1}{2}an^2} - \frac{1}{2}a + \frac{a \times 2n}{2}$$

$$= u - \frac{a}{2} + an$$

$$= u + an - \frac{a}{2}$$

$$S_{nth} = u + \frac{a}{2} [2n - 1]$$

* The equations of motion are applicable only for uniform (constant) acceleration.

* For non-uniform acceleration calculus-based methods are used.

$$\text{e.g. } v = \int a \, dt \quad \text{and} \quad s = \int v \, dt$$

$$\text{also, } v = \frac{dx}{dt} \quad \text{and} \quad a = \frac{dv}{dt} = \frac{d}{dt} \left(\frac{dx}{dt} \right)$$

CHAPTER: MOTION IN A PLANE

Q3 State triangle law of vector addition and derive a formula for magnitude of resultant of two vectors.

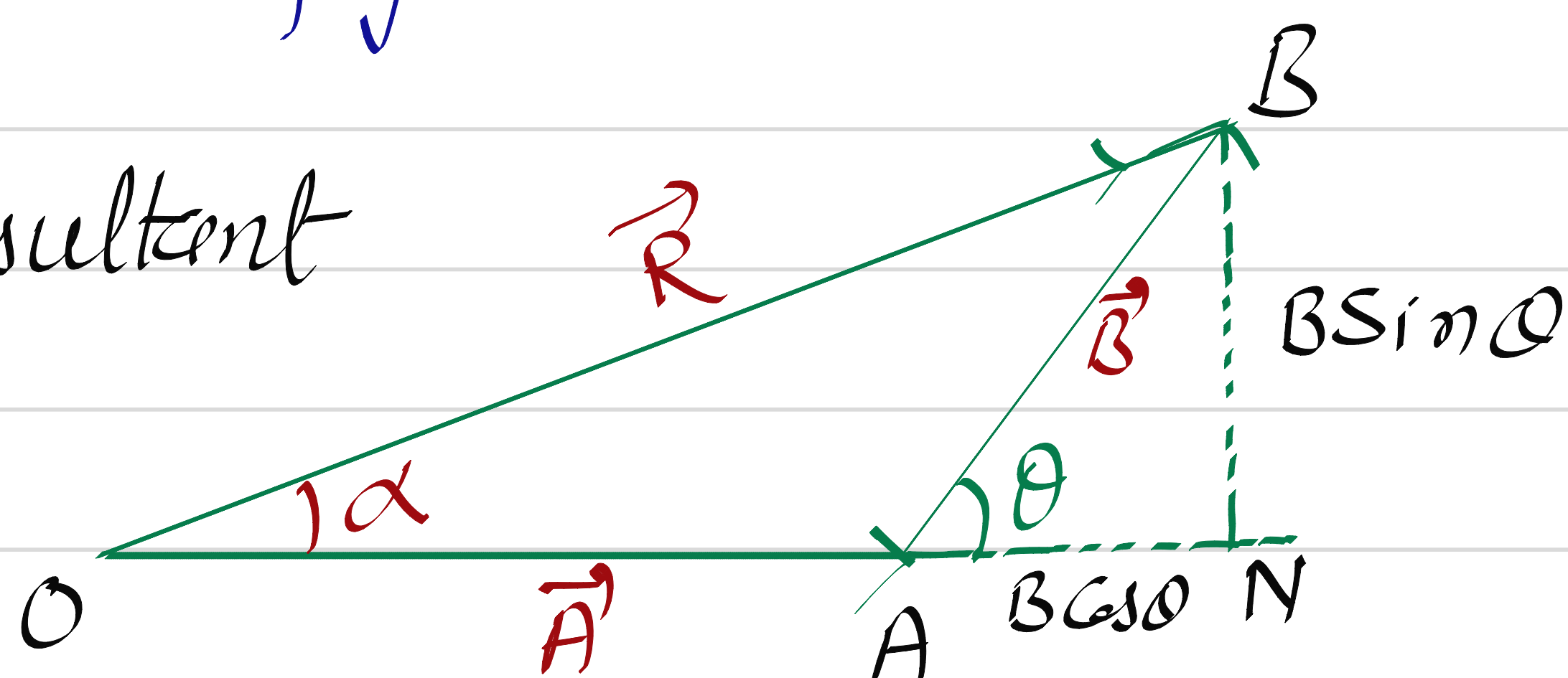
A. Statement of triangle law

If two vectors are represented by two sides of a triangle, taken in an order, then the resultant vector is represented by the third side of the triangle in the opposite direction.

Derivation for magnitude and direction of resultant vector:

Consider two vectors $\vec{A}$ and $\vec{B}$ inclined at an angle θ as shown in fig.

$\vec{R}$ is the resultant vector



$$\vec{OA} = \vec{A}$$

$$\vec{AB} = \vec{B}$$

$$\vec{OB} = \vec{R}$$

$$BN = B \sin \theta$$

$$AN = B \cos \theta$$

Extend OA and draw BN such that $BN \perp AN$

In $\triangle ANB$ (Right-angled triangle)

$$\sin \theta = \frac{BN}{AB} = \frac{BN}{B} \Rightarrow BN = B \sin \theta$$

$$\text{and } \cos \theta = \frac{AN}{AB} = \frac{AN}{B} \Rightarrow AN = B \cos \theta$$

Magnitude of $\vec{R}$

Now in $\triangle ONB$ (Right-angled triangle)

$$OB^2 = ON^2 + BN^2$$

$$R^2 = (OA + AN)^2 + BN^2$$

$$= (A + B \cos \theta)^2 + (B \sin \theta)^2$$

$$= A^2 + B^2 \cos^2 \theta + 2AB \cos \theta + B^2 \sin^2 \theta$$

$$= A^2 + B^2 (\cos^2 \theta + \sin^2 \theta) + 2AB \cos \theta$$

$$R^2 = A^2 + B^2 + 2AB \cos \theta \quad [\because \cos^2 \theta + \sin^2 \theta = 1]$$

OR $R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$

This is the relation for magnitude of resultant vector $\vec{R}$.

Direction of $\vec{R}$ with $\vec{A}$

In $\triangle ONB$ (Right-angled triangle)

$$\tan \alpha = \frac{BN}{ON} = \frac{BN}{OA + AN} \quad [\alpha \text{ is angle b/w } \vec{A} \text{ \& } \vec{R}]$$

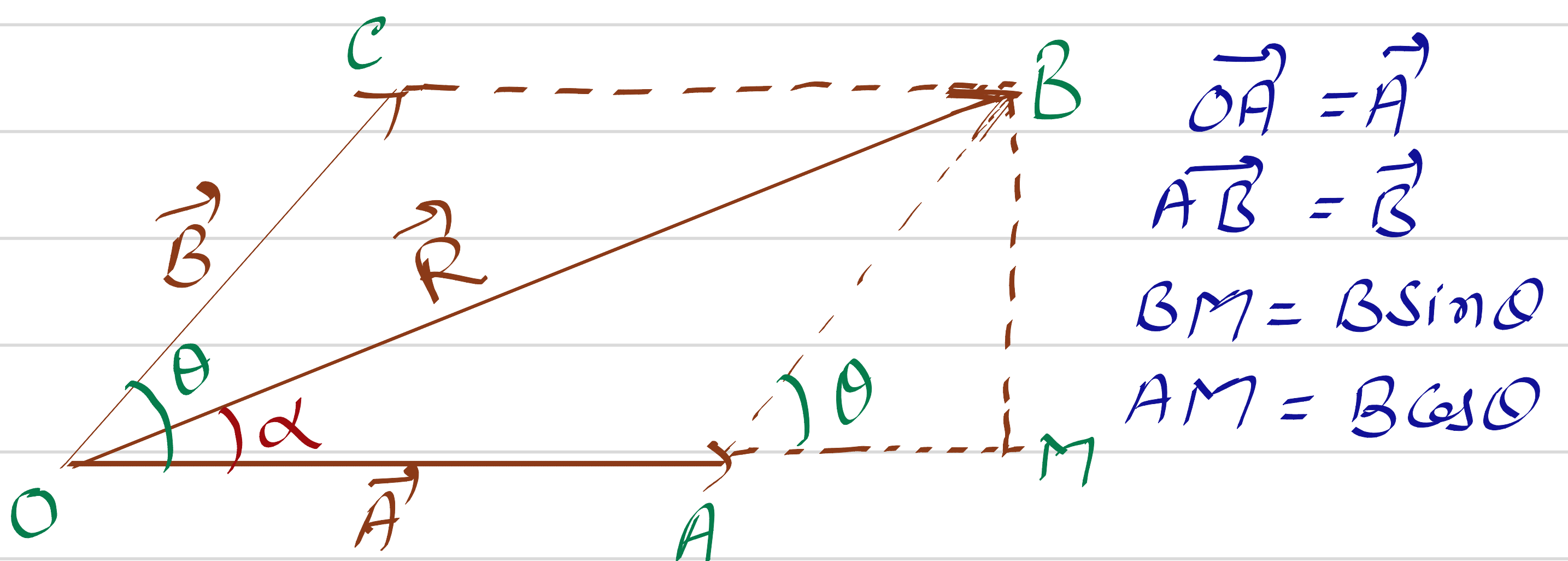
OR $\tan \alpha = \frac{B \sin \theta}{A + B \cos \theta} \quad \left[\begin{array}{l} BN = B \sin \theta \\ AN = B \cos \theta \end{array} \right]$

OR $\alpha = \tan^{-1} \left(\frac{B \sin \theta}{A + B \cos \theta} \right)$

$\vec{R}$ makes an angle α with $\vec{A}$.

Q.4. State parallelogram law of vector addition and derive a formula for magnitude and direction for resultant vector.

A. "According to parallelogram law of vector addition if two vectors are taken as the adjacent sides of a parallelogram, then the resultant of two vectors is given by the vector that is diagonal passing through the point of contact of the two vectors."



consider two vectors $\vec{A}$ and $\vec{B}$ inclined at an angle θ as shown in fig. Let $\vec{R}$ is the resultant of $\vec{A} + \vec{B}$.

* Derivation for magnitude and direction of $\vec{R}$ is same as for triangle law.

Magnitude of $\vec{R}$

In $\triangle OBM$

$$OB^2 = OM^2 + BM^2$$

$$R^2 = (OA + AM)^2 + BM^2$$

$$= (A + B \cos \theta)^2 + (B \sin \theta)^2$$

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$$= A^2 + B^2 \cos^2 \theta + 2AB \cos \theta + B^2 \sin^2 \theta$$

$$= A^2 + B^2 (\cos^2 \theta + \sin^2 \theta) + 2AB \cos \theta$$

$$R^2 = A^2 + B^2 + 2AB \cos \theta \quad [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

Direction of $\vec{R}$

If $\vec{R}$ makes an angle α with $\vec{A}$, then

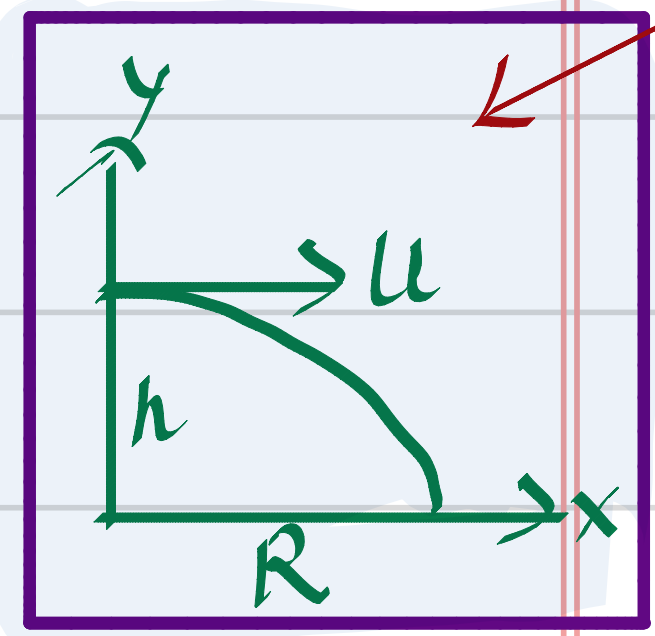
$$\tan \alpha = \frac{BM}{OM} = \frac{BM}{OA + AM} = \frac{B \sin \theta}{A + B \cos \theta}$$

$$\text{or } \alpha = \tan^{-1} \left(\frac{B \sin \theta}{A + B \cos \theta} \right)$$

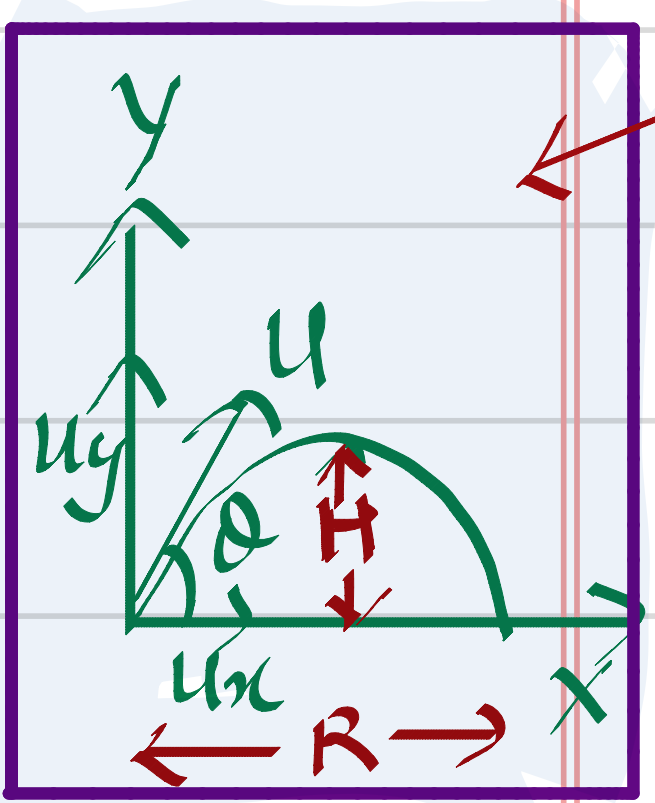
Q.5. What is projectile motion? Differentiate between horizontal projectile and oblique projectile.

A. Motion of an object thrown into the air moving along a curved path due to gravity is called projectile motion.

Projectile motion $\begin{cases} \rightarrow \text{Horizontal Projectile} \\ \rightarrow \text{Oblique or angular Projectile} \end{cases}$



Horizontal Projectile: Object is projected with an initial velocity along the horizontal axis only.
e.g. bullet fired from a gun, Airplane dropping a package.



Oblique or Angular Projectile: Object is projected with an initial velocity at an angle to the horizontal.
e.g. Football kick, rocket launch, throwing a ball etc.

- Q.6 A body is thrown horizontally from a height h .
- (i) Show that its path is parabolic (trajectory)
 - (ii) Derive expression for time of flight and horizontal range.

(iii) Find its velocity when it hits the ground.

A. Consider a body is projected with initial velocity ' u ', along the x -axis from height ' h ' as shown in fig-

Path of Projectile:

Motion along x -axis

$$x = u_x t + \frac{1}{2} a_x t^2$$

$$u_x = u, \quad a_x = 0$$

$$x = u t + 0$$

$$t = \frac{x}{u} \quad \text{--- (1)}$$

Motion along y -axis

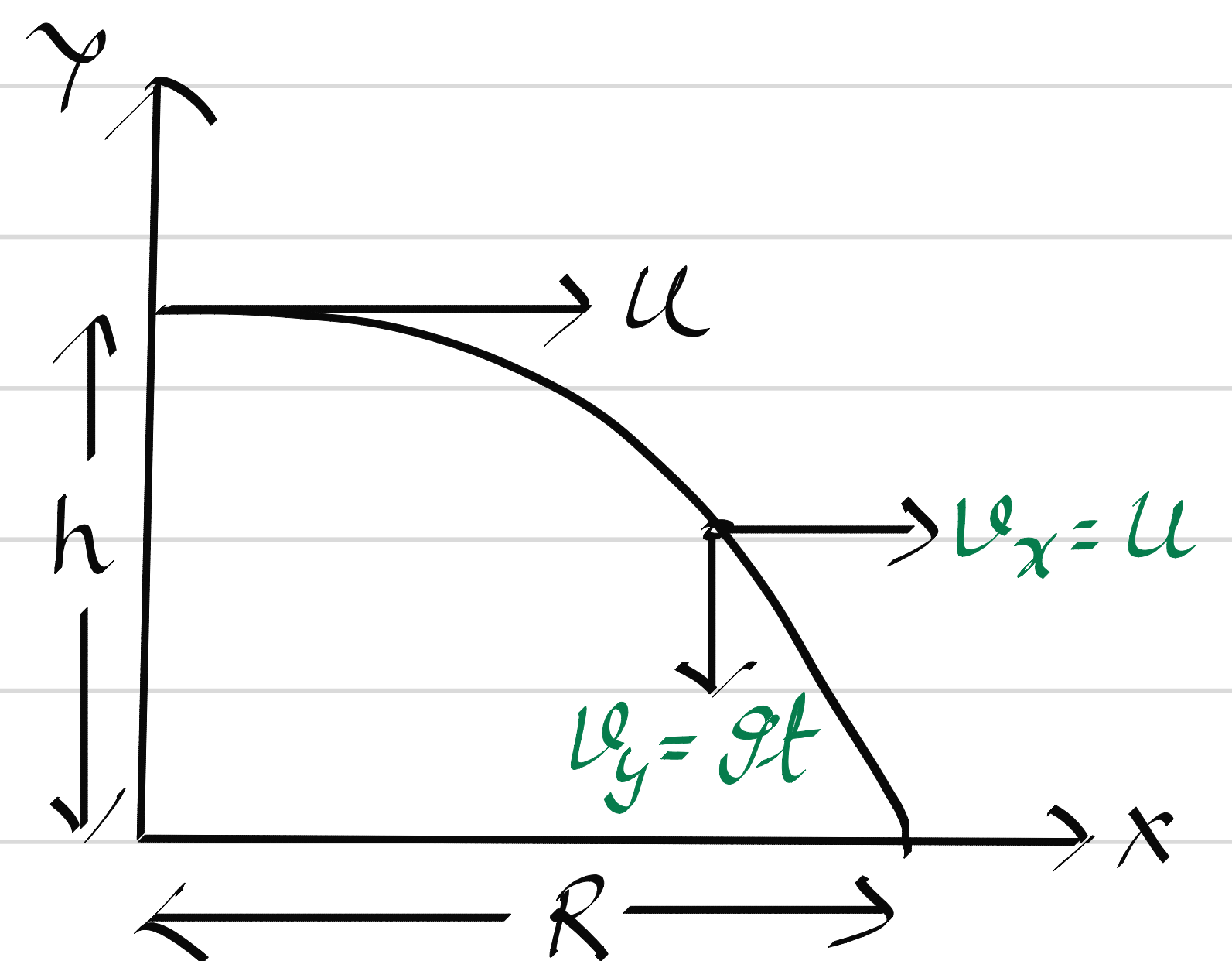
$$y = u_y t + \frac{1}{2} a_y t^2$$

$$y = h, \quad u_y = 0, \quad a_y = g$$

$$y = \frac{1}{2} g \left(\frac{x}{u} \right)^2$$

$$[\text{from eq}^n (1) \quad t = \frac{x}{u}]$$

$$y = \frac{g}{2u^2} x^2$$



$$u_x = u, \quad a_x = 0$$

$$u_y = 0, \quad a_y = g$$

Which is a quadratic equation of parabola. Thus path of projectile is parabolic.

Time of flight (T)

Total time for which the projectile remains in air is called time of flight.

$$y = u_y t + \frac{1}{2} a_y t^2$$

here $y = h$, $u_y = 0$, $a_y = g$, $t = T$

$$h = 0 + \frac{1}{2} g T^2$$

$$\text{or } T^2 = \frac{2h}{g}$$

$$\text{or } \boxed{T = \sqrt{\frac{2h}{g}}}$$

Horizontal Range (R)

Maximum horizontal distance travelled by the projectile is called its horizontal range.

$$x = u_x t + \frac{1}{2} a_x t^2$$

here $x = R$, $u_x = u$, $a_x = 0$, then

$$R = uT + 0$$

$$\text{or } \boxed{R = u \sqrt{\frac{2h}{g}}} \quad \left[\because T = \sqrt{\frac{2h}{g}} \right]$$

Velocity at any instant

Along x-axis

$$v_x = u_x + a_x t$$

here $u_x = u$, $a_x = 0$, then

$$v_x = u \quad \text{--- (1)}$$

Along y-axis

$$v_y = u_y + a_y t$$

Here $u_y = 0$, $a_y = g$, then

$$v_y = gt \quad \text{---(1)}$$

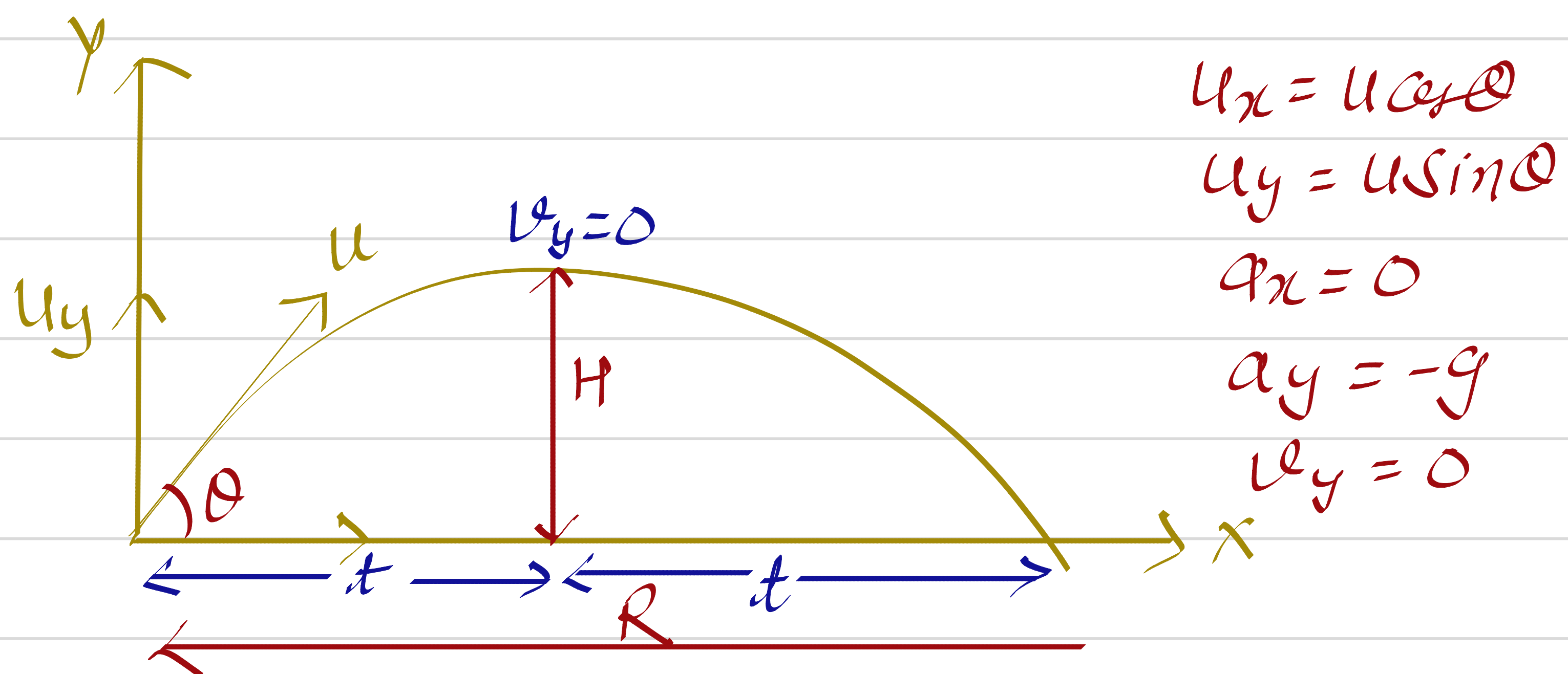
$$\text{as } v = \sqrt{v_x^2 + v_y^2}$$

$$v = \sqrt{u^2 + (gt)^2} \quad [\text{from (1) and (11)}]$$

$$\text{or } \boxed{v = \sqrt{u^2 + g^2 t^2}}$$

Q.7 A projectile is launched at an angle θ with an initial velocity u . Show that it follows a parabolic path. Derive expressions for (i) Time of flight (ii) Maximum height (iii) Horizontal range

A Consider an object projected with velocity u at an angle θ with horizontal as shown in fig.



Path of Projectile

Along x-axis

$$x = u_x t + \frac{1}{2} a_x t^2$$

here $u_x = u \cos \theta$, $a_x = 0$, then

$$x = u \cos \theta t + 0$$

$$\text{or } t = \frac{x}{u \cos \theta} \quad \text{---(1)}$$

Along y-axis

$$y = u_y t + \frac{1}{2} a_y t^2$$

here $u_y = u \sin \theta$, $a_y = g$ and $t = \frac{x}{u \cos \theta}$, then

$$y = u \sin \theta \cdot \frac{x}{u \cos \theta} + \frac{1}{2} (-g) \left(\frac{x}{u \cos \theta} \right)^2$$

or $y = x \tan \theta - \frac{g}{2u^2 \cos^2 \theta} x^2$

This is quadratic eqⁿ which show the path of projectile is parabolic.

Time of flight (T)

Total time for which the projectile remains in air is called time of flight.

$$v_y = u_y + a_y t$$

here $v_y = 0$, $u_y = u \sin \theta$, $a_y = -g$, then

$$0 = u \sin \theta - g t$$

or $t = \frac{u \sin \theta}{g}$

$$T = 2t = \frac{2u \sin \theta}{g}$$

i.e Time of flight

$$T = \frac{2u \sin \theta}{g}$$

Maximum height (H)

$$v_y^2 = u_y^2 + 2a_y y$$

here $v_y = 0$, $u_y = u \sin \theta$, $a_y = -g$, $y = H$

$$0^2 = (u \sin \theta)^2 - 2gH$$

or $2gH = u^2 \sin^2 \theta$

$$\text{or } H = \frac{u^2 \sin^2 \theta}{2g}$$

* For $H_{\max}$, $\sin^2 \theta = 1 \Rightarrow \sin \theta = 1 = \sin 90^\circ$
i.e. $\theta = 90^\circ$

$$\text{i.e. } H_{\max} = \frac{u^2}{2g}$$

Horizontal Range (R)

Maximum horizontal distance travelled by the projectile.

$$x = u_x t + \frac{1}{2} a_x t^2$$

here $x = R$, $u_x = u \cos \theta$, $a_x = 0$, $t = T$, then

$$R = u_x T$$

$$= u \cos \theta \cdot \frac{2u \sin \theta}{g} \quad \left[T = \frac{2u \sin \theta}{g} \right]$$

$$\text{or } R = \frac{u^2 (2 \sin \theta \cos \theta)}{g}$$

$$\text{or } R = \frac{u^2 \sin 2\theta}{g} \quad [\because 2 \sin \theta \cos \theta = \sin 2\theta]$$

For $R_{\max}$, $\sin 2\theta = 1 = \sin 90^\circ$

$$\text{or } 2\theta = 90^\circ$$

i.e. $\theta = 45^\circ$ for maximum range.

and

$$R_{\max} = \frac{u^2}{g}$$

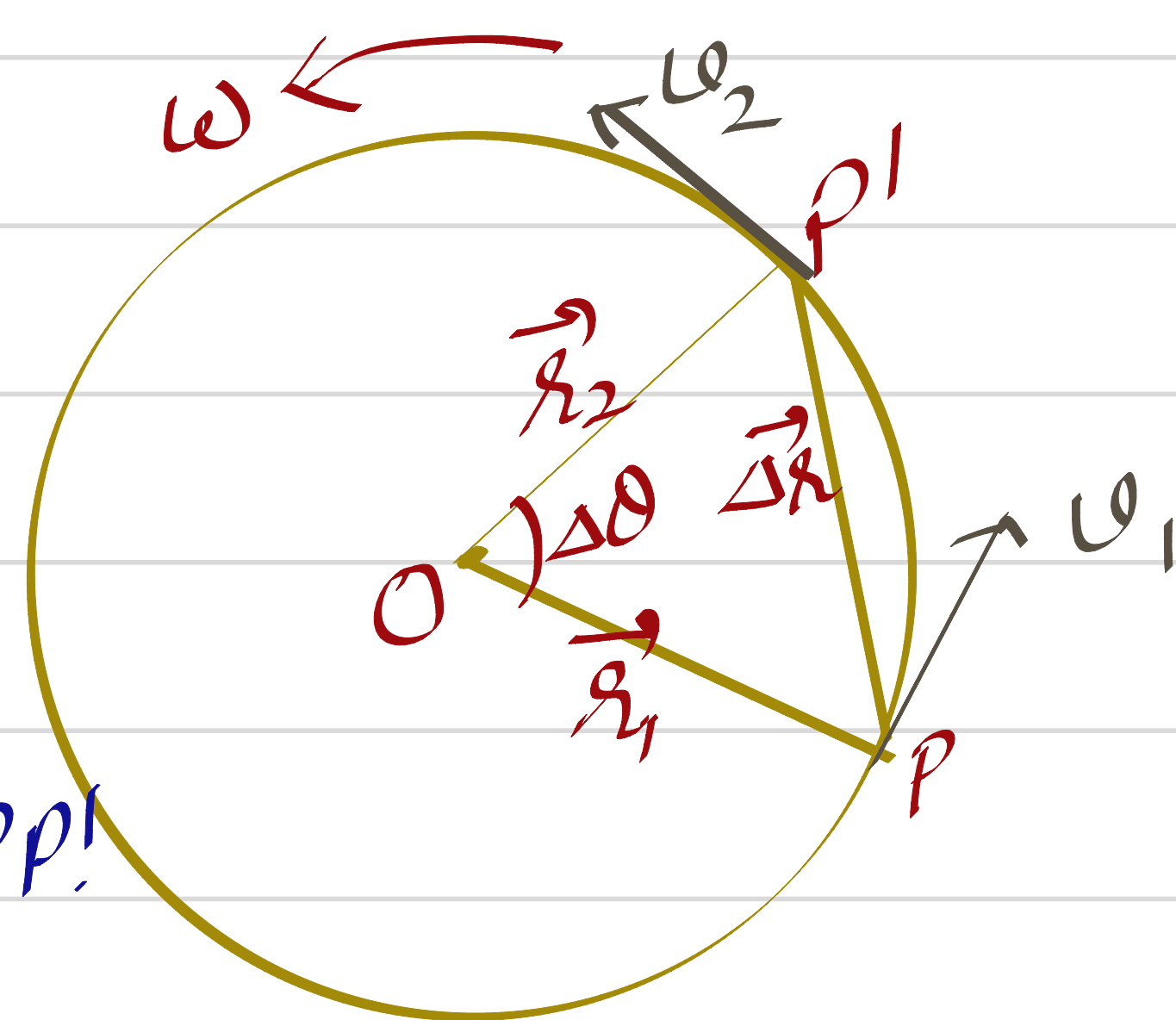
Q.8 A particle is moving on uniform circular path of radius r . Derive following relations -
 (a) $v = r\omega$ (b) $a = \frac{v^2}{r}$ (c) $F = \frac{mv^2}{r}$

here ω is angular velocity, a is linear acceleration and v is linear velocity of the particle.

A. Uniform circular motion (UCM): Motion of an object along a circular path with a constant speed is called circular motion.

consider a particle is moving in uniform circular motion as shown in fig.

Let in Δt time its angular displacement is $\Delta\theta$ for arc PP' .



(a) $v = r\omega$

We know $\text{angle} = \frac{\text{arc}}{\text{radius}}$

$$|\vec{r}_1| = |\vec{r}_2| = r$$

$$|\vec{v}_1| = |\vec{v}_2| = v$$

$$\Delta\theta = \frac{PP'}{r} = \frac{\Delta s}{r} \quad [PP' = \text{linear displacement}]$$

divide by Δt on both sides, then

$$\frac{\Delta\theta}{\Delta t} = \frac{\Delta s}{\Delta t} \cdot \frac{1}{r}$$

$$\omega = \frac{v}{r} \quad \left[\because \frac{\Delta\theta}{\Delta t} = \omega \text{ and } \frac{\Delta s}{\Delta t} = v \right]$$

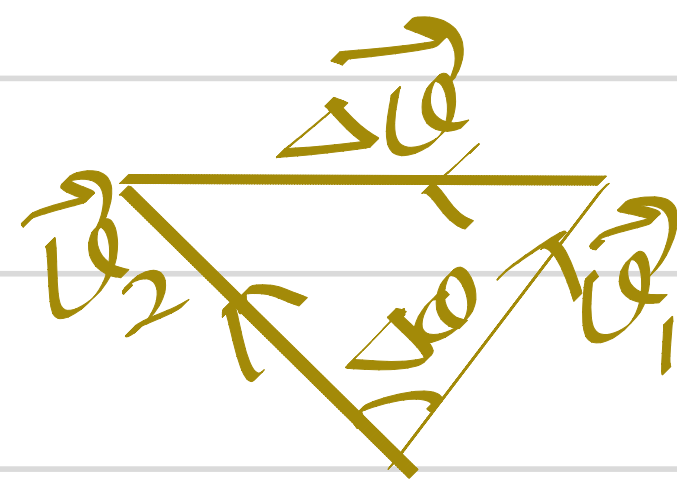
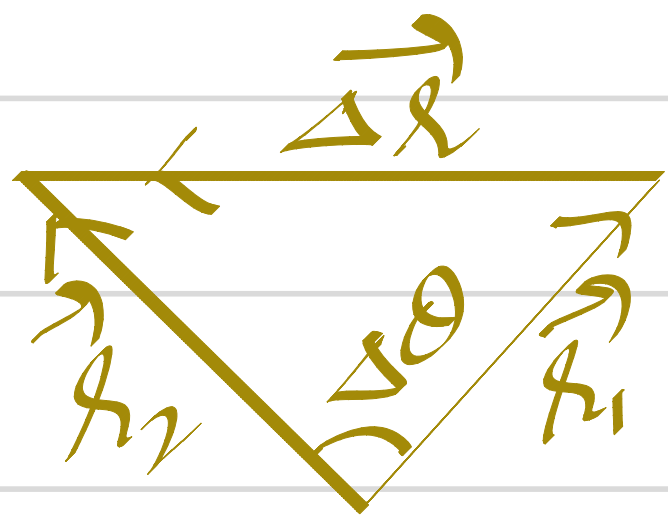
or $\boxed{v = r\omega}$

Vector form

$$\boxed{\vec{v} = \vec{\omega} \times \vec{r}}$$

$$(b) \quad a = \frac{v^2}{r}$$

From fig. we can take similar triangles for radius vectors and velocity vectors



In similar Δ 's

$$\frac{\Delta r}{r} = \frac{\Delta v}{v}$$

$$|\vec{r}_1| = |\vec{r}_2| = r$$

$$|\vec{v}_1| = |\vec{v}_2| = v$$

divide both sides by Δt

$$\frac{1}{r} \left(\frac{\Delta r}{\Delta t} \right) = \left(\frac{\Delta v}{\Delta t} \right) \frac{1}{v}$$

$$\text{or} \quad \frac{v}{r} = \frac{a}{v} \quad \left[\frac{\Delta r}{\Delta t} = v, \quad \frac{\Delta v}{\Delta t} = a \right]$$

$$\text{or} \quad \boxed{a = \frac{v^2}{r}}$$

$a \rightarrow$ centripetal acceleration

$$\rightarrow \text{but } v = r\omega, \quad a = \frac{(r\omega)^2}{r} \Rightarrow \boxed{a = r\omega^2}$$

$$(c) \quad F = \frac{mv^2}{r}$$

$F \rightarrow$ centripetal force
(necessary force for circular motion)

from part (b)

$$a = \frac{v^2}{r}$$

By newton's II law

$$F = ma$$

$$\text{or} \quad F = m \left(\frac{v^2}{r} \right)$$

$$\text{or} \quad \boxed{F = \frac{mv^2}{r}}$$

$F \rightarrow$ centripetal force.

* on putting $v = r\omega$

$$F = m (r\omega)^2$$

$$F = m r \omega^2$$

CHAPTER- LAWS OF MOTION AND FRICTION

Q.9 Show that newton's 2nd law is real law.

A. Newton's 2nd law is called the real law because it contains newton's 1st law and 3rd law.

Derivation of newton's 1st law from 2nd law

We know the 2nd law

$$F = ma$$

If no net force acts on an object, then
 $F = 0$

therefore $ma = 0$

$$\text{or } a = 0 \quad [m \neq 0]$$

i.e. velocity of the object remains constant. This means the object either stay in rest or moves with constant velocity. This is the 1st law of newton.

So newton's 1st law is a special case of 2nd law where $F = 0$.

Derivation of newton's 3rd law from 2nd law:

Consider two objects A and B interacting with each other in an isolated system.

Let F_{AB} = force on object A by object B

and F_{BA} = force on object B by object A

Then from Newton's 2nd law

$$\vec{F}_{AB} = \frac{d\vec{p}_A}{dt} \quad [\text{rate of change of momentum of object A}]$$

and $\vec{F}_{BA} = \frac{d\vec{p}_B}{dt}$ [rate of change of momentum of object B]

Now

$$\begin{aligned}\vec{F}_{AB} + \vec{F}_{BA} &= \frac{d\vec{p}_A}{dt} + \frac{d\vec{p}_B}{dt} \\ &= \frac{d(\vec{p}_A + \vec{p}_B)}{dt}\end{aligned}$$

As there is no external force, so no change in momentum, thus

$$\frac{d(\vec{p}_A + \vec{p}_B)}{dt} = 0$$

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Therefore $\vec{F}_{AB} + \vec{F}_{BA} = 0$

$$\boxed{\vec{F}_{AB} = -\vec{F}_{BA}} \rightarrow \text{Newton's 3rd law}$$

Hence Newton's 2nd law contains the 3rd law.

Q. 10 Explain the law of conservation of linear momentum.

A. (a) Law of conservation of linear momentum:

The total linear momentum of a system remains constant if the net external force acting on the system is zero.

According to 2nd law of motion

$$\vec{F} = \frac{d\vec{p}}{dt}$$

If $F = 0$, then $\frac{d\vec{p}}{dt} = 0$

or $\vec{p} = \text{constant}$
 or $m\vec{v} = \text{constant}$
 or initial momentum = final momentum
 $m_1\vec{u}_1 = m_2\vec{u}_2$

for n system of particles

$$\vec{F} = \frac{d}{dt} (\vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \dots + \vec{p}_n)$$

If $\vec{F} = 0$

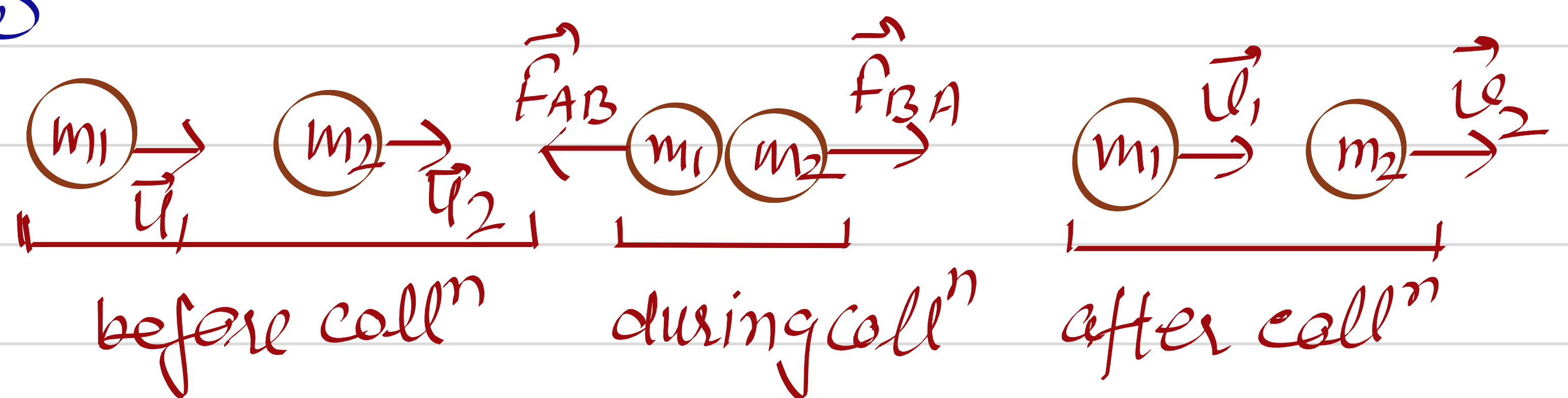
$$p_1 + p_2 + p_3 + \dots + p_n = \text{constant}$$

$$p_{\text{net}} = \text{constant}$$

This is the principle of conservation of linear momentum.

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(b) Conservation of linear momentum by Newton's 3rd law



$$\text{Total momentum before collision} = m_1\vec{u}_1 + m_2\vec{u}_2$$

$$\text{Total momentum after collision} = m_1\vec{v}_1 + m_2\vec{v}_2$$

Change in momentum of body A = $\vec{F}_{AB} \Delta t$
 from 2nd law

$$\vec{F}_{AB} \Delta t = m_1\vec{v}_1 - m_1\vec{u}_1 \quad [p_f - p_i]$$

similarly

$$\vec{F}_{BA} \Delta t = m_2\vec{v}_2 - m_2\vec{u}_2$$

From Newton's 3rd law

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

$$\text{or } \vec{F}_{AB} \Delta t = -\vec{F}_{BA} \Delta t$$

so

$$m_1 \vec{u}_1 - m_1 \vec{u}_1 = -(m_2 \vec{u}_2 - m_2 \vec{u}_2)$$

$$\text{or } m_1 \vec{u}_1 + m_2 \vec{u}_2 = m_1 \vec{u}_1 + m_2 \vec{u}_2$$

momentum after collⁿ = momentum before collⁿ

This is the law of conservation of momentum.

e.g. Speed of recoiling of a Gun

$$\text{by } m\vec{u} = M\vec{V}$$

$$\boxed{V = \frac{m\vec{u}}{M}}$$

$m \rightarrow$ mass of bullet

$M \rightarrow$ mass of gun

$u \rightarrow$ speed of bullet

$V \rightarrow$ speed of gun

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Q. 11 Find expression for maximum velocity of a car on a level or flat road to take safe turn.

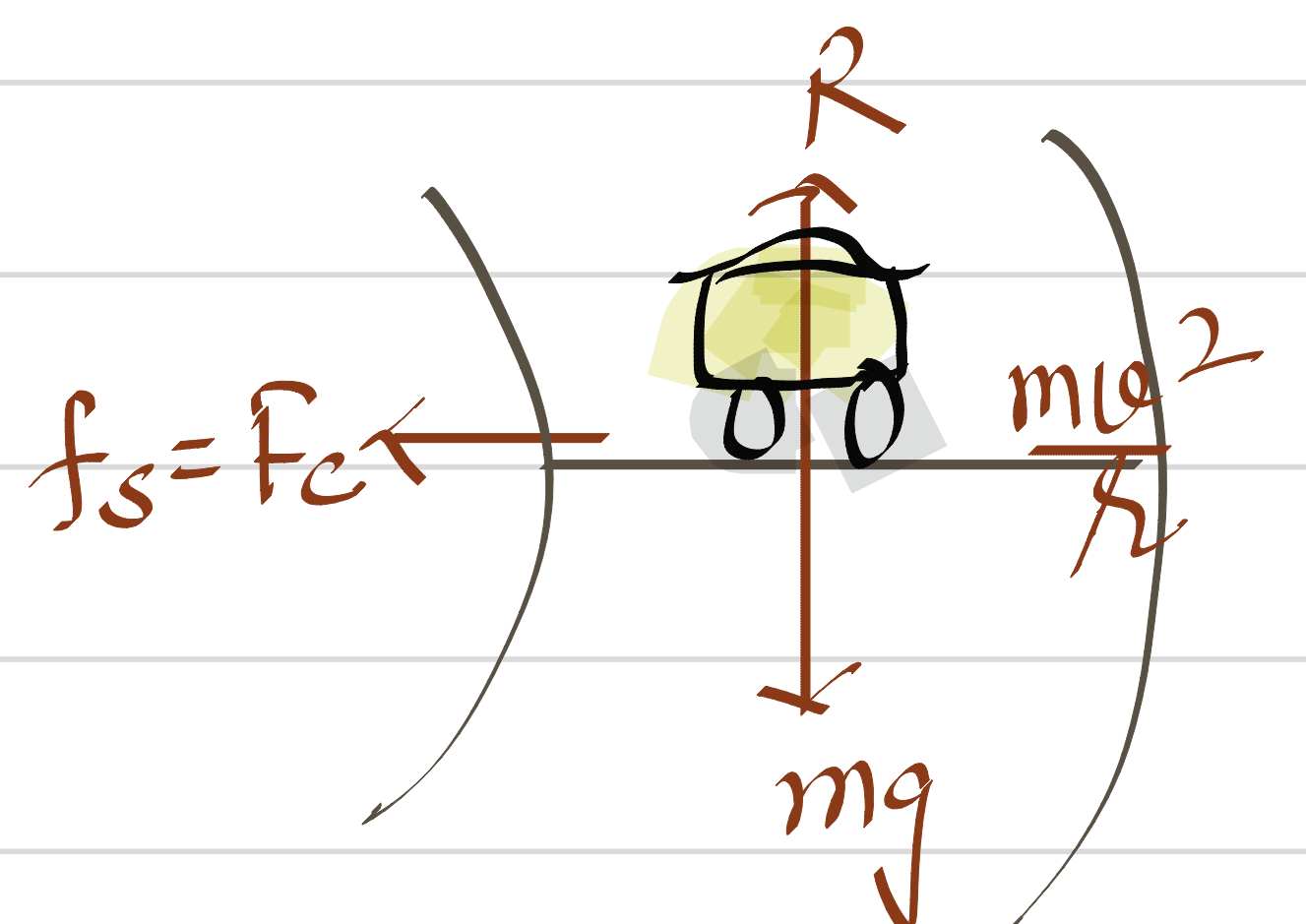
Car on a level circular road

Consider a car of mass m moving with a constant speed u on a circular road of radius R . Various forces acting on the car are -

- (i) Weight of car mg (downwards)
- (ii) Normal reaction R (upwards)
- (iii) Friction b/w road and tyres of car

In vertical direction

$$R = mg \quad \text{--- (1)}$$



$F_c \rightarrow$ centripetal force

$f_s \rightarrow$ friction force

In horizontal direction
the centripetal force is provided by friction force

$$F_c = f_s = \frac{mv^2}{r}$$

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here $f_s = \mu R = \mu mg$, then

$$\mu mg = \frac{mv^2}{r}$$

$$v^2 = \mu r g$$

$$v = \sqrt{\mu r g}$$

This is the maximum speed of car in circular motion
to take safe turn - $v_{\max} = \sqrt{\mu r g}$

* $v_{\max}$ is independent of mass of car.

Q.12 What is banking of roads? Obtain an expression
for the maximum safety speed of a vehicle moving
along a curved horizontal road.

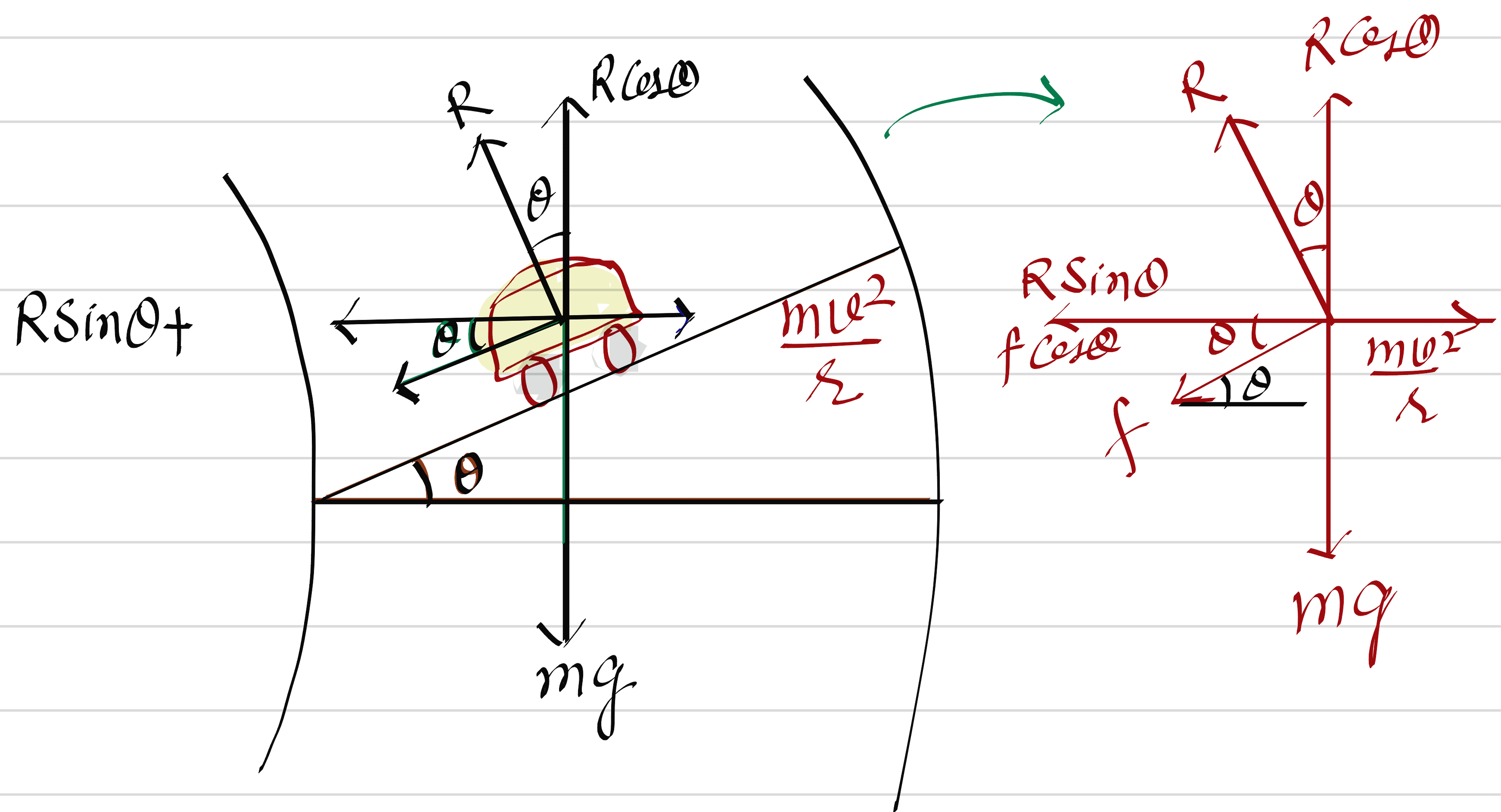
A. Banking of roads: To avoid risk of skidding
and to reduce wear and tear of the tyres,
the outer edge of the circular road is raised
a little above the inner edge. This is called
banking of a road.

Motion of a car on a banked circular road
(taking friction)

The various forces involve in this motion
are -

- (i) Weight of a car (downwards)
- (ii) Normal reaction R (upward $\perp$ to the surface)

(iii) Force of friction b/w road and tyres of car.



From fig
For vertical direction

$$R \cos \theta = mg + f \sin \theta$$

$$\text{or } mg = R \cos \theta - f \sin \theta \quad (1)$$

and for horizontal direction

$$R \sin \theta + f \cos \theta = \frac{mv^2}{r}$$

$$\text{or } \frac{mv^2}{r} = R \sin \theta + f \cos \theta \quad (2)$$

Divide eqn (2) by eqn (1)

$$\frac{mv^2/r}{mg} = \frac{R \sin \theta + f \cos \theta}{R \cos \theta - f \sin \theta}$$

$$\text{or } \frac{v^2}{rg} = \frac{R \sin \theta + \mu R \cos \theta}{R \cos \theta - \mu R \sin \theta}$$

$$\frac{v^2}{rg} = \frac{\sin \theta + \mu \cos \theta}{\cos \theta - \mu \sin \theta}$$

divide and multiply by $\cos \theta$ on R.H.S., we get.

$$\frac{v^2}{rg} = \frac{\frac{\sin \theta}{\cos \theta} + \mu \frac{\cos \theta}{\cos \theta}}{\frac{\cos \theta}{\cos \theta} - \mu \frac{\sin \theta}{\cos \theta}}$$

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$$\Rightarrow \frac{v^2}{rg} = \frac{\tan \theta + \mu}{1 - \mu \tan \theta}$$

$$\Rightarrow v^2 = rg \left[\frac{\mu + \tan \theta}{1 - \mu \tan \theta} \right]$$

$$\Rightarrow v = \sqrt{rg \left(\frac{\mu + \tan \theta}{1 - \mu \tan \theta} \right)}$$

$$v = \left[rg \left(\frac{\mu + \tan \theta}{1 - \mu \tan \theta} \right) \right]^{1/2}$$

For this speed wear and tear will be minimum.
Case I: If there is no friction, $\mu = 0$, then

$$v = \sqrt{rg \left(\frac{0 + \tan \theta}{1 - 0} \right)}$$

$$\text{or } v = \sqrt{rg \tan \theta} \Rightarrow \theta = \tan^{-1} \frac{v^2}{rg}$$

Case II: If road is horizontal (flat), i.e. $\theta = 0$, then

$$v = \sqrt{\mu rg}$$

* Safe velocity on banked road

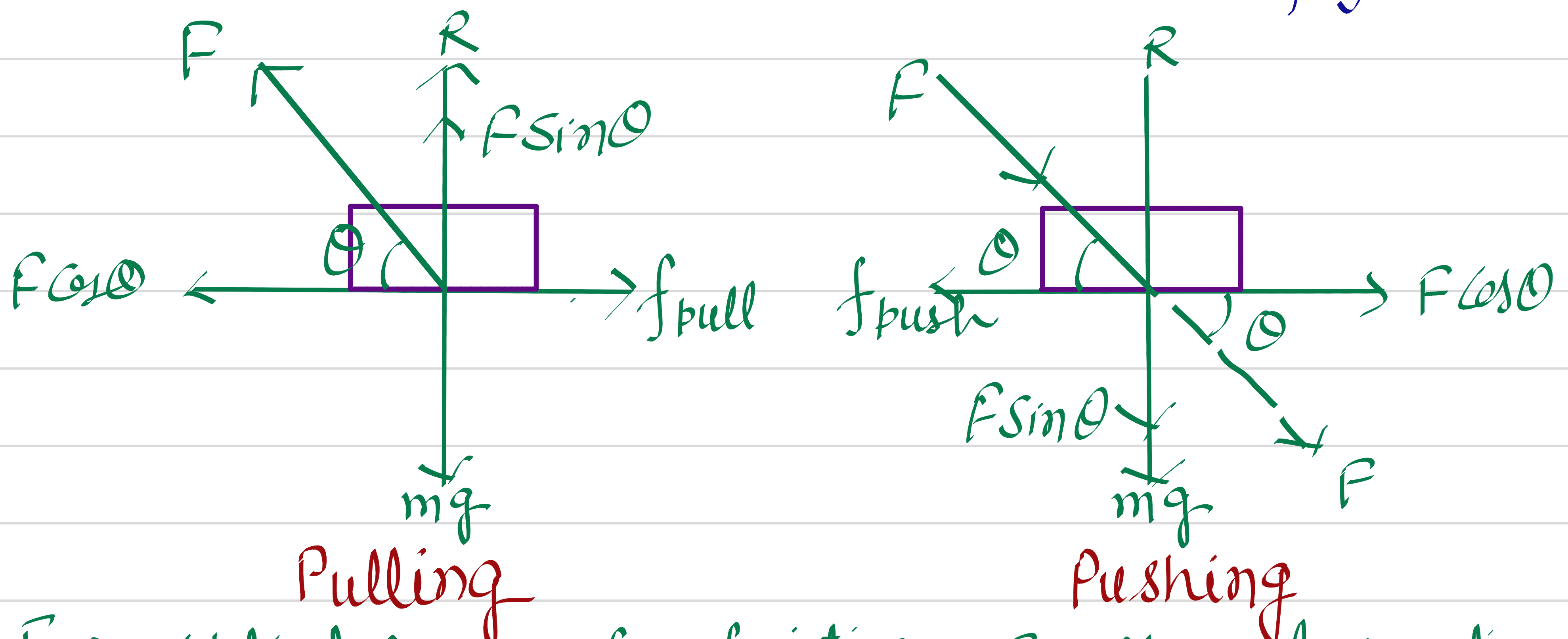
$$v_{\max} = \sqrt{rg \tan \theta}$$

speed will be maximum when
 $\tan \theta = 1 \Rightarrow \theta = 45^\circ$

Q.12 Show that pulling is easier than pushing.
OR

Pulling a lawn roller is easier than pushing it.
Why?

A. Consider both the cases as shown in fig



$F \rightarrow$ applied force, $f \rightarrow$ friction, $R \rightarrow$ Normal reaction

For vertical direction

In pulling

$$R + F \sin \theta = mg$$

or

$$R = mg - F \sin \theta$$

$$f_{\text{pull}} = \mu R$$

$$= \mu (mg - F \sin \theta)$$

In pushing

$$R = mg + F \sin \theta$$

$$f_{\text{push}} = \mu R$$

$$f_{\text{push}} = \mu (mg + F \sin \theta)$$

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It is clear that $f_{\text{pull}} < f_{\text{push}}$
i.e. Pulling is easier than pushing.

CHAPTER: WORK, ENERGY AND POWER

Q. Prove work energy theorem for constant force.

A. Work-Energy theorem - The work done by a force on a body is equal to the change in kinetic energy of the body. i.e.
change in K.E = Work done

Proof: For rectilinear motion

$$v^2 - u^2 = 2as \quad [\text{III eq}^n \text{ of motion}]$$

here $u \rightarrow$ initial speed, $v \rightarrow$ final speed
 $a \rightarrow$ acceleration, $s \rightarrow$ distance.

Multiply both sides by $\frac{1}{2}m$, we get

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}m(2as)$$

$$\text{Final K.E} - \text{Initial K.E} = mas$$

$$K_f - K_i = FS \quad [\because F = ma]$$

or $\Delta K = W \quad [FS = W]$

"Change in kinetic energy is equal to the work done." This is the WE theorem.

Q. Prove work energy theorem for variable force.

A. For variable force we use calculus method.

The time rate of change of kinetic energy is

$$\frac{dK}{dt} = \frac{d}{dt} \left(\frac{1}{2}mv^2 \right)$$

$$= \frac{1}{2}m(2v) \cdot \frac{dv}{dt}$$

$$= m \frac{dv}{dt} v$$

$$= (ma)v$$

$$\text{or } \frac{dK}{dt} = F v$$

$$\frac{dK}{dt} = F \frac{dx}{dt} \quad [v = \frac{dx}{dt}]$$

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$$\text{or } dK = F dx$$

on integrating, we get

$$\int_{K_i}^{K_f} dK = \int_{x_i}^{x_f} F dx$$

$$[K]_{K_i}^{K_f} = W \quad \left[\int F dx = W \right]$$

$$\text{or } K_f - K_i = W$$

$$\text{or } \Delta K = W$$

This is work energy theorem for variable force.

Q. Show that mechanical energy of a body during the free fall under the action of gravity, remain conserved.

A. Consider a body of mass 'm' falling freely from point 'A' at height 'h' from point 'B' at the ground as shown in fig. Let the body covers the distance x from point 'A' to reach at point 'P'.

At point 'A' (At highest point)

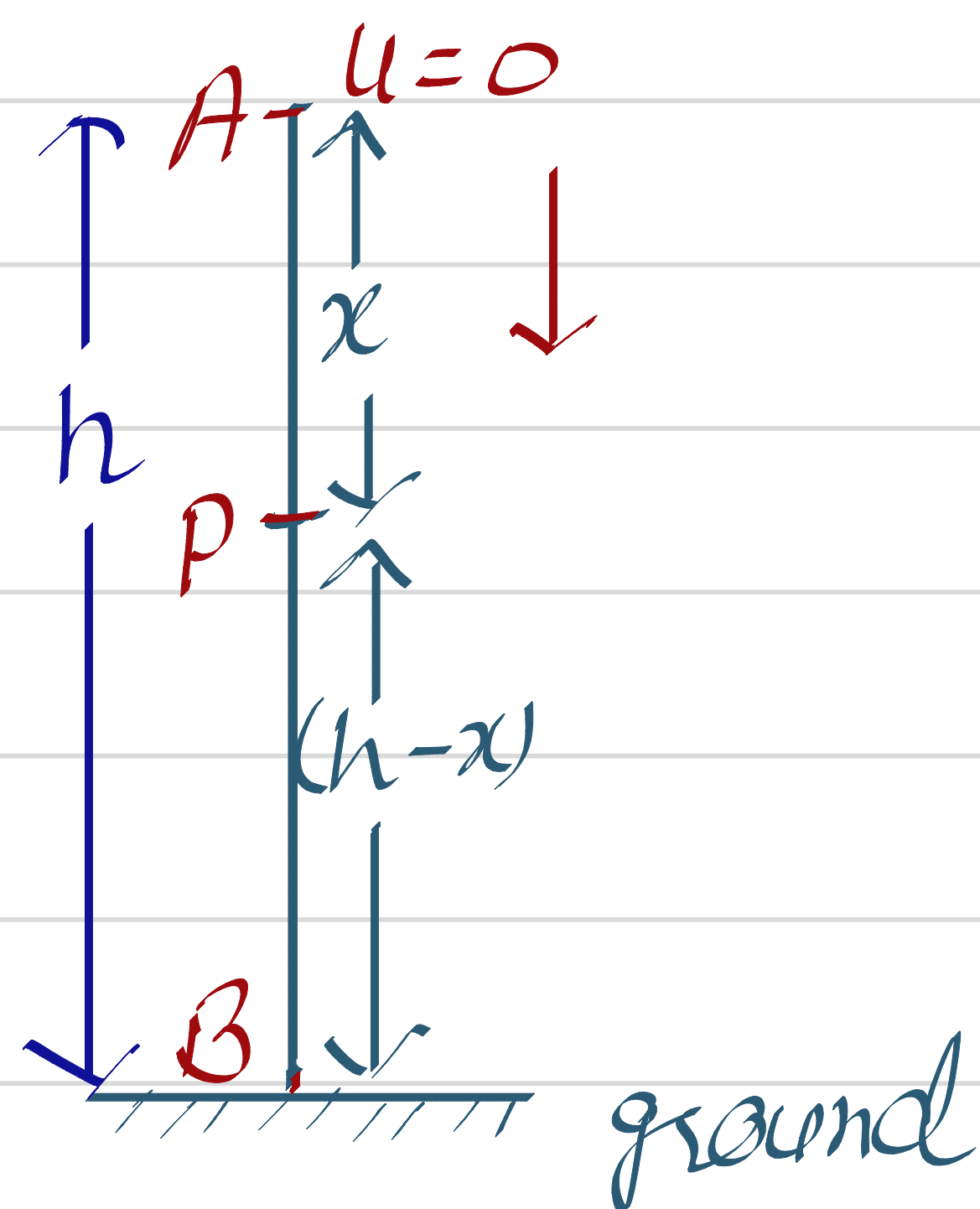
$$K.E = \frac{1}{2} m (0)^2 \quad [v=0 \text{ at 'A'}]$$

$$K.E = 0$$

$$P.E = mgh$$

Total energy i.e mechanical energy at point 'A'

$$(M.E)_A = K.E + P.E$$



$$\text{or } (M.E.)_A = 0 + mgh$$

$$(M.E.)_A = mgh \quad - (1)$$

Now at point 'P' (Any point on AB)

$$\text{by } v^2 = u^2 + 2gh$$

$$\text{here } u = 0, \quad h = x$$

$$\text{so, } v^2 = 2gx$$

$$\text{Now } K.E. = \frac{1}{2} m (2gx)$$

$$K.E. = mgx$$

$$\text{and } P.E. = mg(h-x)$$

hence

$$\begin{aligned} (M.E.)_P &= K.E. + P.E \\ &= mgx + mg(h-x) \\ &= mgx + mgh - mgx \end{aligned}$$

$$(M.E.)_P = mgh \quad - (2)$$

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At point 'B' (on ground)

$$\text{by } v^2 = u^2 + 2gh$$

$$u = 0, \quad h = h, \text{ then}$$

$$v^2 = 2gh$$

$$K.E. = \frac{1}{2} m (2gh)$$

$$K.E. = mgh$$

$$\text{And } P.E. = mg(0) \quad [\text{at ground } h=0]$$

$$P.E. = 0$$

$$\begin{aligned} \text{So } (M.E.)_B &= K.E. + P.E \\ &= mgh + 0 \end{aligned}$$

$$(M.E.)_B = mgh \quad - (3)$$

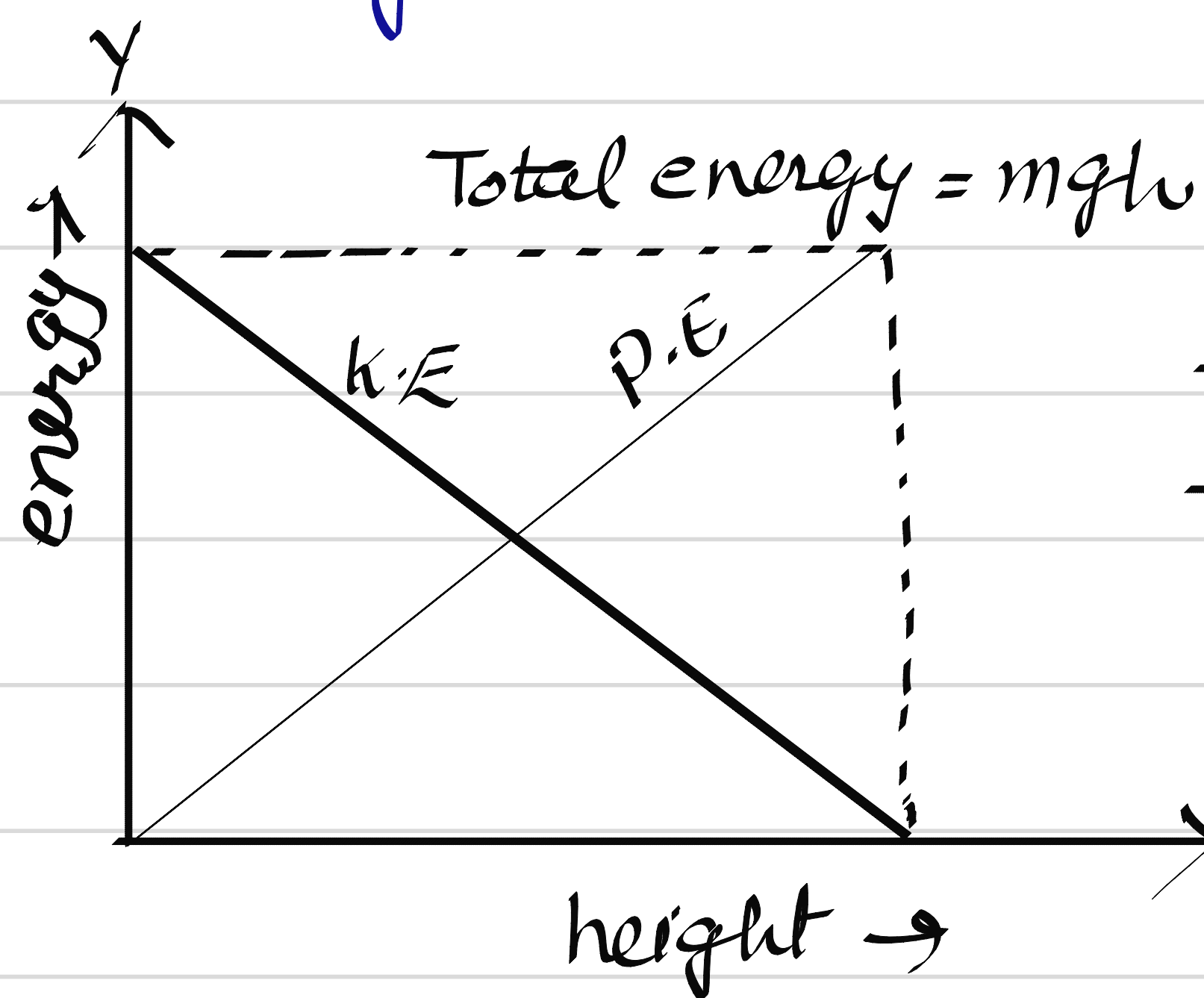
From eqⁿ (1), (2) and (3)

$$(M.E)_A = (M.E)_P = (M.E)_B = mgh$$

i.e Mechanical energy is conserved at every point.

* Total M.E of a system is conserved if forces doing work on it are conserved (here gravitational force is conservative force)

Variation of M.E, K.E and P.E with the height -



- P.E is max. at highest point.
- During journey towards ground P.E decreases and K.E increases
- K.E is max. when body hits the ground.

Q. Define spring constant. Find expression for energy stored in a spring.

A Spring constant (K) The restoring force per unit displacement is called force constant.

$$F \propto x$$

$$F = Kx$$

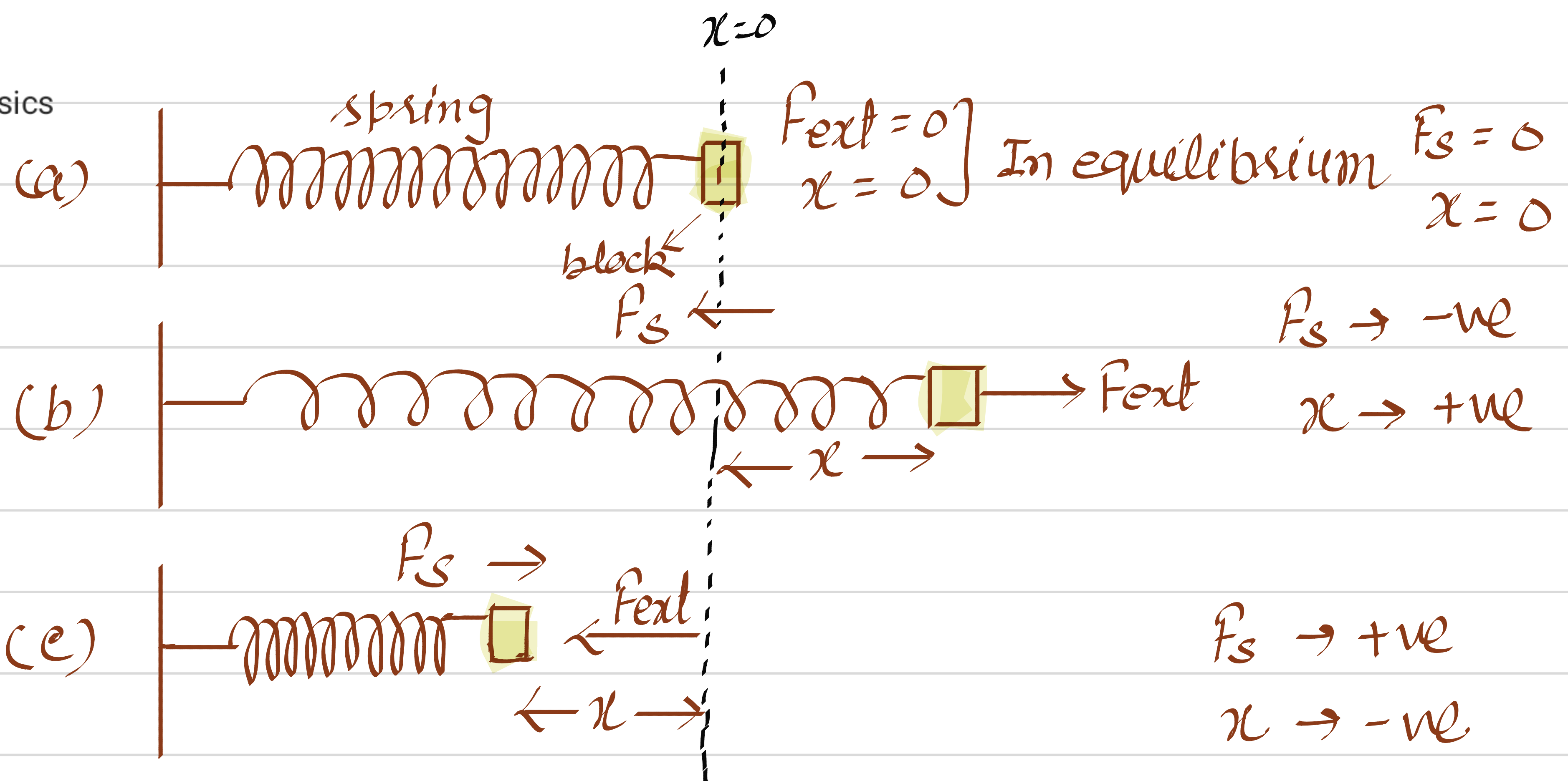
$$K = \frac{F}{x}$$

Spring is stiff if K is large
spring is soft if K is small

SI unit - Nm^{-1} , CGS unit $dyne\ cm^{-1}$
dimension $[MT^{-2}]$

Elastic Potential Energy of a spring

When a spring is stretched or compressed from its equilibrium position ($x=0$), then a restoring force is developed in the spring to bring it back.



$F_{ext} \rightarrow$ External force on spring

$F_s \rightarrow$ Restoring force of spring.

$x \rightarrow$ displacement of spring from mean position.

According to force law of spring (Hooke's law)

$$F_s = -kx \quad [-ve \text{ sign shows } F_s \& x \text{ are opposite}]$$

When block is pulled outward, the work done by the spring is

$$W_s = \int_0^x F_s dx$$

$$= -\int_0^x kx dx$$

$$W_s = -\frac{1}{2} kx^2$$

The same is true when spring is compressed.

Work done by applied force is equal to the elastic potential energy stored in the spring

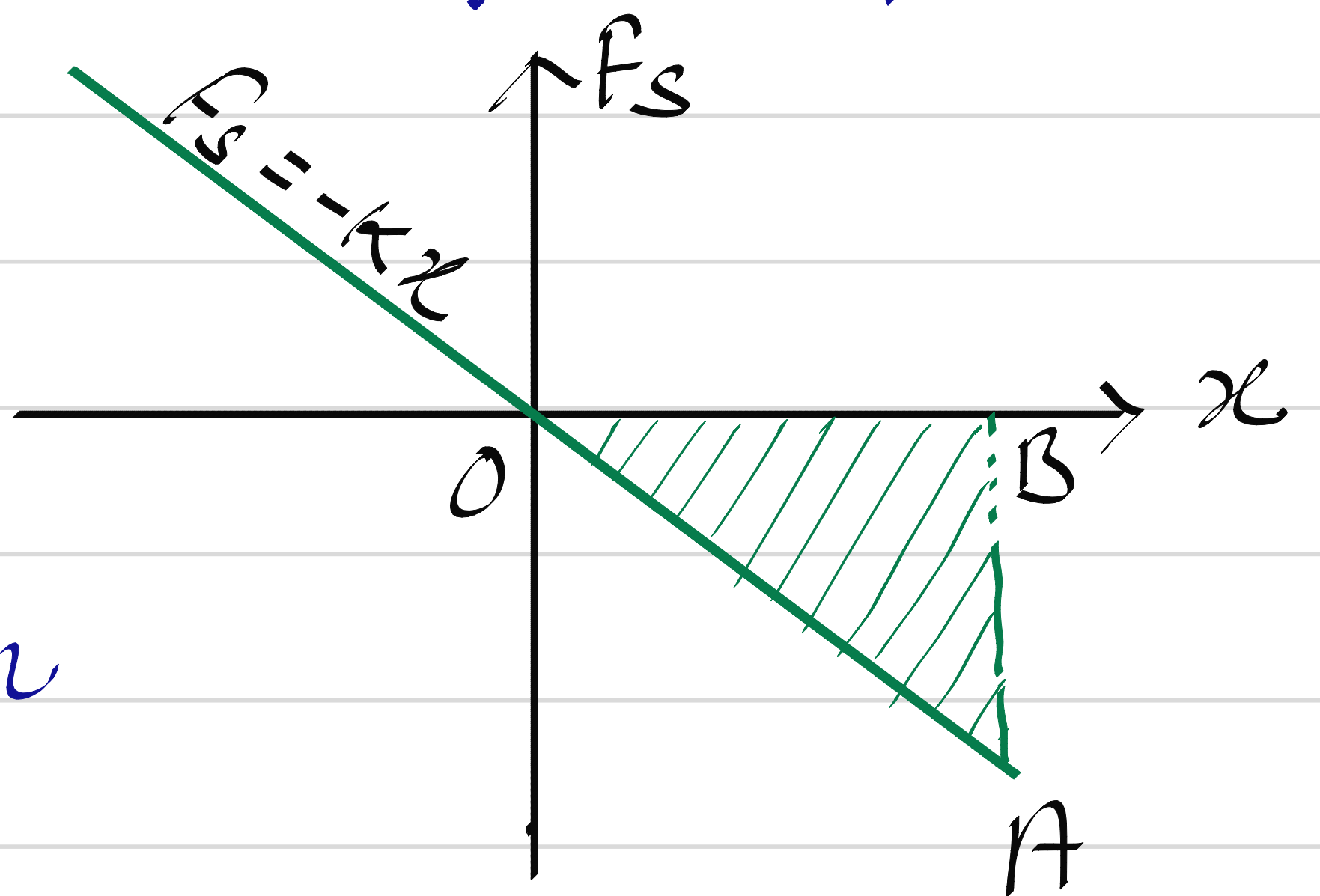
$$W_{ext} = U = \frac{1}{2} kx^2$$

It is always +ve

* Spring force is conservative force because work done by spring force depends only on the end point.

Graphical Method - Fig shows the force displacement graph.

Work done in stretching or compressing the spring
 $W = \text{Area under the graph}$
 $= \text{Area of shaded portion}$
 $= \text{Area of } \triangle OAB$
 $= \frac{1}{2} \times OB \times AB$
 $= \frac{1}{2} \times x \times F_s$



$$\text{or } W = -\frac{1}{2} kx^2$$

or $W = -\frac{1}{2} kx^2$. [-ve sign shows that it is the energy released by the spring]

This is the work done by the spring.

Now for potential energy stored in the spring,

$$U = \frac{1}{2} kx^2$$

This is the energy that spring has stored due to its compression or extension.

* It is always positive.

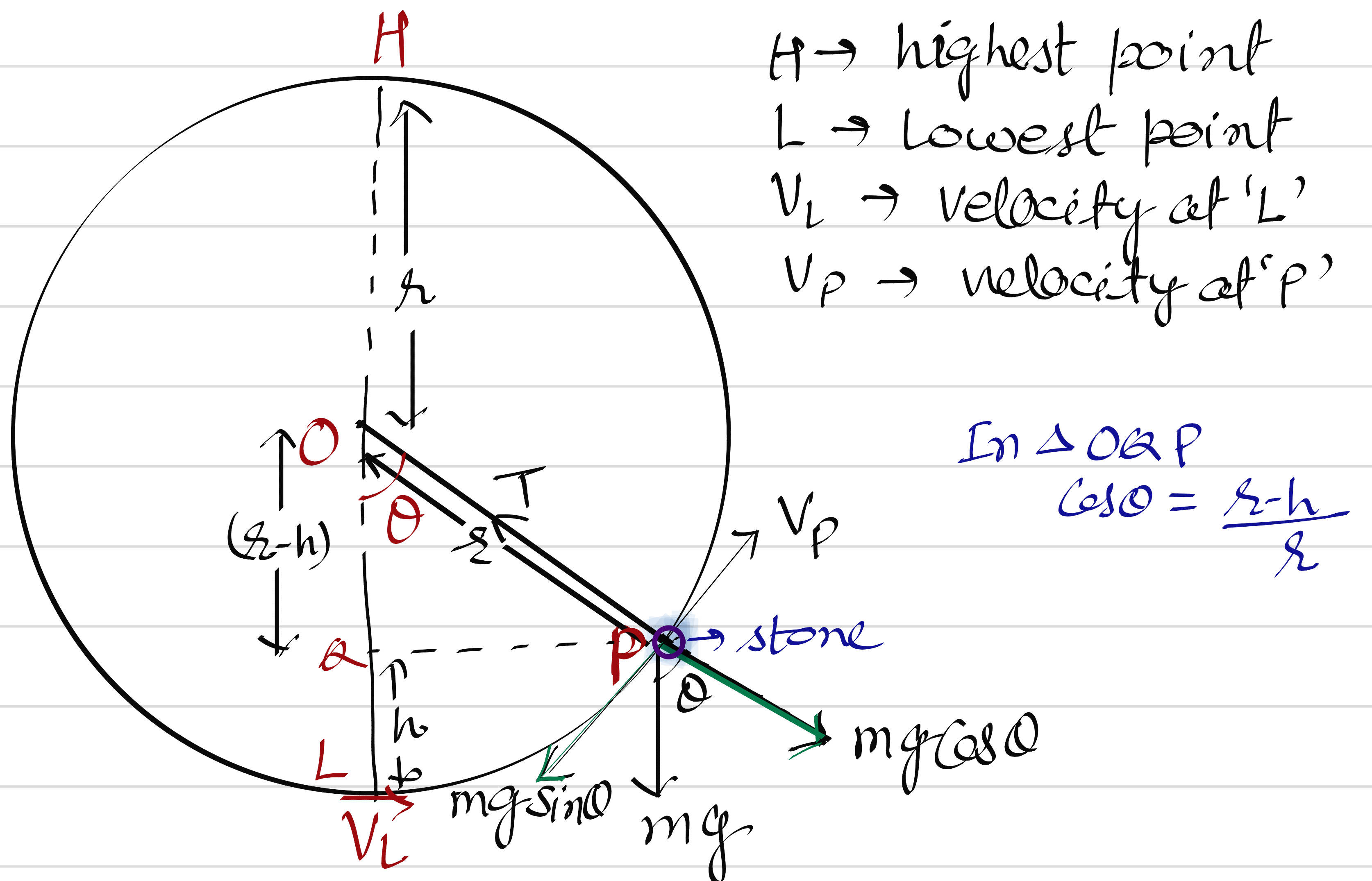
Q. * (Do according to your syllabus)

Explain the forces involved in vertical circular motion. How velocities affects motion at different points in the circular path?

A. Vertical circular motion: It is the motion of an object when it moves along a circular path in vertical plane.

e.g. Roller coasters, pendulum, bucket of water swung in vertical circle etc.

* Vertical circular motion is an example of non uniform circular motion



Total energy at point 'L'

$$\begin{aligned}
 E &= K + U \\
 &= \frac{1}{2} m V_L^2 + 0 \\
 E &= \frac{1}{2} m V_L^2
 \end{aligned}$$

Total energy at 'P'

$$E = \frac{1}{2} m V_P^2 + mgh \quad \text{--- (1)}$$

According to law of conservation of energy

$$\frac{1}{2} m V_P^2 + mgh = \frac{1}{2} m V_L^2$$

divide eqⁿ by $\frac{1}{2} m$

$$V_P^2 + 2gh = V_L^2$$

or

$$V_P^2 = V_L^2 - 2gh \quad \text{--- (2)}$$

i.e

$$V_P > V_L$$

Various forces on the string at point 'P'.

- (i) Tension T along PO
- (ii) Weight mg downwards
- (iii) $mg \cos \theta$ and $mg \sin \theta$ are the components of mg

Expression for Tension

From fig- $T - mg \cos \theta = \frac{mv_P^2}{r}$ ($\frac{mv^2}{r} \rightarrow$ centripetal force)

$$\text{or } T = \frac{mv_P^2}{r} + mg \cos \theta$$

From eqⁿ (2) $v_P^2 = v_C^2 - 2gh$, on putting this we get

$$T = \frac{m(v_C^2 - 2gh)}{r} + mg\left(\frac{r-h}{r}\right) \left[\cos \theta = \frac{r-h}{r}\right]$$

i.e

$$T = \frac{m}{r} [v_C^2 - 2gh + rg - gh]$$

$$\boxed{T = \frac{m}{r} [v_C^2 - 3gh + rg]} \quad \text{--- (3)}$$

Tension at lowest point 'L'

At 'L' $h=0$, then from eqⁿ (3)

$$\boxed{T_L = \frac{m}{r} [v_C^2 + rg]} \quad \text{--- (4)}$$

Tension at highest point 'H'

At H, $h = 2r$ (diameter), then from eqⁿ (3)

$$\begin{aligned} T &= \frac{m}{r} [v_C^2 - 3g(2r) + rg] \\ &= \frac{m}{r} [v_C^2 - 6rg + rg] \end{aligned}$$

$$\text{or } \boxed{T_H = \frac{m}{r} [v_C^2 - 5rg]} \quad \text{--- (5)}$$

Difference in tensions

$$T_L - T_H = \frac{m}{r} [v_L^2 + gr - v_L^2 + 5rg]$$

$$T_L - T_H = \frac{m}{r} [6rg]$$

or

$$T_L - T_H = 6mg$$

Speed of the stone

(i) At lowest point 'L' (v_L)

In order to keep moving in circular path

$$\text{i.e. } \frac{m}{r} [v_L^2 - 5rg] \geq 0 \quad [T_H \geq 0] \quad [\text{from eq}^n (5)]$$

$$\text{or } v_L^2 \geq 5rg$$

∴ minimum speed at 'L'

$$v_L^2 = 5rg$$

$$\text{or } v_L = \sqrt{5rg}$$

(ii) At highest point 'H' (v_H) - At 'H', $h = 2r$

From eqⁿ (2)

$$v_H^2 = v_L^2 - 2gh$$

$$\text{but } v_L^2 = 5rg$$

$$v_H^2 = 5rg - 2g(2r) \quad [h = 2r]$$

$$= 5rg - 4rg$$

$$v_H^2 = rg$$

$$v_H = \sqrt{rg}$$

→ Velocity of the stone at highest point for looping.

Practical Applications

- (i) Water in bucket do not spill if it is rotated in vertical motion above a certain velocity.
- (ii) Globe of death or loop the loop: The motor cyclist drives along a vertical circular path.

Q. Derive relation b/w power and force. Write different units of power.

Power of a machine It is defined as the rate at which work is done or the rate at which energy is transferred by a machine.

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$$P = \frac{W}{t} = \frac{\text{Energy}}{\text{time}}$$

Power in terms of Velocity
Instantaneous power,

$$P = \frac{dW}{dt} = \frac{d(\vec{F} \cdot \vec{s})}{dt} \\ = \vec{F} \cdot \frac{d\vec{s}}{dt}$$

$$P = \vec{F} \cdot \vec{v} = Fv \cos \theta$$

Units —

SI unit $\rightarrow$ Watt or J/s

C.G.S $\rightarrow$ Erg/sec

* Common unit

Horse Power (hp)

1 hp = 746 watts

θ is the angle
b/w $\vec{F}$ & $\vec{v}$

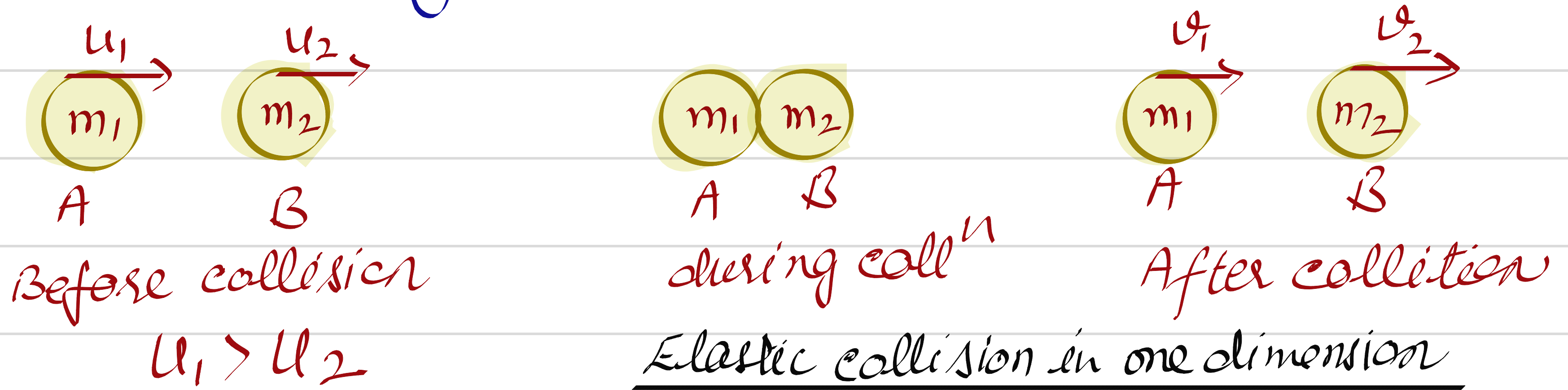
Q. Define elastic and inelastic collisions. Show that for elastic collision in one dimension the relative velocity of approaching = relative velocity of separation.

A. Elastic and Inelastic collisions

Elastic collision $\rightarrow$ Momentum (p) and K.E remains conserved. eg. collision of two billiard balls.

Inelastic collision $\rightarrow$ Only momentum is conserved
e.g. - car crash.

Derivation of $u_1 - u_2 = v_2 - v_1$



For elastic collision linear momentum is conserved
so, $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

$$\text{or } m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad \text{--- (1)}$$

Kinetic energy is also conserved, so

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$\text{or } m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2) \quad \text{--- (2)}$$

divide eqⁿ (2) by eqⁿ (1)

$$\frac{m_1(u_1^2 - v_1^2)}{m_1(u_1 - v_1)} = \frac{m_2(v_2^2 - u_2^2)}{m_2(v_2 - u_2)}$$

$$\text{or } \frac{(u_1 + v_1)(u_1 - v_1)}{(u_1 - v_1)} = \frac{(v_2 + u_2)(v_2 - u_2)}{(v_2 - u_2)}$$

$$\text{or } u_1 + v_1 = v_2 + u_2$$

$$\text{or } \boxed{u_1 - u_2 = v_2 - v_1}$$

Thus Relative velocity of approach = Relative velocity of separation

Q. Derive expression for velocities after collision.
A. For elastic collision,

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \quad \text{--- (1)}$$

also we have

$$u_1 - u_2 = v_2 - v_1$$

$$\text{or } v_2 = u_1 - u_2 + v_1 \quad \text{--- (2)}$$

put the value of u_2 from eqⁿ(2) in eqⁿ(1)

$$m_1 u_1 + m_2 u_2 = m v_1 + m_2 (u_1 - u_2 + v_1)$$

$$\text{or } m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 u_1 - m_2 u_2 + m_2 v_1$$

$$\text{or } m_1 u_1 - m_2 u_1 + 2m_2 u_2 = (m_1 + m_2) v_1$$

$$\text{or } (m_1 - m_2) u_1 + 2m_2 u_2 = (m_1 + m_2) v_1$$

$$\text{or } v_1 = \frac{(m_1 - m_2) u_1}{(m_1 + m_2)} + \frac{2m_2 u_2}{(m_1 + m_2)}$$

similarly from $u_1 - u_2 = v_2 - v_1$
 $v_1 = v_2 - u_1 + u_2$

put this value in eqⁿ (1)

$$m_1 u_1 + m_2 u_2 = m_1 (v_2 - u_1 + u_2) + m_2 v_2$$

$$\text{or } m_1 u_1 + m_2 u_2 = m_1 v_2 - m_1 u_1 + m_1 u_2 + m_2 v_2$$

$$\text{or } 2m_1 u_1 + m_2 u_2 - m_1 u_2 = (m_1 + m_2) v_2$$

$$\text{or } 2m_1 u_1 + (m_2 - m_1) u_2 = (m_1 + m_2) v_2$$

$$\text{or } v_2 = \frac{(m_2 - m_1) u_2}{(m_1 + m_2)} + \frac{2m_1 u_1}{m_1 + m_2}$$

Special cases

(1) If $m_1 = m_2 = m$

$$v_1 = 0 + \frac{2m u_2}{2m}$$

$$v_1 = u_2$$

and

$$v_2 = 0 + \frac{2m u_1}{2m}$$

or

$$v_2 = u_1$$

i.e. velocities exchanges after collision.

$$(1) \text{ If } u_2 = 0$$

$$u_1 = u_1 \frac{m_1 - m_2}{m_1 + m_2}$$

$$\text{and } u_2 = \frac{2m_1 u_1}{m_1 + m_2}$$

CHAPTER: SYSTEM OF PARTICLE AND ROTATIONAL MOTION

Q. Define relation between torque and angular momentum.

A. For a rotating body angular momentum of a particle is given by

$$\vec{L} = \vec{r} \times \vec{p}$$

$\vec{r} \rightarrow$ position vector

$\vec{p} \rightarrow$ linear momentum

on differentiating w.r.to t

$$\frac{d\vec{L}}{dt} = \frac{d}{dt} (\vec{r} \times \vec{p})$$

$$= \vec{r} \times \frac{d\vec{p}}{dt} + \frac{d\vec{r}}{dt} \times \vec{p}$$

$$= \vec{r} \times \vec{F} + \vec{v} \times (m\vec{v}) \quad [\because \vec{p} = m\vec{v}]$$

$$= \vec{\tau} + m(\vec{v} \times \vec{v})$$

$$\text{or } \frac{d\vec{L}}{dt} = \vec{\tau} + 0 \quad [\vec{v} \times \vec{v} = 0]$$

$$\text{or } \boxed{\frac{d\vec{L}}{dt} = \vec{\tau}}$$

So that the time rate of change of angular momentum is equal to the torque.

Q. Show that if resultant external torque on a body is zero then the angular momentum of the body remains conserved.

A. We know angular momentum of a body is given by

$$L = mvr$$

$$= m(r\omega)r$$

$$= mr^2\omega$$

$$L = I\omega \quad [I = mr^2]$$

on differentiating w.r. to t , we get

$$\frac{dL}{dt} = I \frac{d\omega}{dt}$$

$$\text{or } \frac{dL}{dt} = I\alpha \quad [\because \frac{d\omega}{dt} = \alpha]$$

$$\text{or } \frac{dL}{dt} = \tau_{\text{ext}}$$

$$\text{If } \tau_{\text{ext}} = 0$$

$$\frac{dL}{dt} = 0$$

$$\therefore L = \text{constant}$$

$$\text{or } L = I\omega = \text{constant}$$

Rotational Power

$$P = \frac{dW}{dt}$$

$$= r \frac{d\theta}{dt} \quad [dW = r d\theta]$$

$$P = \tau\omega$$

Relation b/w τ and α

$$L = I\omega$$

$$\frac{dL}{dt} = I \frac{d\omega}{dt}$$

$$\tau = I\alpha \quad [\frac{d\omega}{dt} = \alpha]$$

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$$I_1\omega_1 = I_2\omega_2$$

CHAPTER : GRAVITATION

Q. How does the acceleration due to gravity (g) change with increasing height above the earth surface? Derive expression.

A. Variation of acceleration due to gravity:

Effect of height (Altitude) -

consider a point 'A' on the earth surface and a point 'B' at height 'h' as shown in fig.

Value of ' g ' at point 'A' (at the surface of earth)

$$g = \frac{GM}{R^2} \quad \text{--- (1)}$$

At point 'B' [At 'h' height]

$$g' = \frac{GM}{(R+h)^2} \quad - (2)$$

From (1) and (2)

$$\frac{g'}{g} = \frac{GM/(R+h)^2}{GM/R^2}$$

$$\frac{g'}{g} = \frac{R^2}{(R+h)^2}$$

→ This formula is applicable when 'h' is comparable to 'R' e.g. for satellite

$$\frac{g'}{g} = \frac{R^2}{R^2(1 + \frac{h}{R})^2}$$

$$\frac{g'}{g} = \left(1 + \frac{h}{R}\right)^{-2}$$

by using binomial expression for $\frac{h}{R} \ll 1$

$$\frac{g'}{g} = 1 - \frac{2h}{R}$$

$$\text{or } g' = g \left[1 - \frac{2h}{R}\right]$$

→ This formula is applicable when $h \ll R$ for small height

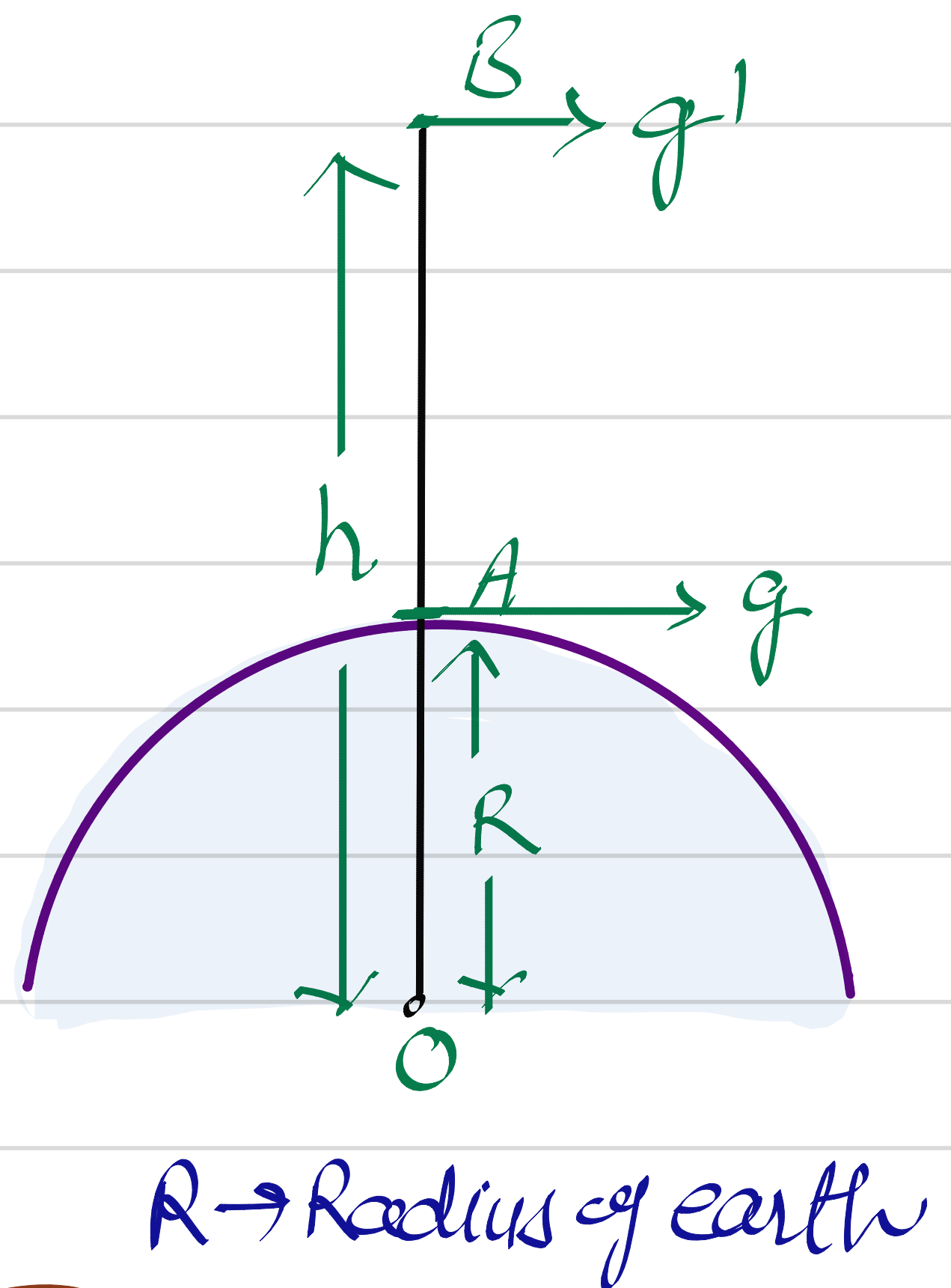
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$$\therefore g' < g$$

Thus on going above the earth surface value of 'g' decreases by a factor $\left(1 - \frac{2h}{R}\right)$.

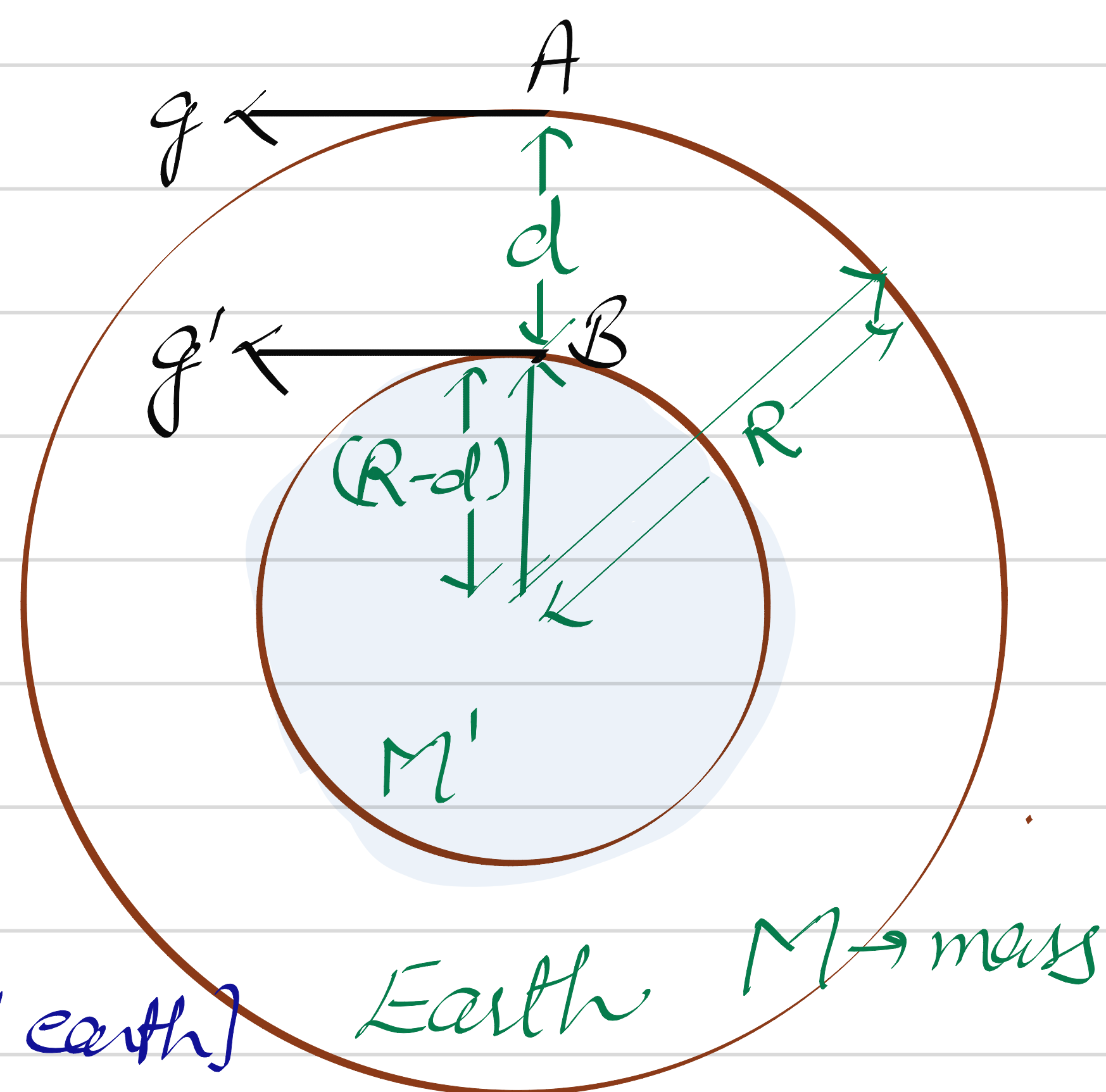
Q. How does the acceleration due to gravity vary on moving inside the earth. Derive expression.
OR

What is the effect of depth on 'g'. Derive expression.



A. Effect of depth on 'g':

In fig. at the surface of earth at a point 'A' the accⁿ due to gravity is 'g'. Inside the earth at depth 'd' it becomes g' at point 'B'.



At the surface (At point 'A')

$$g = \frac{GM}{R^2} \quad [M \rightarrow \text{Mass of earth}]$$

$$= \frac{G \left(\frac{4}{3} \pi R^3 \rho \right)}{R^2} \quad [M = V \cdot \rho = \frac{4}{3} \pi R^3 \rho]$$

$$g = \frac{4}{3} G \pi R \rho \quad \text{--- (1)}$$

At depth 'd' (at point 'B')

$$g' = \frac{GM'}{(R-d)^2} \quad [M' \text{ is mass of shaded part of earth}]$$

$$= \frac{G \frac{4}{3} \pi (R-d)^3 \rho}{(R-d)^2} \quad [M' = V \cdot \rho = \frac{4}{3} \pi (R-d)^3 \rho]$$

$$g' = \frac{4}{3} G \pi (R-d) \rho \quad \text{--- (2)}$$

Divide eqⁿ (2) from eqⁿ (1)

$$\frac{g'}{g} = \frac{\frac{4}{3} G \pi (R-d) \rho}{\frac{4}{3} G \pi R \rho}$$

$$\text{or } \frac{g'}{g} = \frac{R-d}{R} = \frac{R(1 - \frac{d}{R})}{R}$$

$$\text{or } \boxed{g' = g \left(1 - \frac{d}{R} \right)} \quad g' < g$$

Thus as we go deeper inside the earth, the value of 'g' decreases by a factor $(1 - \frac{d}{R})$

* At the centre of earth $d=R$, then $g' = 0$

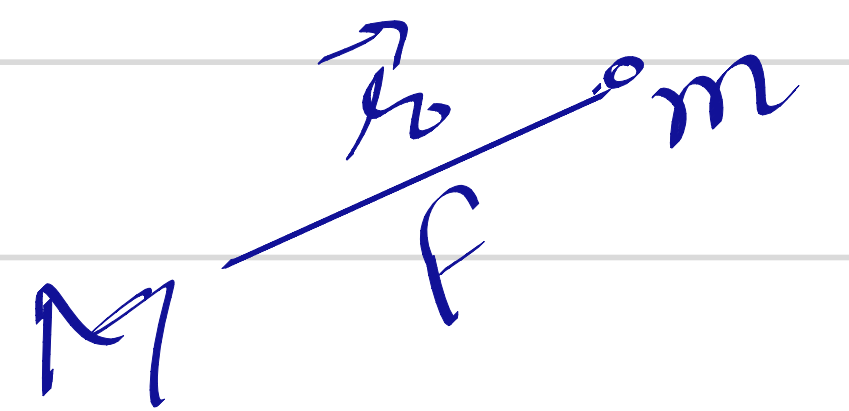
* Value of g is maximum at the earth surface. It decrease with altitude and depth.

Q. Show that intensity of gravitational field is numerically equal to the acceleration due to gravity.

A. Intensity of gravitational field at any point is defined as the gravitational force per unit mass in the gravitational field of any other mass.

$$I = \frac{F}{m}$$

$$= \frac{GMm/r^2}{m}$$



$$\text{or } I = \frac{GM}{r^2}$$

$$\text{or } \boxed{I = g} \quad \left[\frac{GM}{r^2} = g \right]$$

$$\text{SI unit} \rightarrow \text{N/Kg} = \text{m/s}^2$$

Q. Define gravitational potential energy. Derive expression for it.

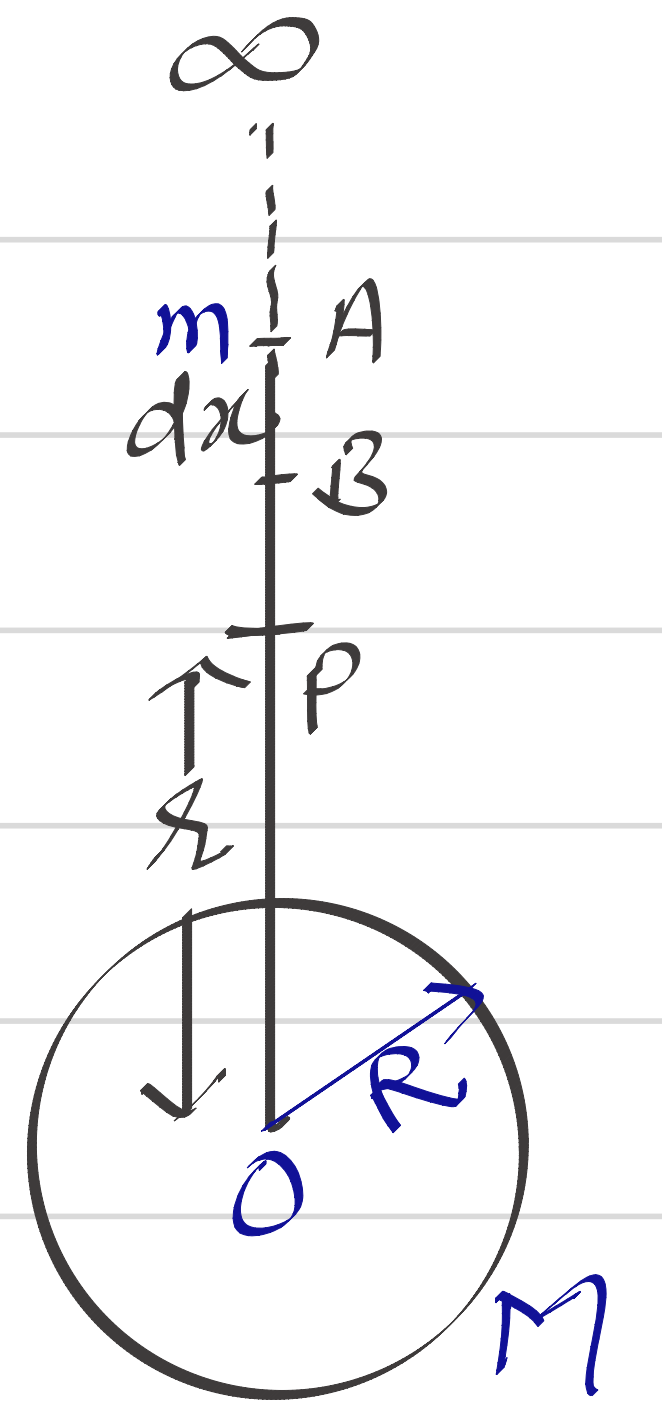
A. It is equal to the work done in bringing an object from infinity to a point in the gravitational field of a body.

Expression:

Fig. shows that a body of mass m is brought to a point P. Distance of P from the centre of earth O is r. Let dx is small displacement.

$$OP = \mathbb{Z} \quad \text{and} \quad \mathbb{Z} \succ \mathbb{R}$$

When the body is at A. By Newton's law of gravitation $F = \frac{GMm}{x^2}$



Work done in moving a distance dx
 $dW = F dx$

$$dW = F dx$$

$$= \frac{G M m}{r^2} dr$$

Total work done in moving the body from ∞ to P. i.e from $x = \infty$ to $x = r$

$$W = \int dW$$

$$= \int_{\infty}^R \frac{GMm}{x^2} dx$$

$$= G M m \int_{\infty}^R \frac{1}{x^2} dx$$

$$= G M m \int_{\infty}^R x^{-2} dx$$

$$= GMm \left[-\frac{1}{x} \right]_{\infty}^R$$

$$= 6Mm \left[-\frac{1}{x} + \frac{1}{\infty} \right]$$

$$W = \frac{-GMm}{r} \quad \left[\frac{1}{\infty} = 0 \right]$$

This work done is the gravitational potential energy. i.e

$$U = - \frac{G M m}{R}$$

* U is maximum at infinity which is zero. ($U_{\infty} = 0$)

Q. Define Escape speed. Derive relation for escape speed for a body to escape from earth's gravitational field.

A. Escape speed: The minimum speed required by an object to escape from the gravitational field of a massive object and never return.

For Earth it is 11.2 km/s .

Expression: By the conservation of energy

$$(K+U)_i = (K+U)_f \quad \begin{array}{l} i \rightarrow \text{initial} \\ f \rightarrow \text{final} \end{array}$$

$$\text{Where } K_i = \frac{1}{2} m v^2 \text{ and } U_i = -\frac{GMm}{R}$$

At infinity $U_f = 0$ and $K_f = 0$

$$\text{so, } \frac{1}{2} m v_e^2 - \frac{GMm}{R} = 0 \quad [v_e \rightarrow \text{escape speed}]$$

$$\text{or } \frac{1}{2} m v_e^2 = \frac{GMm}{R}$$

$$\text{or } v_e^2 = \frac{2GM}{R}$$

$$\text{or } v_e = \sqrt{\frac{2GM}{R}}$$

put $GM = gR^2$, then

$$v_e = \sqrt{\frac{2gR^2}{R}}$$

$$\text{or } v_e = \sqrt{2gR}$$

This is the expression for escape speed.
For Earth $g = 9.8 \text{ m/s}^2$ $R = 6.4 \times 10^6 \text{ m}$, then
$$v_e = \sqrt{2 \times 9.8 \times 6.4 \times 10^6}$$

$$\begin{aligned} \text{or } v_e &= \sqrt{2 \times 2 \times 49 \times 64 \times 10^4} \\ &= 2 \times 7 \times 8 \times 10^2 \\ &= 112 \times 10^2 \text{ m/s} \end{aligned}$$

$$v_e = 11.2 \text{ km/s}$$

Thus if a body is thrown with minimum speed of 11.2 km/s it will not return to earth.
* For moon escape speed is 2.4 km/s which is very less. This is the reason why moon has no atmosphere.

Q. Define orbital speed and derive the formula for orbital speed of a satellite. Also find relation b/w escape speed and orbital speed.

A. Orbital Speed: The minimum speed required to put a body into a given orbit around a massive object.

For Earth the minimum speed required to put a satellite into a given orbit around the earth is called orbital velocity for earth.

Expression:

Consider a satellite moving in a circular orbit of distance $(R+h)$. $R \rightarrow$ Radius of earth, $h \rightarrow$ height of satellite from earth surface.

Here the centripetal force is provided by the gravitational force b/w earth and satellite.

$$\text{So } \frac{mv_o^2}{(R+h)} = \frac{GMm}{(R+h)^2}$$

$$\text{or } v_o^2 = \frac{GM}{(R+h)}$$

$$\text{or } v_0 = \sqrt{\frac{GM}{R+h}}$$

for $h=0$ (At earth surface)

$$v_0 = \sqrt{\frac{GM}{R}}$$

$$\text{or } v_0 = \sqrt{\frac{gR^2}{R}}$$

$$[\because GM = gR^2]$$

$$v_0 = \sqrt{gR}$$

$$\Rightarrow v_0 = \sqrt{9.8 \times 6.4 \times 10^6} = 79.2 \times 10^2 \text{ m/s}$$

$$v_0 = 7.92 \approx 8 \text{ km/s}$$

Relation b/w escape speed and orbital speed

We know the escape speed

$$v_e = \sqrt{2gR} \quad (1)$$

and orbital speed

$$v_0 = \sqrt{gR} \quad (2)$$

From (1) and (2)

$$v_e = \sqrt{2} v_0$$

i.e. the escape speed is $\sqrt{2}$ times of orbital speed.