

Physics Annual Exams

@Jyotisharmaphysics

Class XI Most Important Questions

(Part-2)

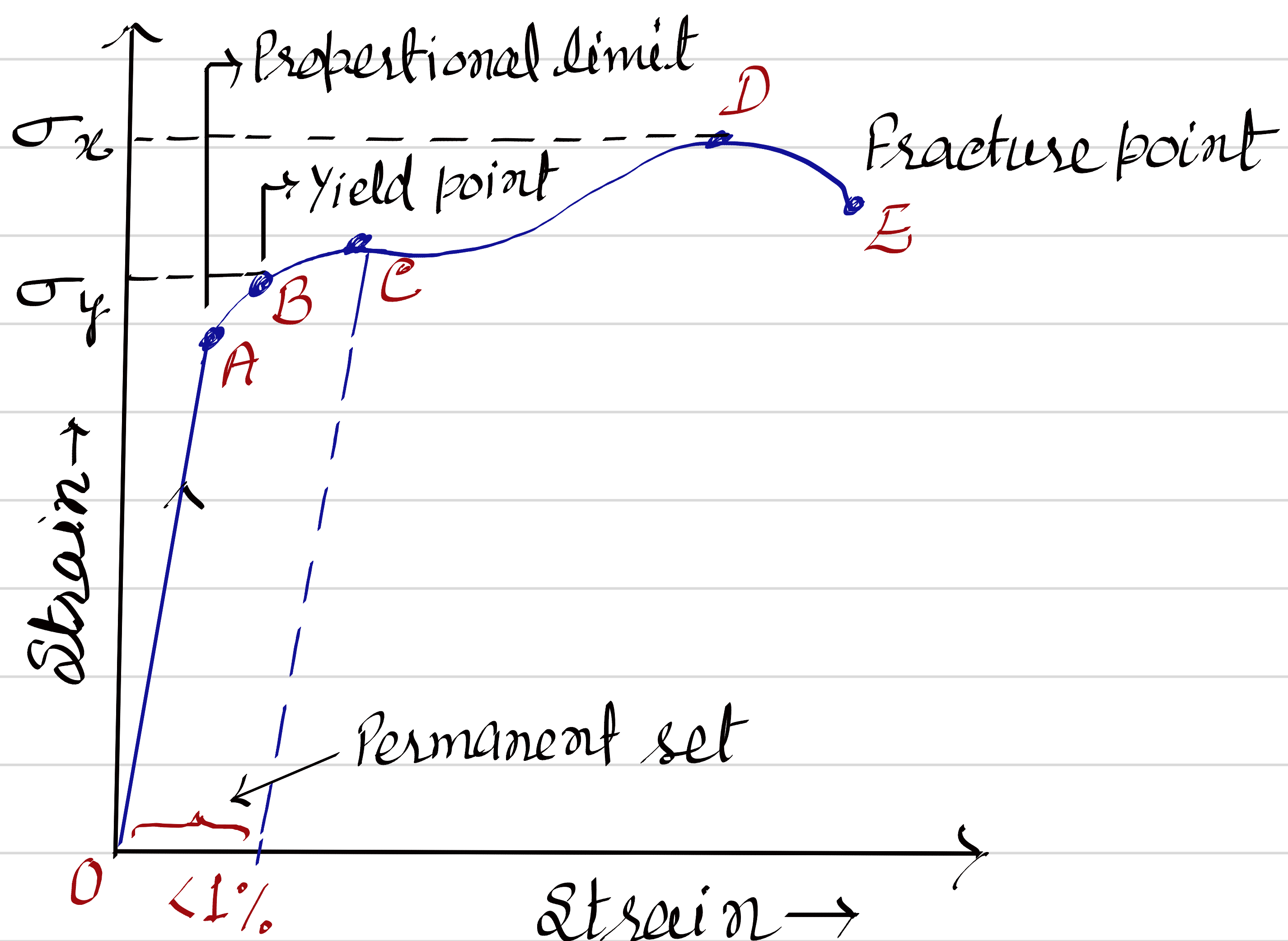
[Mechanical Properties of solids ch-8]

Q.1. State Hooke's law. Also plot the stress-strain curve for a metal showing the different regions:

- (i) Elastic limit
- (ii) Permanent set
- (iii) Yield point
- (iv) Fracture point

A. Hooke's law: For small deformations the stress and strain are proportional to each other. This is known as Hooke's law.

Stress-strain curve:



Stress-strain curve for metal

From the graph, Region:

O to A → Stress $\propto$ Strain.

→ Curve is linear.

→ Hooke's law is obeyed.

A to B → Stress is not proportional strain

→ 'B' represent the yield point. i.e. body regains original dimensions when load is removed.

→ ' σ_y ' is yield strength or Elastic limit

B to D → Strain increases rapidly even for small change in stress

→ At any point between B and D (say C) when load is removed, body does not regain original dimensions.

→ Strain is not zero for zero stress.

→ Material is said to have permanent set.

→ Deformation is called plastic deformation.

→ Point 'D' is called ultimate tensile strength.

D to E → Beyond 'D' additional strength is produced even on reducing load.

→ Fracture point or Breaking point occurs at point E.

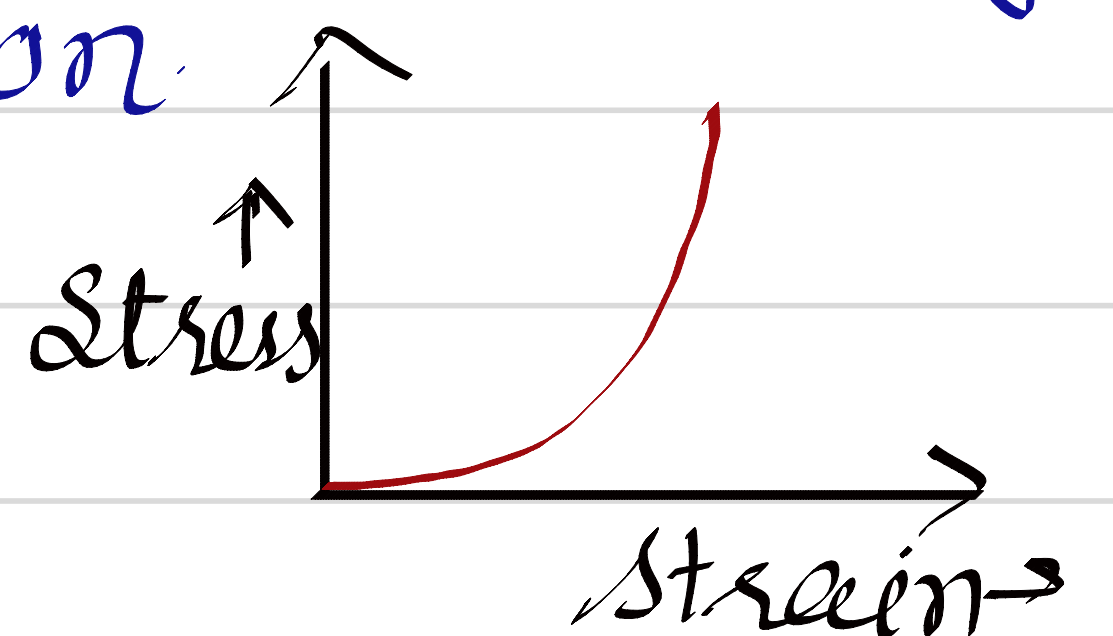
* If D and E are close, body is brittle.

* If D and E are far apart, body is ductile.

* σ_u is ultimate tensile strength.

* If stress-strain curve is like material does not obey Hooke's law over most of the region and no well defined plastic region.

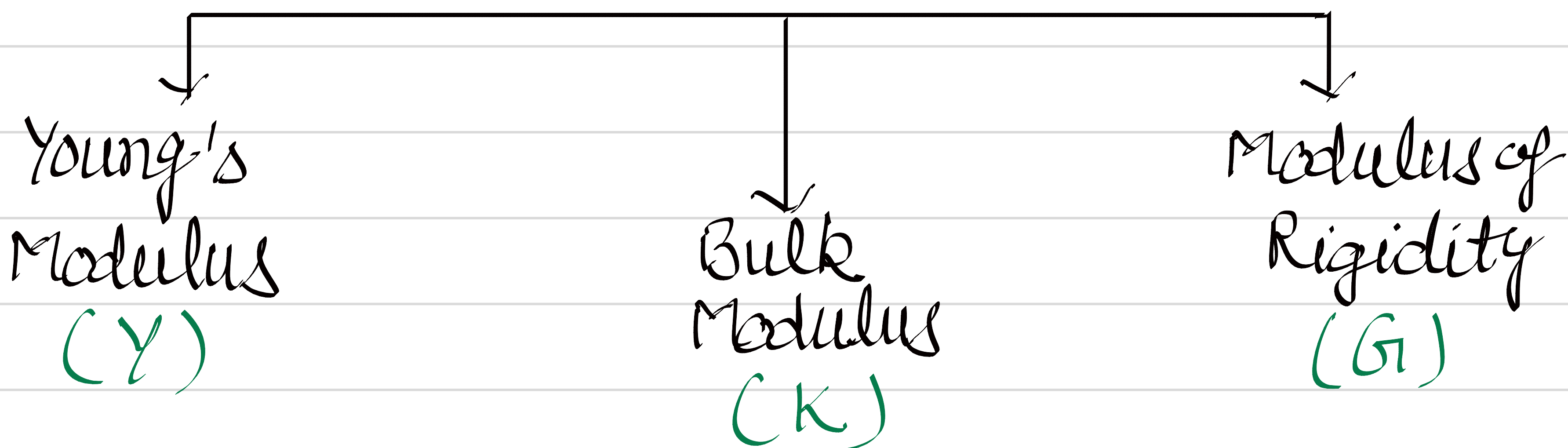
Then substances are called Elastomers. e.g. Rubber, aorta etc.



Q. Explain the types of Modulus of Elasticity with their formula.

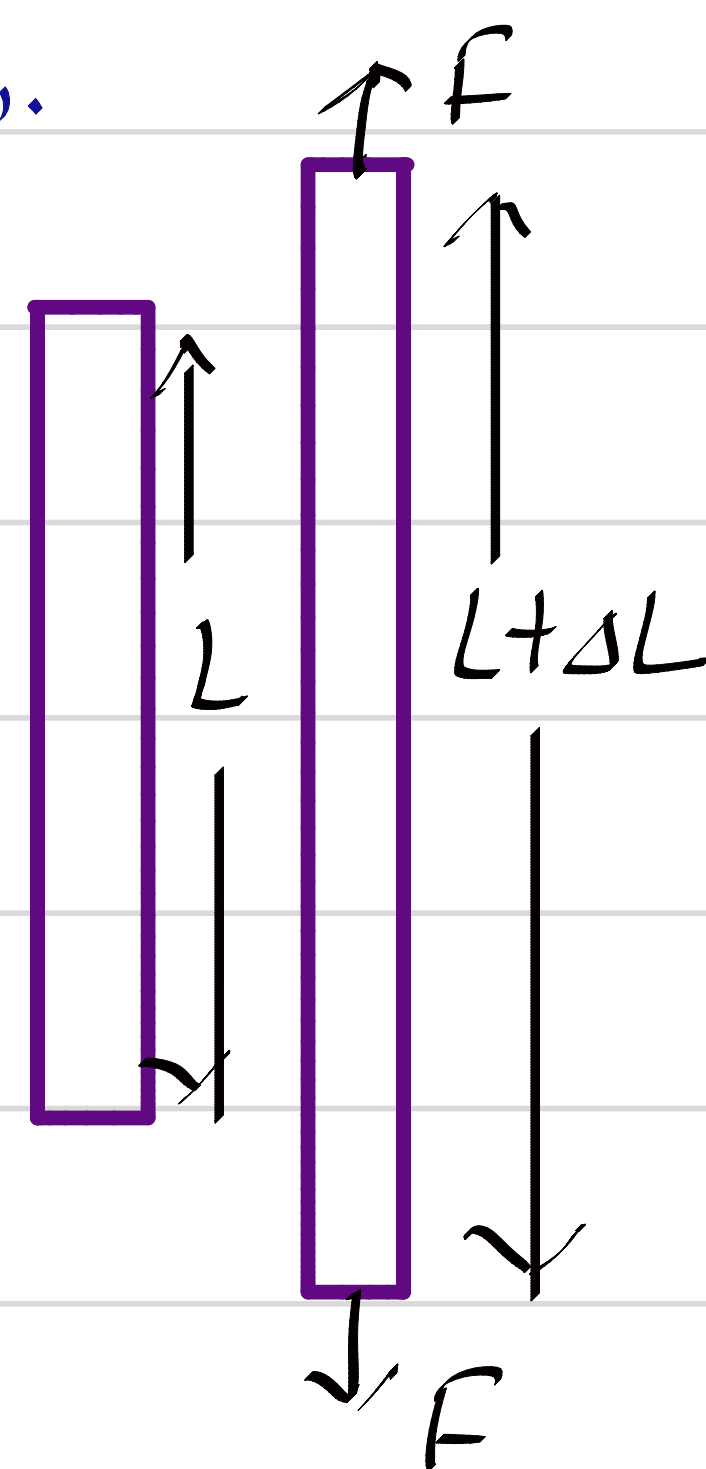
A.

Elastic Moduli



1. Young Modulus (Y) → Measure the resistance of a solid to change in its length.

$$Y = \frac{\text{Normal stress}}{\text{Longitudinal strain}} = \frac{\sigma}{\epsilon}$$



or

$$Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A \Delta L}$$

For wire of radius r and mass m

$$Y = \frac{mgL}{\pi r^2 \Delta L}$$

L → Length
 r → Radius
 ΔL → Change in L

SI unit - N/m^2 or Pa.

* Steel is more elastic than copper, Aluminium and Iron its value of Y is larger.

* Normal Stress

Tensile (stretching, $L \uparrow$) Compressive (compressing, $L \downarrow$)

Y
 The ratio of normal stress to longitudinal strain

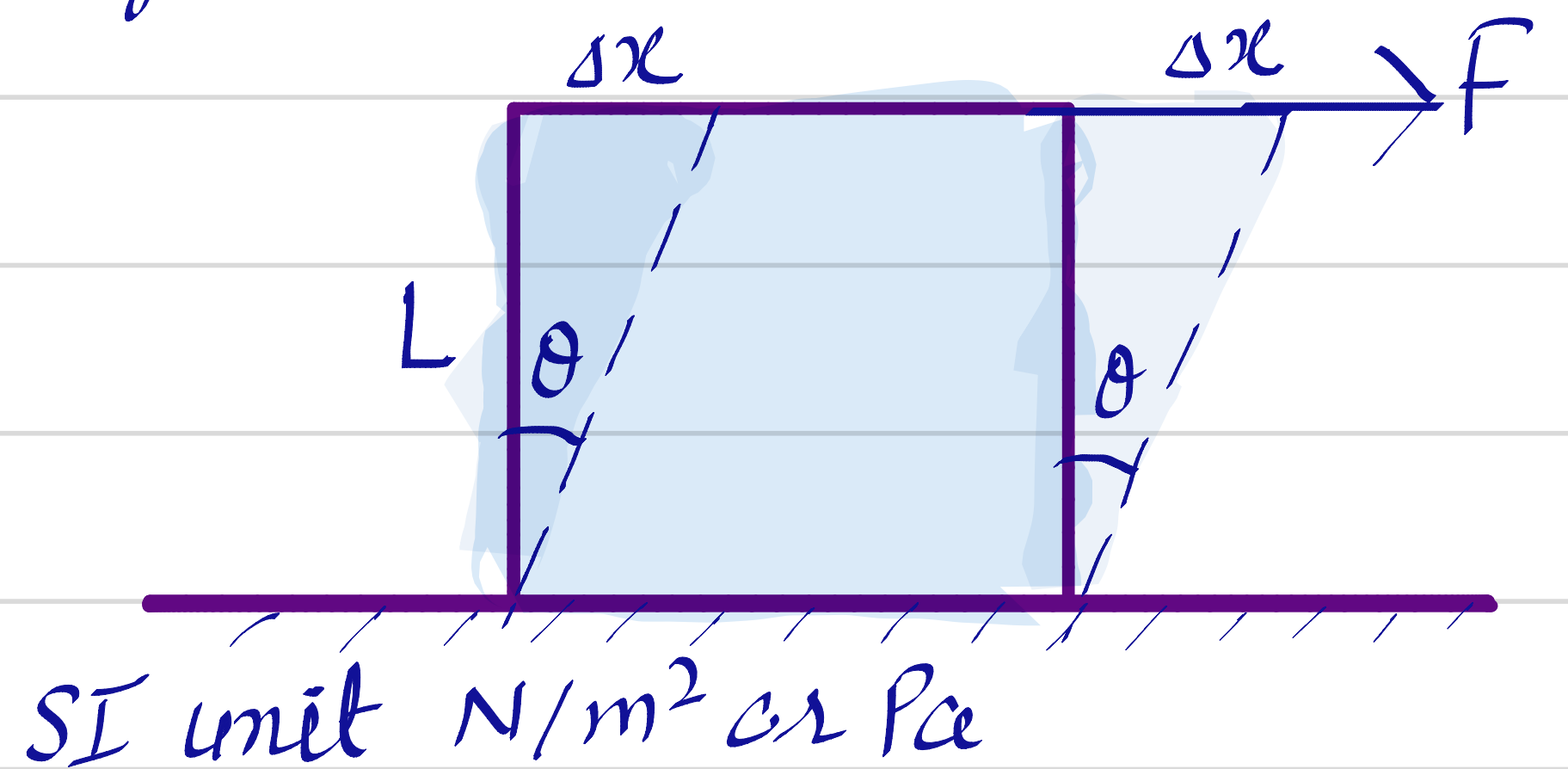
2. Shear Modulus or Modulus of Rigidity (G)
Measure the resistance of motion of the planes within a solid parallel to each other.

$$\text{Shear Modulus } G = \frac{\text{Shearing Stress}}{\text{Shearing Strain}}$$

The ratio of shearing stress to the shearing strain

$$G = \frac{F/A}{\Delta x/L}$$

$$G = \frac{F}{A\theta}$$



* Shear modulus generally less than Young's modulus. For most of the materials $G = \frac{Y}{3}$

3. Bulk Modulus (B): Measure the resistance of solids and liquids to changes their volume.

$$B = \frac{-p}{\Delta V/V}$$

@Jyotisharmaphysics

$$B = \frac{-pV}{\Delta V}$$

-ve sign shows that when pressure P increases the volume V decreases. SI unit N/m^2 or Pa.

Q.3 Derive expression for elastic potential energy.

A. Elastic potential energy: When a wire is stretched work has to be done against the restoring force. This work done is stored in the form of elastic potential energy.

Work done in stretching the wire

$$W = F_{av} \times l$$

$$= \frac{0+F}{2} \times l \quad [F_{av} = \frac{0+F}{2}]$$

$$W = \frac{F \times l}{2}$$

$$= \frac{F}{2} \times l \times \frac{AL}{AL} \quad [\text{multiply and divide by } AL]$$

$$= \frac{1}{2} \frac{F}{A} \cdot \frac{l}{L} \times AL$$



$$W = \frac{1}{2} \text{ stress} \times \text{strain} \times \text{Volume}$$

This work done is equal to the elastic potential energy

$$U = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

$$\text{or } U = \frac{1}{2} \times \frac{\text{stress}}{\text{strain}} \times \text{strain}^2 \times \text{volume}$$

$$\text{or } U = \frac{1}{2} \times Y \times \text{strain}^2 \times \text{volume}$$

$$\text{Energy density } u = \frac{U}{\text{Volume}}$$

$$\text{so } u = \frac{1}{2} \times Y \times \text{strain}^2$$

$$\text{or } u = \frac{1}{2} \times \text{stress} \times \text{strain}$$

$$u = \frac{1}{2} \times \frac{F}{A} \times \frac{l}{L}$$

[Mechanical Properties of Fluids ch. 9]

Q4. Show that pressure transmit undiminished through a closed system of fluid. (Pascal's law)

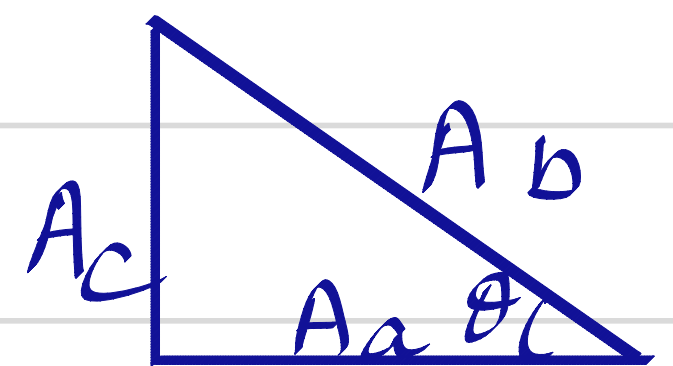
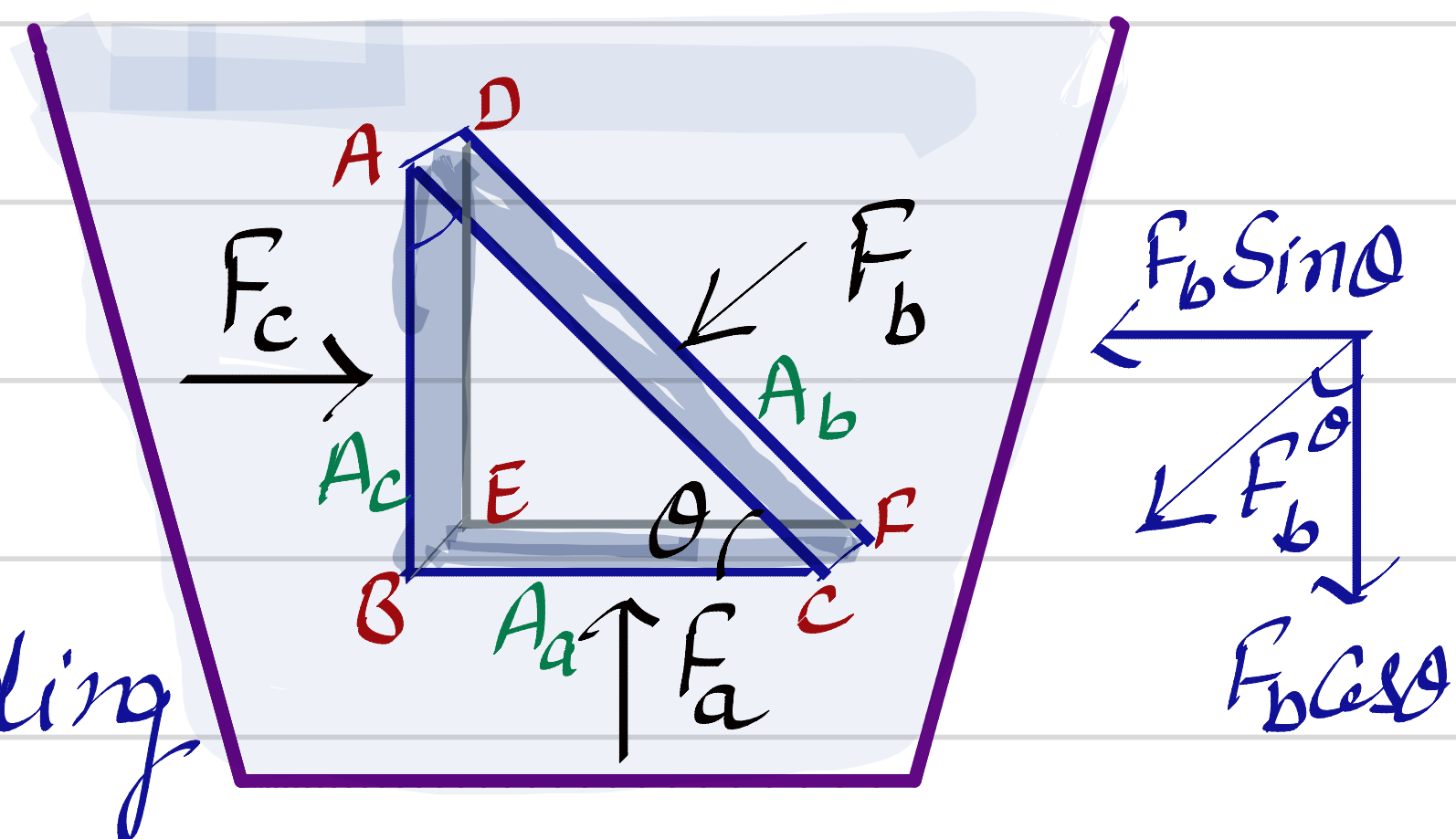
A. Pascal's law: The pressure exerted is same in all directions in a fluid at rest.

Proof: The fig shows an element in the interior of a fluid at rest. The element is so small that effect of gravity can be ignored.

Let A_a , A_b and A_c are the area of faces of element with corresponding forces F_a , F_b and F_c .

In equilibrium condition (force)

$$F_b \sin \theta = F_c \quad (1), \quad F_b \cos \theta = F_a$$



And by geometry (Area)

$$A_b \sin \theta = A_c \quad (3), \quad A_b \cos \theta = A_a \quad (4)$$

On dividing (1) ÷ (3) and (2) ÷ (4), we get

$$\begin{cases} \sin \theta = \frac{A_c}{A_b} \\ \cos \theta = \frac{A_a}{A_b} \end{cases}$$

$$\frac{F_b \sin \theta}{A_b \sin \theta} = \frac{F_c}{A_c} \quad \text{and} \quad \frac{F_b \cos \theta}{A_b \cos \theta} = \frac{F_a}{A_a}$$

$$\text{or} \quad \frac{F_b}{A_b} = \frac{F_c}{A_c} \quad \text{and} \quad \frac{F_b}{A_b} = \frac{F_a}{A_a}$$

Thus,

$$\frac{F_a}{A_a} = \frac{F_b}{A_b} = \frac{F_c}{A_c}$$

$$\text{or} \quad \boxed{P_a = P_b = P_c}$$

This is the Pascal's law.

Q.5. Explain equation of continuity.

A. Equation of continuity: The equation of continuity is a principle of fluid flow.

It states that in a steady flow the rate at which mass enters a system is equal to the rate at which mass leaves.

Mathematically

$$A_1 v_1 = A_2 v_2$$

Where A_1 and A_2 are the cross section areas of the pipe. v_1 and v_2 are velocities of fluid.

Proof: Let mass of fluid entering at 'P' is m_1 and leaving from 'Q' is m_2 .

By conservation of mass
mass entering = mass leaving

$$m_1 = m_2$$

$$v_1 \rho = v_2 \rho \quad [m = v \rho]$$

Volume = Area $\times$ distance

$$V = A \times v \Delta t$$

so

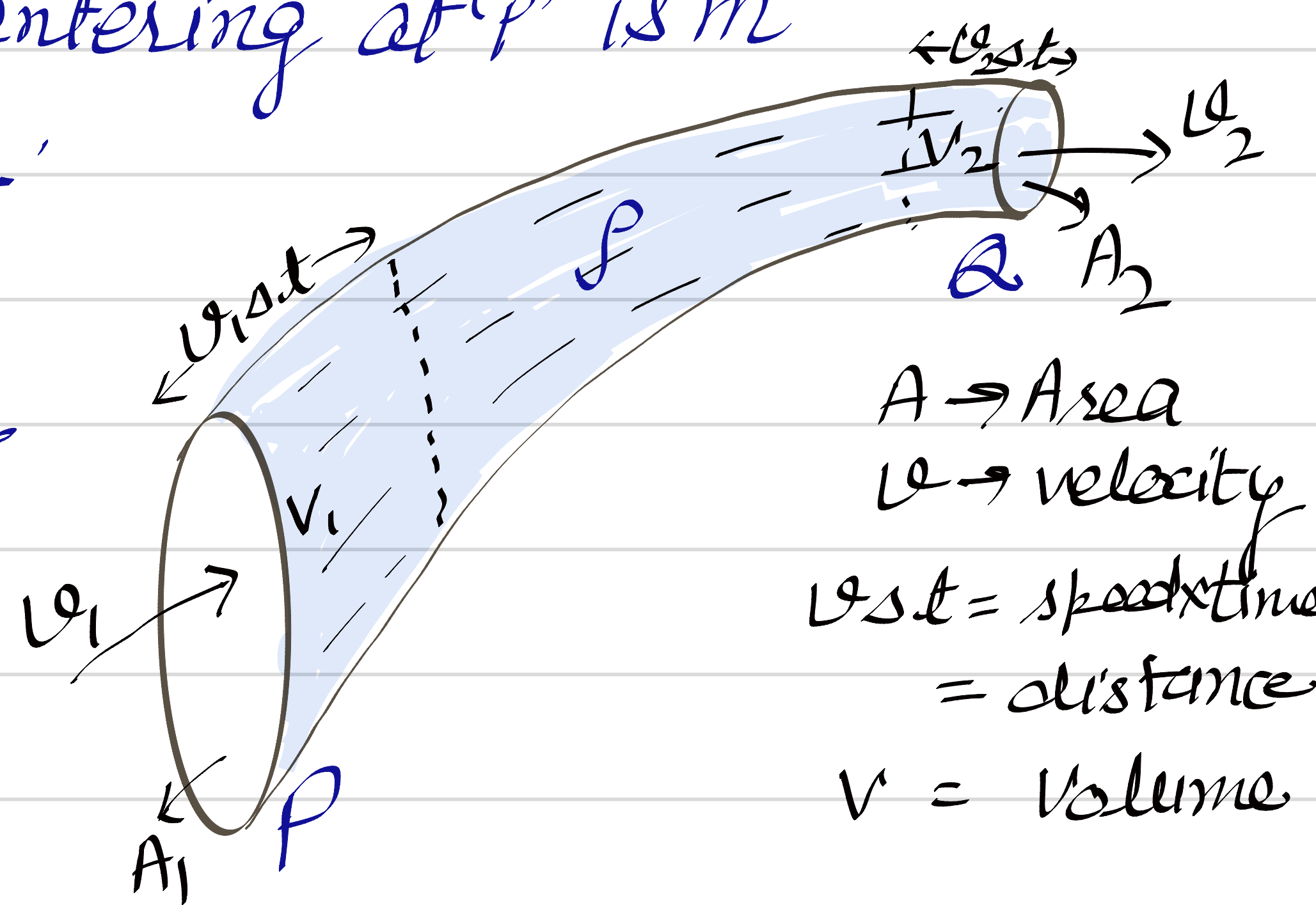
$$A_1 v_1 \Delta t = A_2 v_2 \Delta t$$

$$\text{or } \boxed{A_1 v_1 = A_2 v_2} \Rightarrow A v = \text{constant}$$

@Jyotisharmaphysics

i.e. if $A \uparrow$, $v \downarrow$ and vice versa

i.e. product of cross-section area and velocity is constant for steady and incompressible fluid flow.



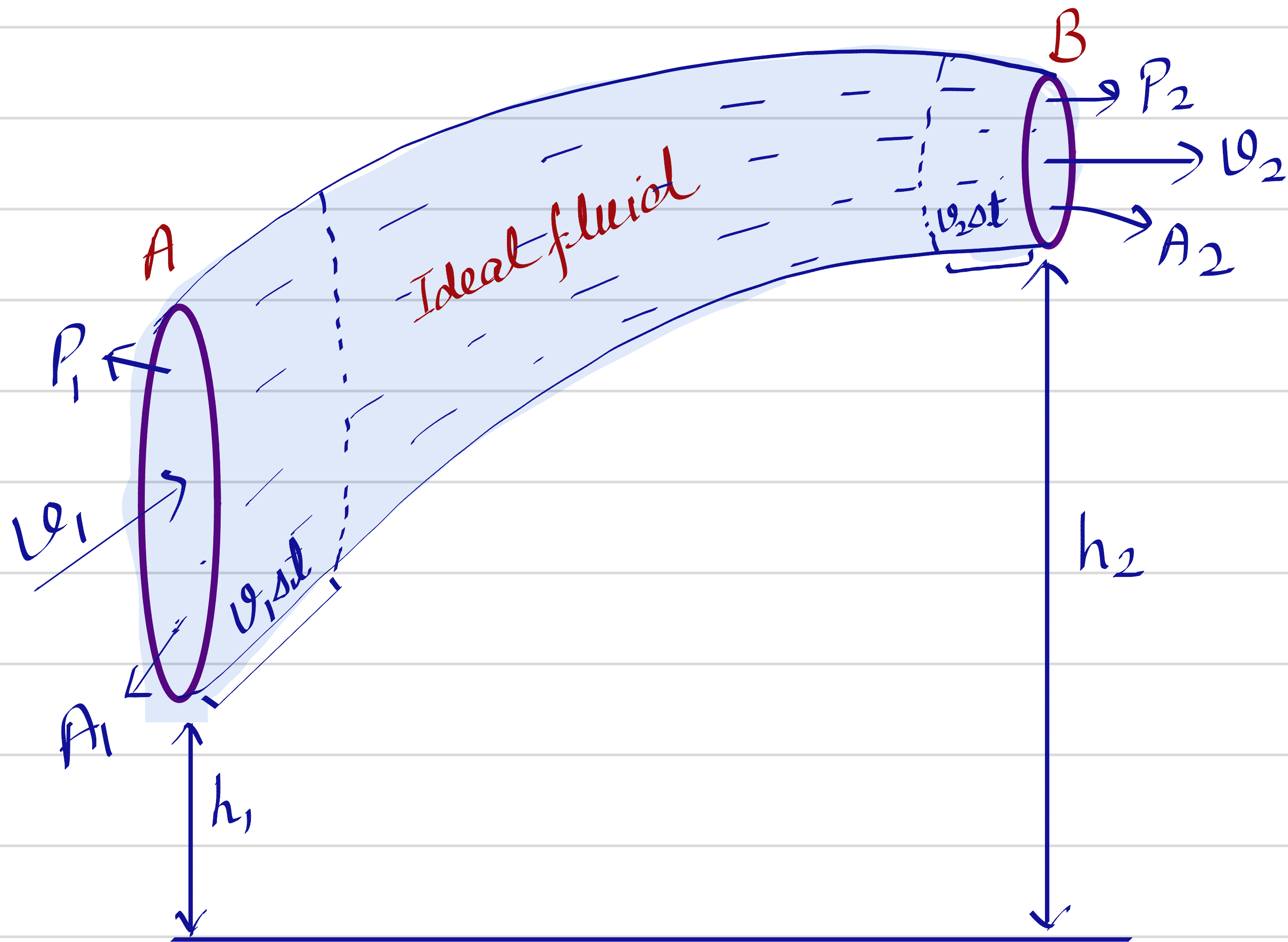
Q.6. State and prove Bernoulli's principle.

A. Bernoulli's Principle: It states that for an incompressible and non-viscous fluid in steady flow, the sum of pressure energy, kinetic energy per unit volume and

potential energy per unit volume remains constant along a streamline.

Mathematically,

$$P + \frac{1}{2} \rho v^2 + \rho gh = \text{Constant}$$



Proof: consider an ideal fluid in streamline
work done on the fluid at 'A'

$$W_1 = F \cdot d$$

$$= P_1 A_1 (v_1 \Delta t)$$

$$[F = PA, d = v_1 \Delta t]$$

$$W_1 = P_1 \Delta V$$

Since ΔV is same at both the ends 'A' and 'B',

work done by the fluid at 'B'

$$W_2 = -P_2 \Delta V \quad [\text{-ve sign show } W_2 \text{ is against the } P_2]$$

Total work done

$$\Delta W = W_1 + W_2$$

$$\Delta W = (P_1 - P_2) \Delta V \quad \text{--- (1)}$$

Now change in kinetic energy from 'A' to 'B'

$$\Delta K = \frac{1}{2} m (v_2^2 - v_1^2) \quad [v_2 > v_1]$$

Change in Potential energy from A' to B'

$$\Delta U = mg(h_2 - h_1) \quad [h_2 > h_1]$$

By conservation of energy

$$\Delta W = \Delta K + \Delta U$$

$$(P_1 - P_2) \Delta V = \frac{1}{2} m (v_2^2 - v_1^2) + mg(h_2 - h_1)$$

or

$$P_1 - P_2 = \frac{1}{2} \frac{m}{\Delta V} (v_2^2 - v_1^2) + \frac{m}{\Delta V} g(h_2 - h_1)$$

$$P_1 - P_2 = \frac{1}{2} \rho (v_2^2 - v_1^2) + \rho g(h_2 - h_1)$$

or

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$$

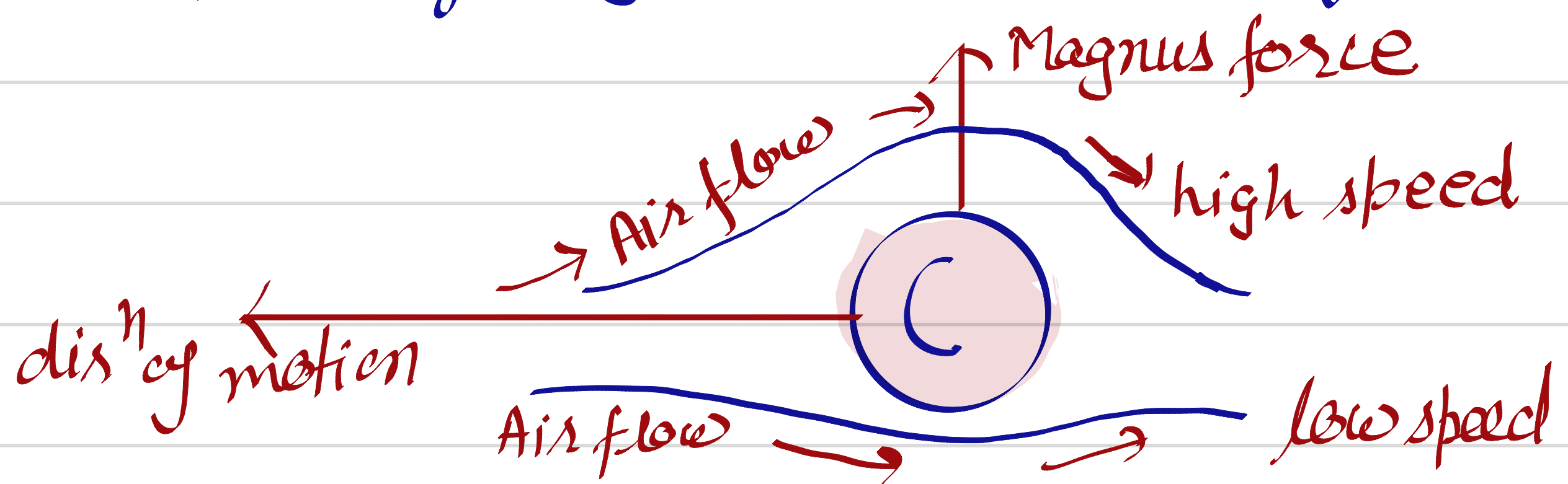
or

$$P + \frac{1}{2} \rho v^2 + \rho g h = \text{Constant}$$

Proved

Applications:

1. Airplane wings (Lift force): Due to the pressure difference above and below the wings generates lift.
2. Venturi meter: Used to measure the flow rate of fluids through a pipe.
3. Spray gun and Atomizers: Pressure difference create a fine spray.
4. Magnus effect: It is a phenomenon that occurs when a spinning object is moving through a fluid (gas or liquid). A lift force act on the spinning object and its path gets deflected.



Q.7. What is viscosity. Define the coefficient of viscosity. Derive its formula. How does the temperature affect viscosity?

A. Viscosity: Viscosity is the property of a fluid that measures its resistance to flow.

It arises due to internal friction between the layers of the moving fluid.

* A fluid with higher viscosity flows slower and with lower viscosity flows faster.

Coefficient of Viscosity (η)

Coefficient of viscosity is defined as the ratio of shear stress and shear strain rate.

$$\eta = \frac{\text{Shear Stress}}{\text{Shear strain rate}} = \frac{\text{Shear stress}}{\text{velocity gradient}}$$

since strain rate = velocity gradient = $\frac{\Delta v}{\Delta x}$
 then, $\eta = \frac{F/A}{\Delta v / \Delta x}$

SI unit is Pa-s or N-s/m²

CGS unit is Poise or dyne-s/m²

Dimension - [ML⁻¹T⁻¹]

According to Newton, A force of viscosity F acting tangentially between two layers is given as,

(1) $F \propto A$ [area of the layers in contact]

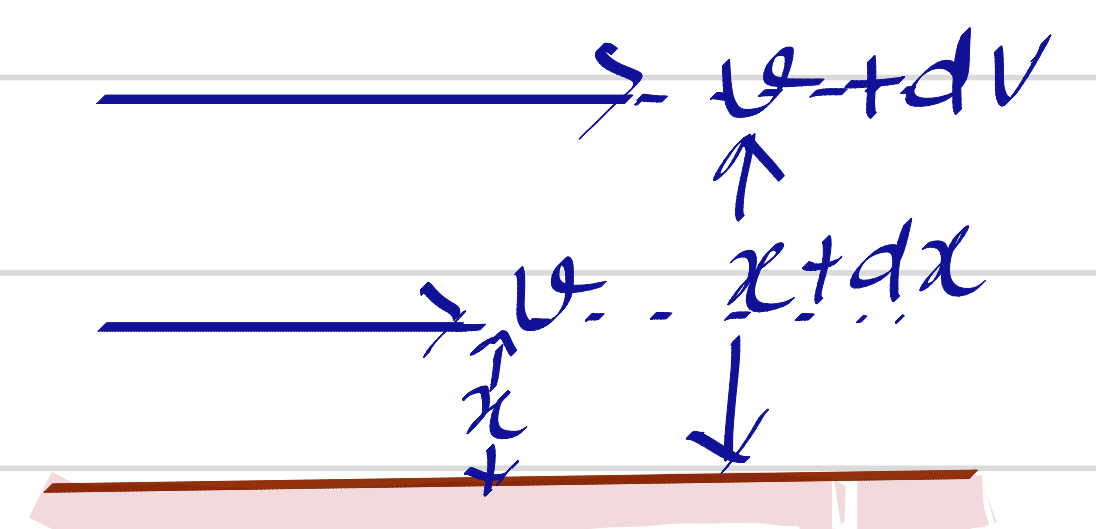
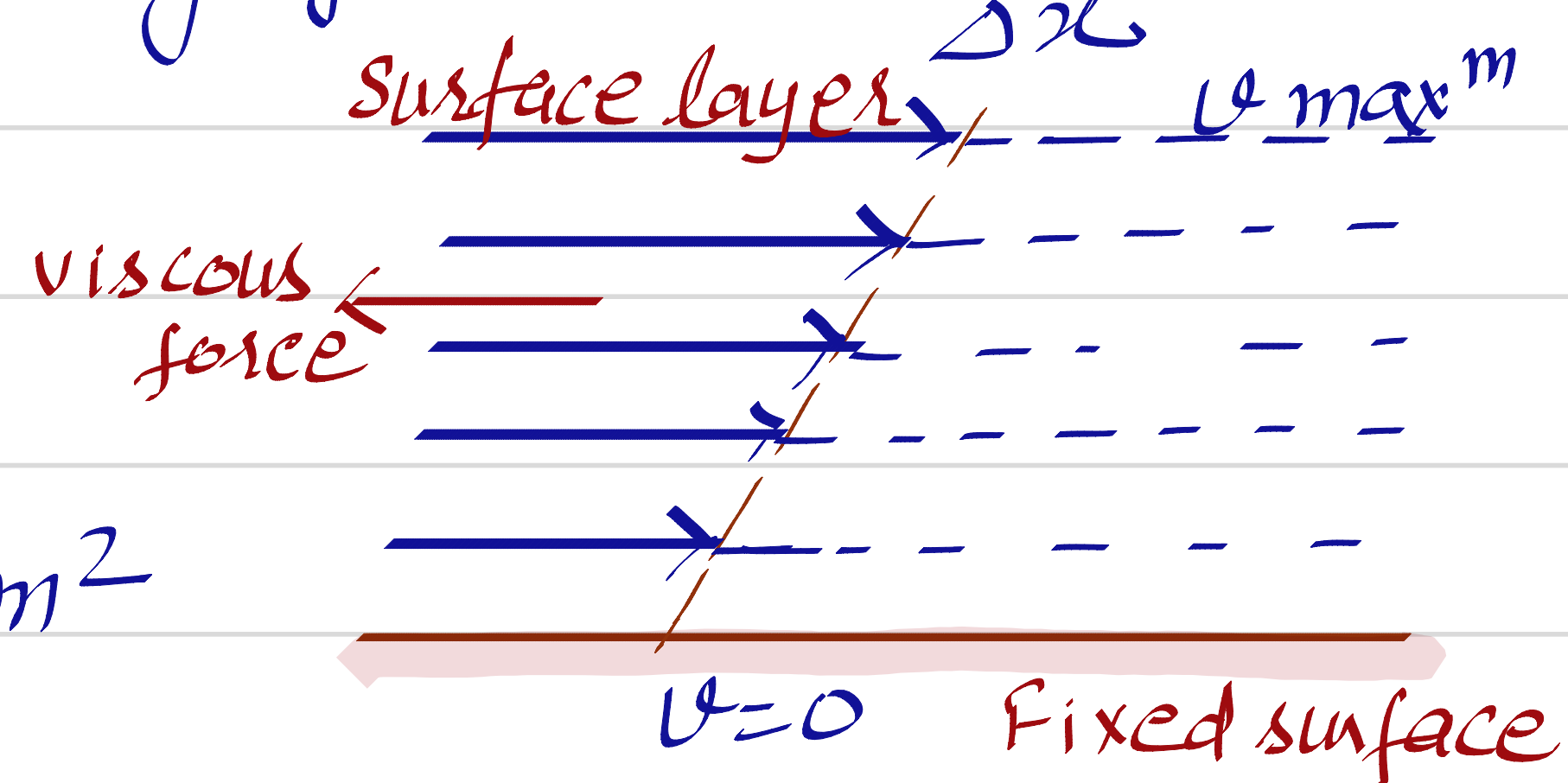
(2) $F \propto \frac{dv}{dx}$

from (1) and (2)

$$F \propto A \frac{dv}{dx}$$

or

$$F = -\eta A \frac{dv}{dx}$$



-ve sign shows the opposite direction of viscous force and motion of fluid.

If $A=1$ and $\frac{dv}{dt}=1$, then

$$F = \eta \quad (\text{numerically})$$

Hence coefficient of viscosity of a liquid may be defined as the tangential viscous force required to maintain a unit velocity gradient between its two parallel layers, each of unit area.

Q.8 State Stokes' law. Deduce Stokes' law by dimensional consideration.

A. Stokes' law: According to Stokes' law magnitude of viscous force (F) acting on a small spherical object moving through a viscous fluid is given by,

$$F \propto r \quad (\text{radius of the sphere})$$

$$F \propto v \quad (\text{velocity of sphere relative to fluid})$$

$$F \propto \eta \quad (\text{coefficient of viscosity})$$

i.e.,

$$F \propto \eta r v$$

or $F = 6\pi\eta r v$ 6π is constant here.

Formula derivation by dimensional method

$$\text{Let } F \propto \eta^a r^b v^c$$

$$\text{or } F = K \eta^a r^b v^c \quad \text{--- (1)}$$

$$[MLT^{-2}] = [ML^{-1}T^{-1}]^a [L]^b [LT^{-1}]^c$$

$$\text{or } [MLT^{-2}] = [M^a L^{-a+b+c} T^{-a-c}]$$

on comparing, the powers

For M

$$a = 1$$

For L, $-a + b + c = 1$

put $a = 1$

$-1 + b + c = 1$

or $b + c = 2$

For T, $-a - c = -2$

or $a + c = 2$

put $a = 1$, we get

$1 + c = 2$

or $\boxed{c = 1}$

then by $b + c = 2$

$b + 1 = 2$

or $\boxed{b = 1}$

so $a = 1$, $b = 1$ and $c = 1$

Now from eqⁿ (1)

$F = k \eta' r' v'$

put $k = 6\pi$

$\boxed{F = 6\pi\eta r v}$

Q. 9 Define terminal velocity and derive expression for it.

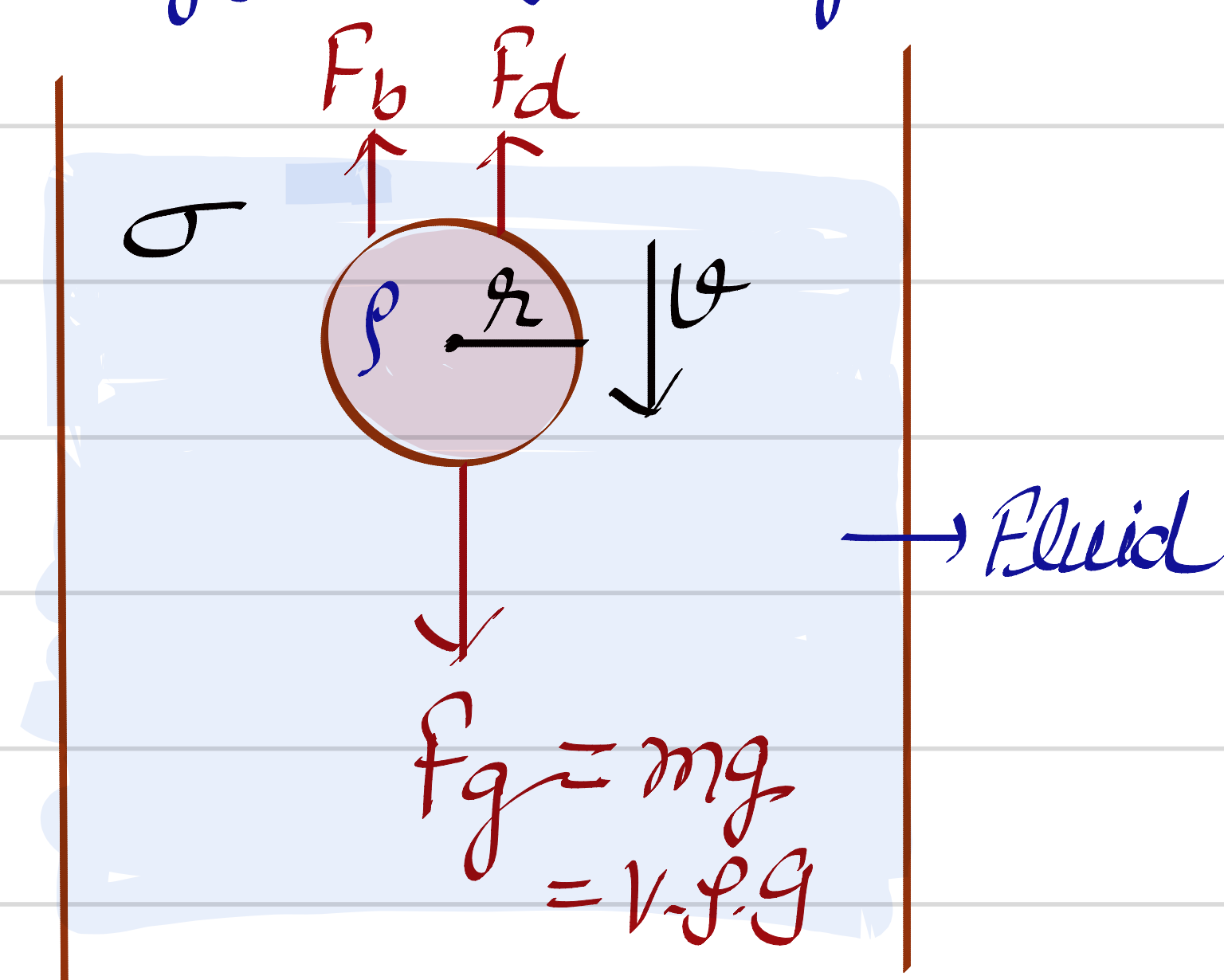
A. Terminal Velocity The maximum constant velocity acquired by a solid during its free fall in any fluid is known as terminal velocity. Consider a spherical body of density ρ is falling in a fluid of density σ . In equilibrium

$F_b + F_d = F_g$

$F_b \rightarrow$ Buoyant force, $F_d \rightarrow$ Drag force

here,

$F_b = \frac{4}{3} \pi r^3 \sigma g$



$$F_d = 6\pi\eta r v_t$$

$$F_g = \frac{4}{3}\pi r^3 \rho g \quad (\text{gravitational force})$$

Now by $F_b + F_d = F_g$

$$\frac{4}{3}\pi r^3 \sigma g + 6\pi\eta r v_t = \frac{4}{3}\pi r^3 \rho g$$

$$\text{or } 6\pi\eta r v_t = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g$$

$$\text{or } 6\pi\eta r v_t = \frac{4}{3}\pi r^3 (\rho - \sigma) g$$

$$\text{or } v_t = \frac{\frac{4}{3}\pi r^3 (\rho - \sigma) g}{6\pi\eta r}$$

$$v_t = \frac{2}{9} \frac{r^2 (\rho - \sigma) g}{\eta}$$

i.e

$$v_t \propto r^2$$

and

$$v_t \propto \frac{1}{\eta}$$

* Effect of temperature on viscosity -

(i) For liquids $\rightarrow$ on increasing temperature viscosity decreases.

(ii) For gas $\rightarrow$ on increasing temperature viscosity increases.

Q.10 Derive relation between surface tension and surface energy.

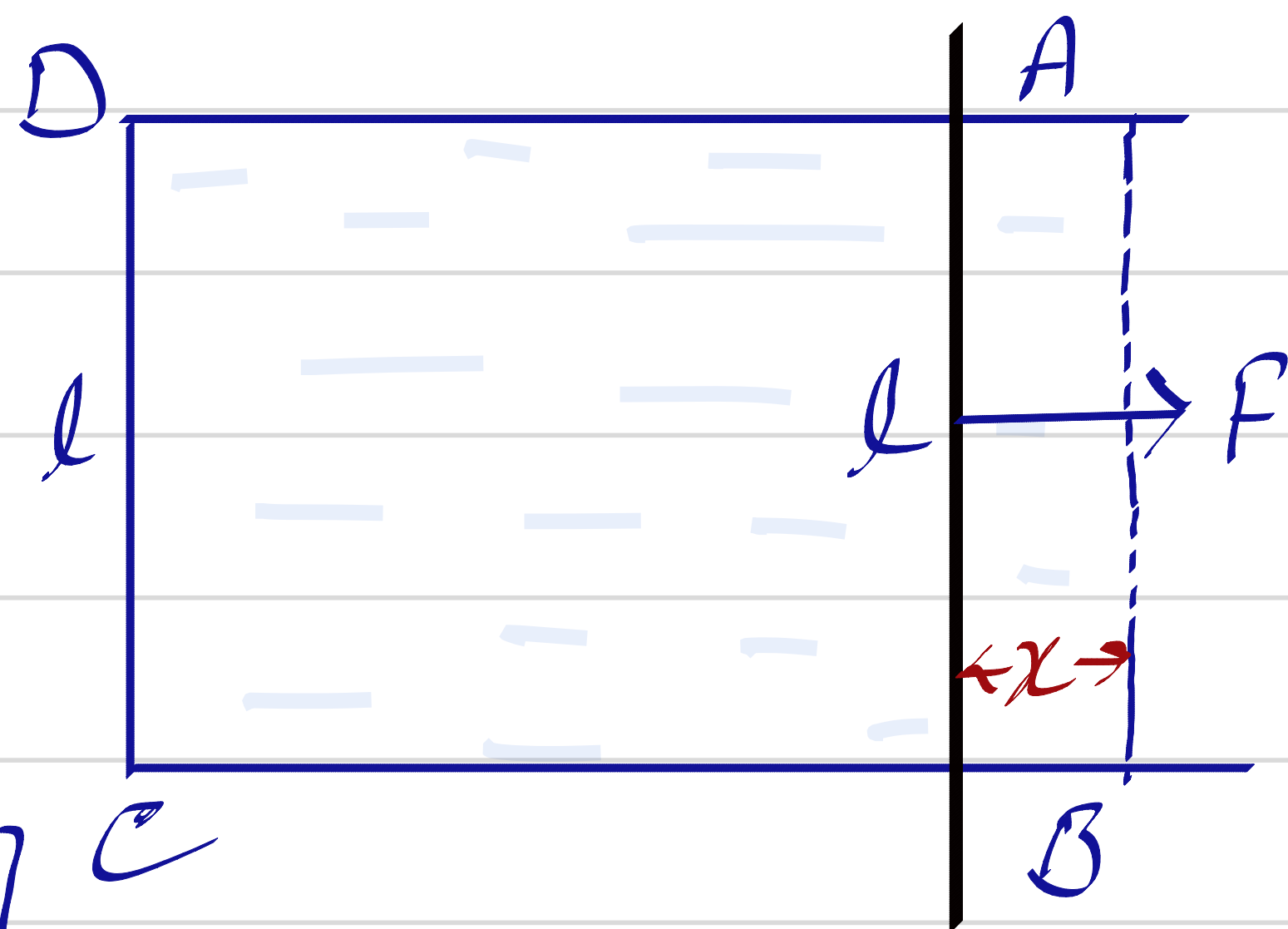
A. Consider a rectangular frame ABCD in which wire AB is movable. A film is formed in this frame and AB is pulled to increase the area of the film. Let x is the increased area.

Work done in increasing area 'x'

$$W = F \cdot x$$

$$= (2lS) \cdot x \quad \left[\because S = \frac{F}{2l} \right]$$

[For two free surfaces of film]
 $S = \frac{F}{2l}$, $S \rightarrow$ surface tension]



then $W = S(2lx)$
 $= S \times \text{increased area}$

or $W = S \Delta A$

This is the work done in increasing the film area.
Surface energy is the work done in increasing the unit area of the film.

$$\text{Surface energy} = \frac{\text{Work done}}{\text{increased area}}$$

$$= \frac{W}{\Delta A}$$

$$= \frac{S \Delta A}{\Delta A} \Rightarrow S$$

ie surface energy = Surface tension
Hence surface energy is numerically to the surface tension.

Q.11. Derive expression for the excess pressure;

- (i) Inside a liquid drop
- (ii) Inside a bubble in air
- (iii) Inside a bubble in liquid.

A. There is an excess pressure inside a liquid drop and bubbles due to the surface tension.

Expression:

(1) Inside a liquid drop:

Consider a liquid drop of radius R . The molecules lying on the surface of liquid drop will experience resultant force acting inward to the surface due to the surface tension.

Let there is a small increase in the radius (ΔR) of the drop.

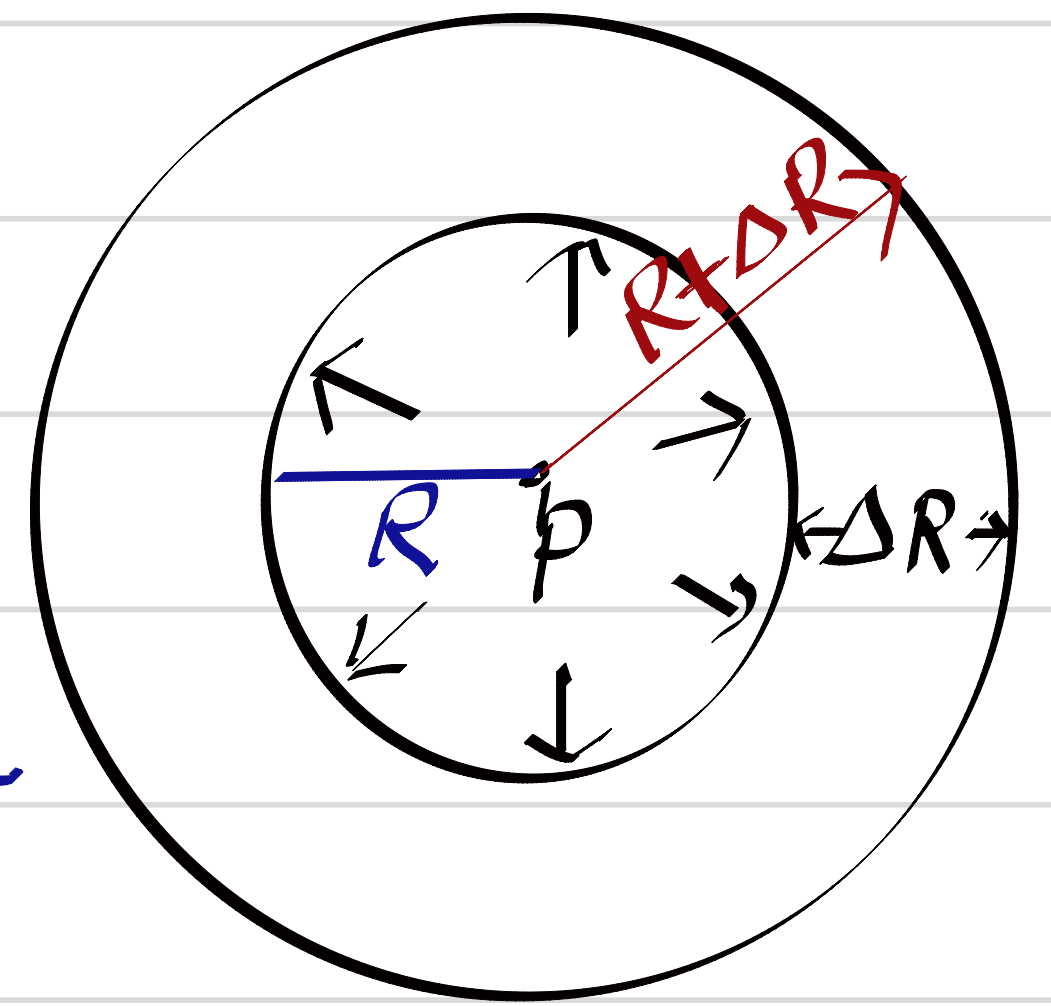
The work done

$$W = \text{Force} \times \text{increased radius}$$

$$= F \times \Delta R$$

$$= (PA) \Delta R \quad P \rightarrow \text{Excess pressure}$$

$$W = P \times 4\pi R^2 \Delta R \quad \text{---(1)}$$



Increase in the surface area of drop

$$\Delta A = \text{Final surface area} - \text{Initial surface area}$$

$$= 4\pi(R + \Delta R)^2 - 4\pi R^2$$

$$= 4\pi(R^2 + \Delta R^2 + 2R \cdot \Delta R) - 4\pi R^2$$

$$= 4\pi[R^2 + \Delta R^2 + 2R \cdot \Delta R - R^2]$$

neglect ΔR^2 as ΔR is very small, then

$$\Delta A = 4\pi(2R \Delta R)$$

$$\Delta A = 8\pi R \Delta R$$

Increase in surface energy = Work done

$$S \Delta A = W$$

$$S \times 8\pi R \Delta R = P \times 4\pi R^2 \Delta R \quad [\text{from eqn (1)}]$$

we get,

$$P = \frac{2S}{R}$$

This is the excess pressure inside a liquid drop.

(I) Inside a bubble in air

Consider a soap bubble of radius R .
Let ΔR is the increase in radius of bubble, then work done

$$\begin{aligned} W &= \text{Force} \times \text{increase in radius} \\ &= F \times \Delta R \\ &= (PA) \Delta R \end{aligned}$$

$$W = p \times 4\pi R^2 \Delta R \quad \text{---(1)} \quad [A = 4\pi R^2]$$

Now increase in area

$$\begin{aligned} \Delta A &= 2[4\pi(R + \Delta R)^2 - 4\pi R^2] \quad \left[\text{Bubble has 2 free surface} \right] \\ \Delta A &= 2 \times 4\pi [R^2 + 2R\Delta R + \Delta R^2 - R^2] \quad \left[\text{so multiply by 2} \right] \end{aligned}$$

$$\Delta A = 8\pi [R^2 + 2R\Delta R + \Delta R^2 - R^2]$$

neglect ΔR^2 as ΔR is very small, then

$$\Delta A = 8\pi (2R\Delta R)$$

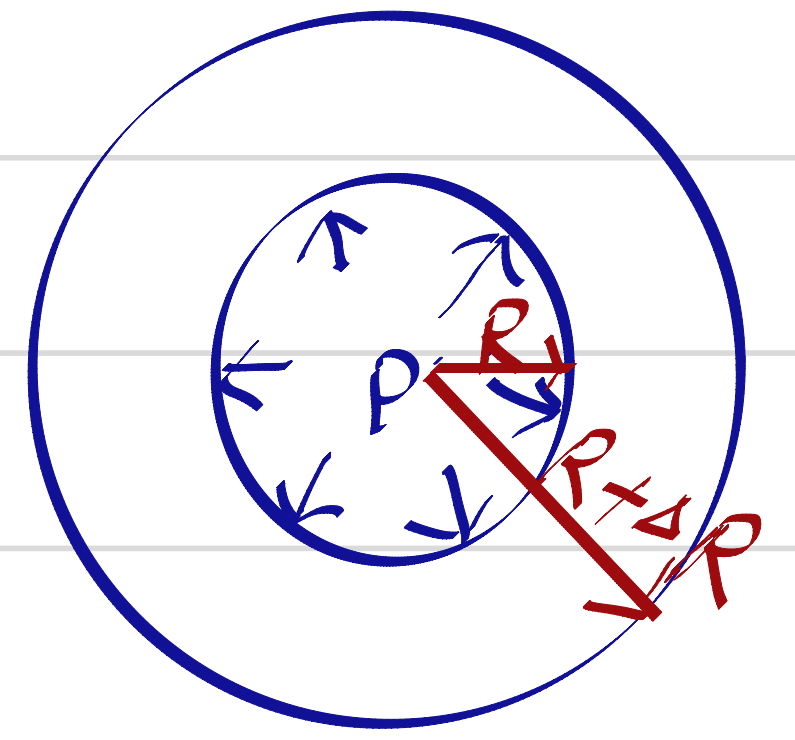
$$\Delta A = 16\pi R \Delta R$$

Now

Work done = Increase in surface energy

$$p \times 4\pi R^2 \Delta R = S \times 16\pi R \Delta R \quad \left[\begin{array}{l} W = p \times 4\pi R^2 \Delta R \\ SE = S \Delta A \end{array} \right]$$

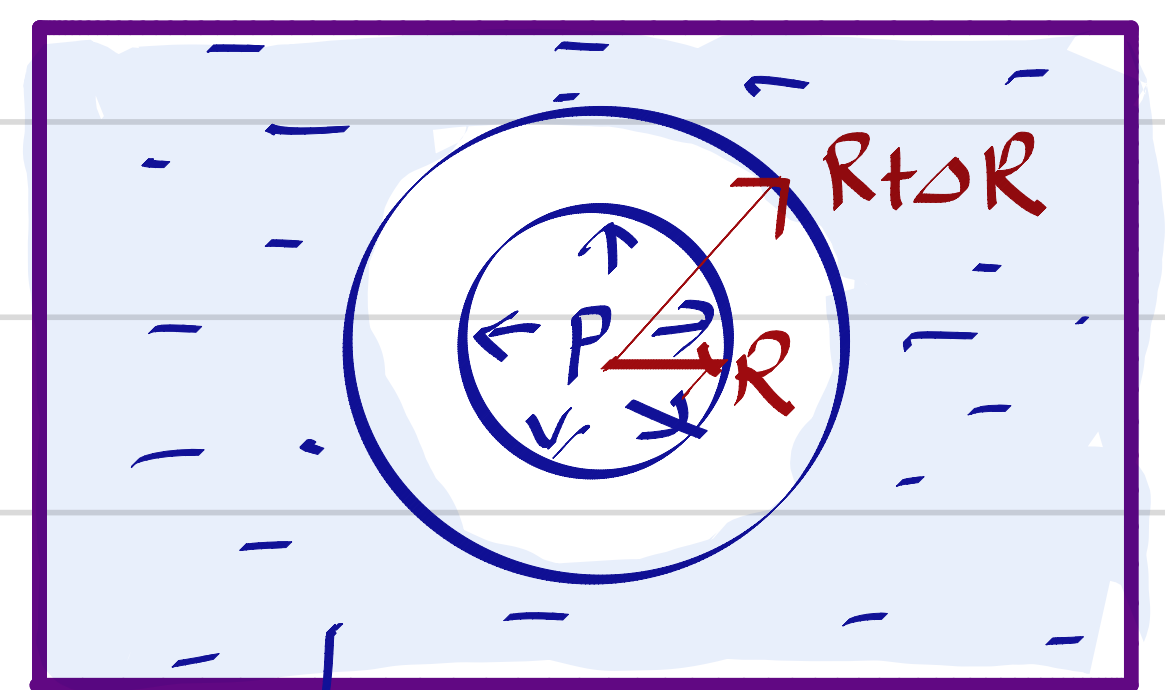
$$p = \frac{4S}{R}$$



(II) Inside a bubble in liquid:

Air bubble inside a liquid has only one free surface.
Therefore excess pressure inside the bubble

$$p = \frac{2S}{R}$$



* Derivation is same in part (i).

(For liquid drop)

OR

Alternative method

(i) Excess pressure inside a liquid drop

Suppose ABC is a half drop of a liquid of radius R.

Let F_1 is the cohesive force acting downwards,

$$F_1 = S \times l = S \times 2\pi R \quad [S \rightarrow \text{surface tension}]$$

Let excess pressure p is acting upwards

therefore upward force F_2 is

$$F_2 = pA = p(\pi R^2)$$

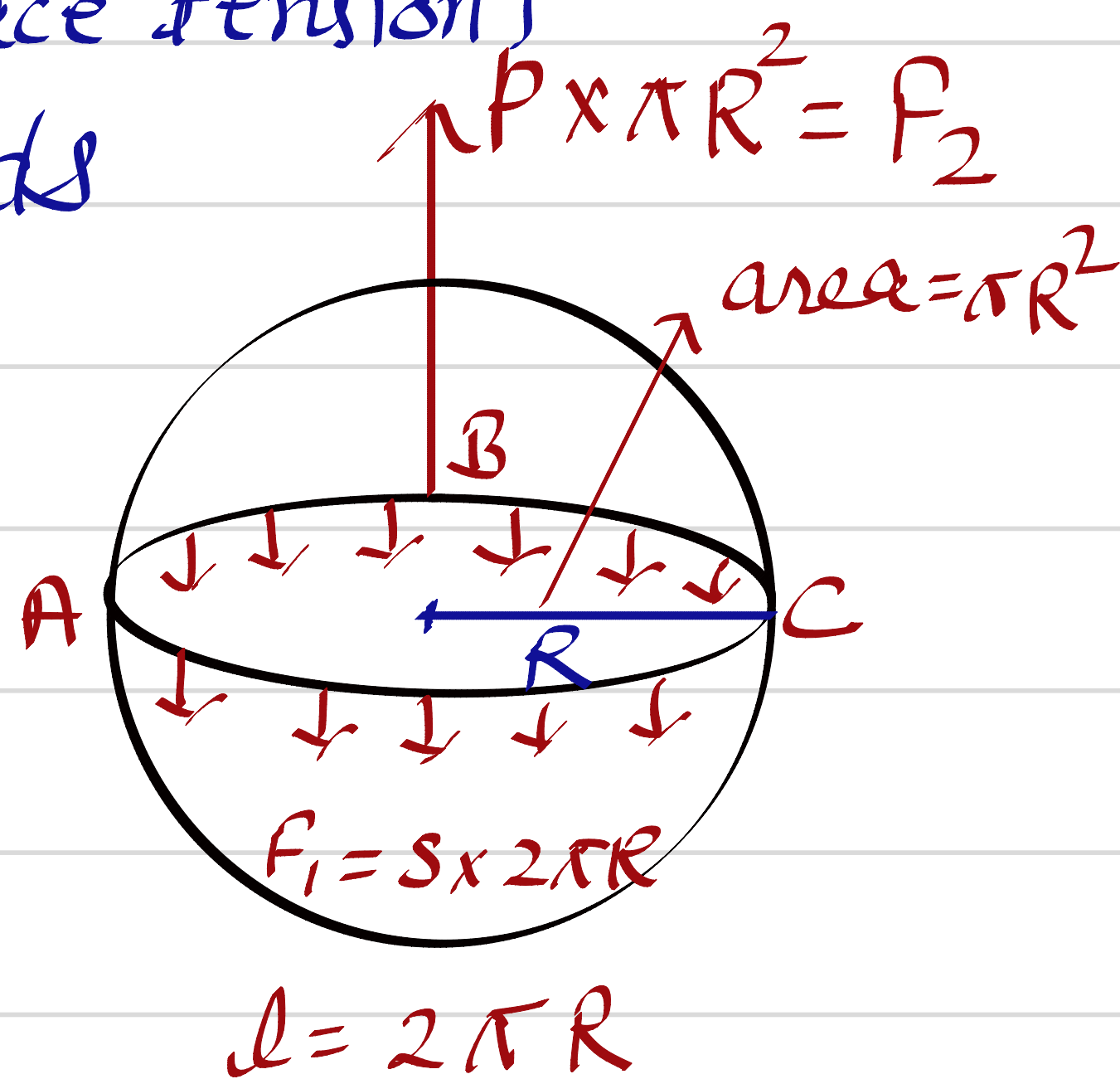
For the equilibrium of the drop

$$F_1 = F_2$$

$$S \times 2\pi R = p \times \pi R^2$$

or

$$p = \frac{2S}{R}$$



(ii) Excess pressure inside a bubble

Air bubble has two free surface therefore

$$F_1 = 2(S \times 2\pi R)$$

$$\text{and } F_2 = p \times \pi R^2$$

then, in equilibrium

$$F_1 = F_2$$

$$2(S \times 2\pi R) = p \times \pi R^2$$

or

$$p = \frac{4S}{R}$$

(iii) Excess pressure in a bubble inside a liquid

Air bubble inside a bubble has only one free surface, therefore

$$p = \frac{2S}{R}$$

Q.12 What is called capillarity? Derive ascent formula for capillary rise.

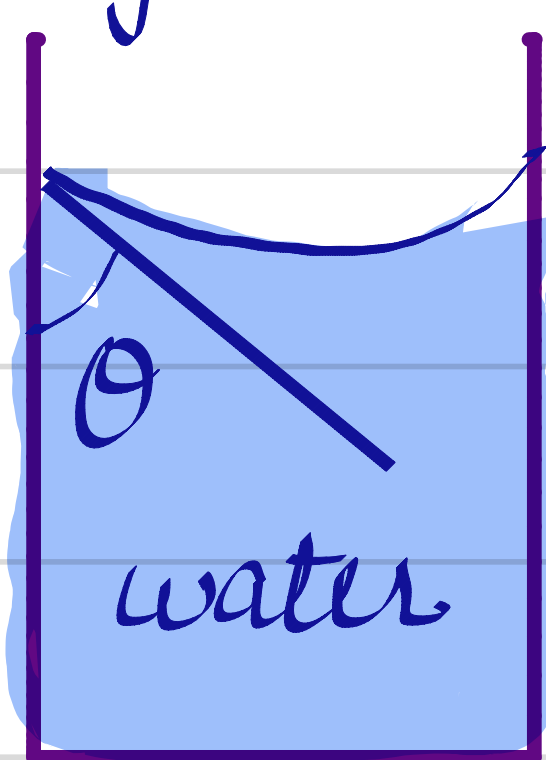
A. Capillarity: When a narrow tube is immersed in a liquid, the surface of the liquid in the tube rises or sinks. This phenomenon is called capillarity.



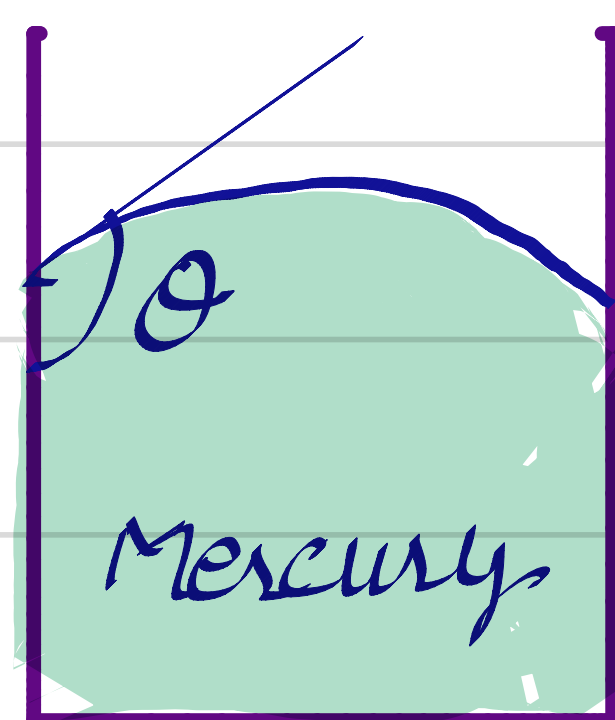
This phenomenon occurs due to properties of cohesiveness and adhesiveness.

Angle of contact: The angle between the tangent to the surface of the liquid at the point of contact and the wall of tube is called the angle of contact.

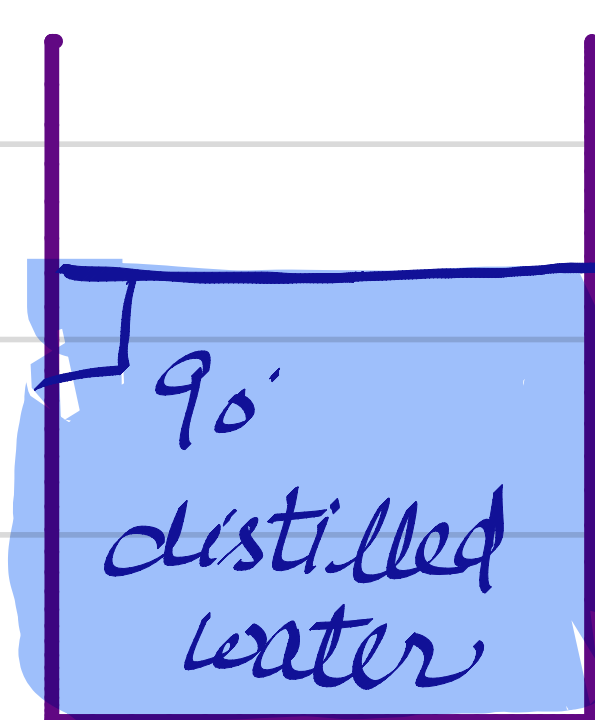
* Angle of contact for water, mercury and distilled water



Acute angle



Obtuse angle

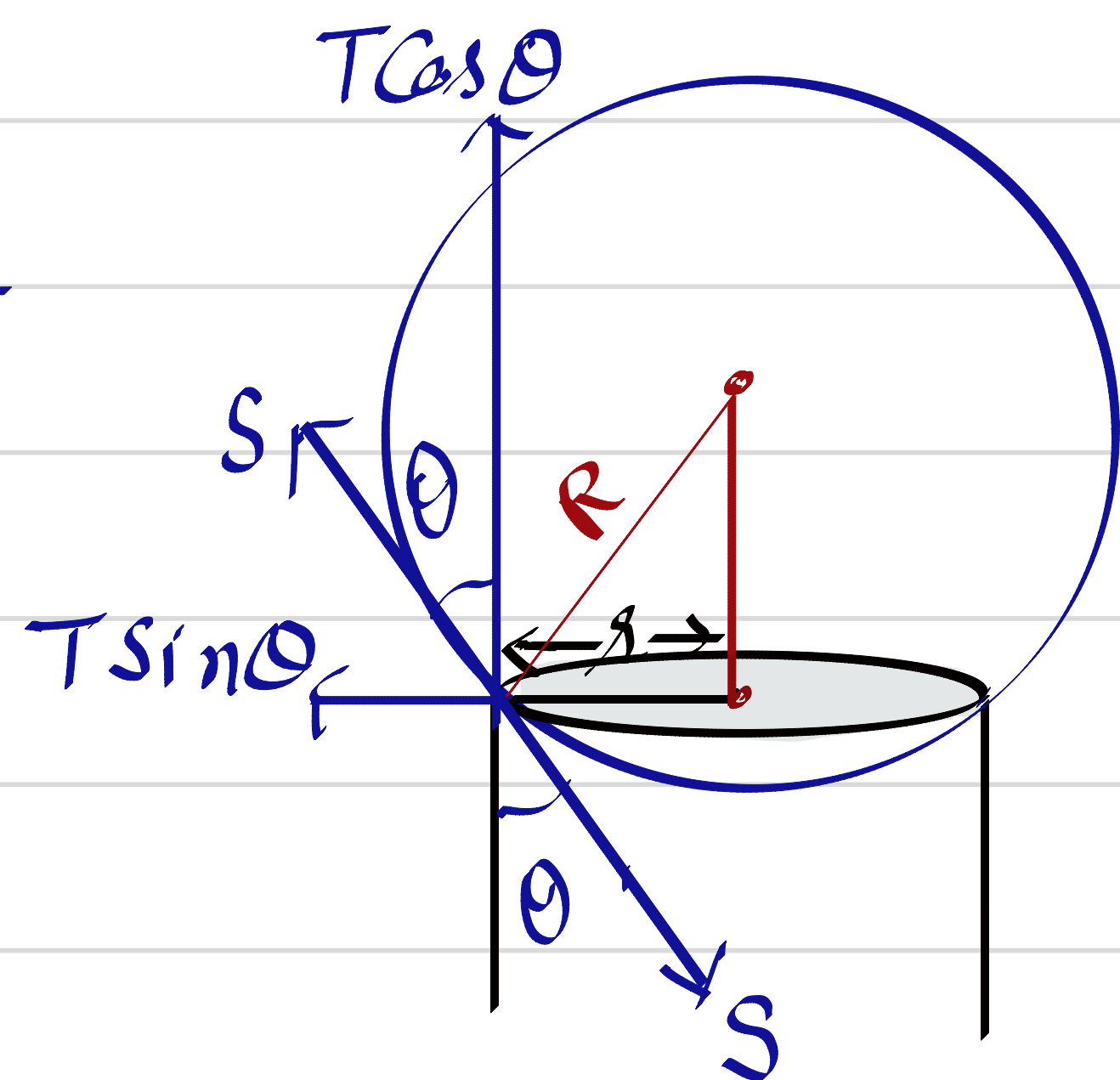
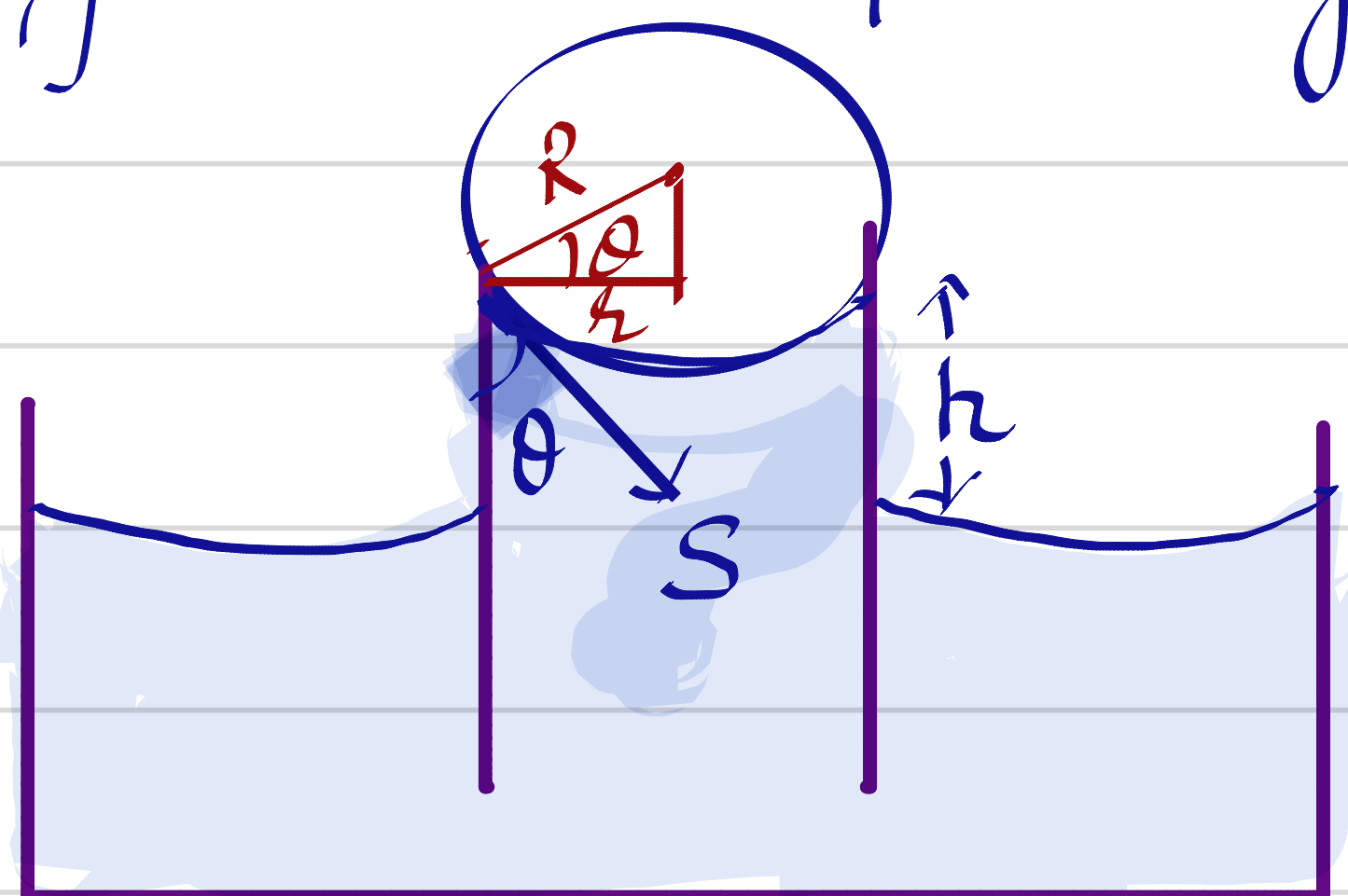


Right angle

Angle of contact (θ) depends on:

- (i) Nature of liquid and solid surface
- (ii) Medium above the liquid
- (iii) Temperature

Ascent formula: capillary rise



The surface tension S can be resolved into two parts

$$S_{\text{horizontal}} = S \sin \theta$$

$$S_{\text{vertical}} = S \cos \theta$$

All horizontal component ($S \sin \theta$) cancel out one another.

All vertical component ($S \cos \theta$) act upwards and

The total upward force

$$F = (S \cos \theta) \cdot l$$

$$[\because S = \frac{F}{l}]$$

$$\text{or } F = S \cos \theta \times 2\pi R$$

$$[l = 2\pi R]$$

Now weight of liquid

$$W = mg$$

$$= (V \cdot \rho)g$$

$$[m = V \cdot \rho]$$

$$W = \pi R^2 h \cdot \rho g$$

In equilibrium state

$$F = W$$

$$\text{i.e. } S \cos \theta \times 2\pi R = \pi R^2 h \rho g$$

or

$$h = \frac{2 S \cos \theta}{R \rho g}$$

This expression is known as Ascent formula.

* If capillary is of insufficient height, the liquid does not overflow. It increases the radius of meniscus to decrease the height.

$$h R = \text{Constant}$$

$R \rightarrow$ Radius of meniscus

$$h_1 R_1 = h_2 R_2$$

Q.13. What are the different units of temperature measurement. Write relation between them.

A. The three most commonly used units are

1. Celsius ($^{\circ}\text{C}$)
2. Fahrenheit ($^{\circ}\text{F}$)
3. Kelvin (K)

Relation:

1. Celsius and Kelvin

$$\text{K} = \text{C} + 273.15 \Rightarrow \boxed{\text{K} = \text{C} + 273}$$

$$\text{or } \boxed{\text{C} = \text{K} - 273}$$

2. Celsius and Fahrenheit

$$\boxed{\text{F} = \frac{9\text{C}}{5} + 32}$$

or

$$\boxed{\text{C} = \frac{5}{9} (\text{F} - 32)}$$

* Convert normal human temperature (98.6°F) into degree Celsius.

$$\text{A. } \text{C} = \frac{5}{9} (\text{F} - 32)$$

$$\text{C} = \frac{5}{9} (98.6 - 32) = \frac{5}{9} \times 66.6 = \underline{37^{\circ}\text{C}}$$

* At what temperature Celsius and Fahrenheit scales show the same value?

$$\text{A. } \text{C} = \frac{5}{9} (\text{F} - 32)$$

but $\text{C} = \text{F} = x$ (given)

$$x = \frac{5}{9} (x - 32)$$

$$9x = 5x - 160$$

$$\text{or } 4x = -160$$

$$\text{or } x = \frac{-160}{4} \Rightarrow x = -40$$

$$\text{so } \boxed{-40^{\circ}\text{C} = -40^{\circ}\text{F}}$$

Q.14. What is thermal expansion? Explain its types and defined the coefficient of thermal expansion. Give daily life examples for each type of expansion.

A. Thermal Expansion: When a substance is heated, it expands. This phenomenon is called thermal expansion.

Types of thermal expansions →

1. Linear Expansion: Expansion in one dimension (length)
e.g. length of wire, rod etc. increases on heating.

- (i) railway tracks expand in summer, so small gap are left between them to prevent bending.
- (ii) Sagging in power lines in summers.

2. Area Expansion: Expansion in two dimension
(surface area)

e.g. Area of sheet like structure increases on heating.

- (i) Glass window develop cracks when temperature increases suddenly.
- (ii) Metal plates/scales expand and precision of readings affected.

3. Volume Expansion: Expansion in three dimensions
(Entire volume increases)

e.g (1) Hot air balloons rises when air inside it heated.

(1) Petrol or diesel expands on hot days.

Coefficients of Thermal Expansions

1. Coefficient of linear expansion (α)

It is the fractional change in length per unit original length per degree change in temperature.

$$\frac{\Delta l}{l_0} = \alpha \Delta T$$



here $l_0 \rightarrow$ original length

$\Delta l \rightarrow$ increase in length

$\alpha \rightarrow$ coefficient of linear expansion

$\Delta T \rightarrow$ increase in temperature

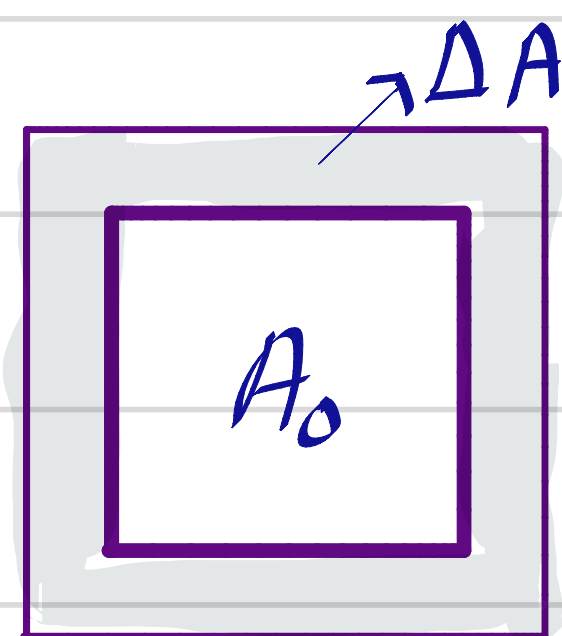
$$\Delta l = l_0 \alpha \Delta T$$

$$\text{or } l - l_0 = l_0 \alpha \Delta T \Rightarrow l = l_0 [1 + \alpha \Delta T]$$

2. Coefficient of area expansion (β)

It is the fractional change in area per unit original area per degree change in temperature.

$$\frac{\Delta A}{A_0} = \beta \Delta T$$



$\Delta A \rightarrow$ increase in area

$A_0 \rightarrow$ original area

$\beta \rightarrow$ coefficient of area expansion

$$\Delta A = A_0 \beta \Delta T$$

$$A - A_0 = A_0 \beta \Delta T \quad \text{or} \quad A = A_0 [1 + \beta \Delta T]$$

coefficient of volume expansion (γ)

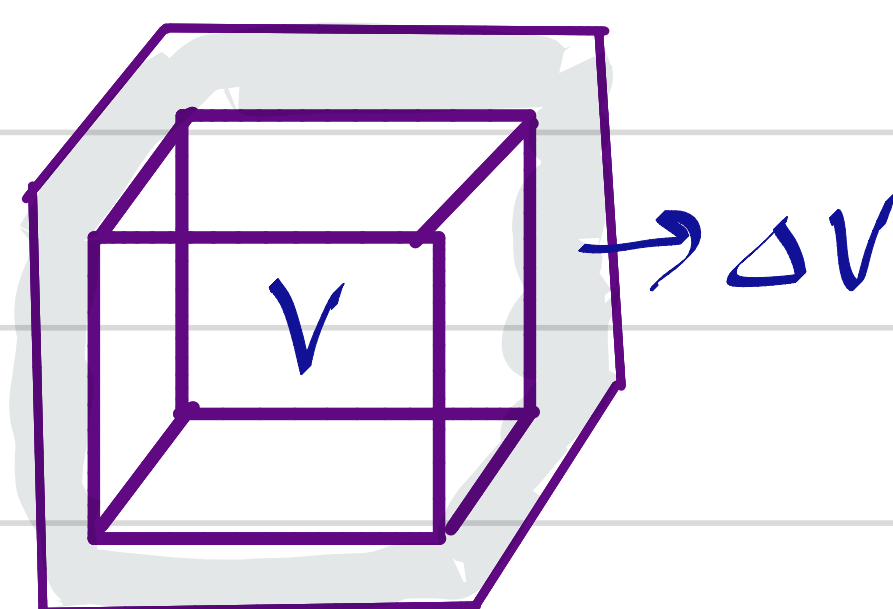
It is the fractional change in volume per unit original volume per degree change in temperature.

$$\frac{\Delta V}{V_0} = \gamma \Delta T$$

or $\Delta V = V_0 \gamma \Delta T$

or $V - V_0 = V_0 \gamma \Delta T$

or $V = V_0 [1 + \gamma \Delta T]$



here $V \rightarrow$ original volume

$\Delta V \rightarrow$ increase in volume

$\gamma \rightarrow$ coefficient of volume expansion

$\Delta T \rightarrow$ increase in temperature

* SI unit of α , β and γ is K^{-1} (per kelvin)
common unit $^{\circ}C^{-1}$ (per degree celsius)

Q. Derive expression between:

(i) α and β

(ii) α and γ

(iii) α , β and γ

A. (i) Relation between α and β :

Consider a square sheet of side l . Let the side is increased by Δl and becomes l' on increasing the temperature by ΔT . i.e.

$$l' = l + \Delta l$$

square both the sides

$$l'^2 = (l + \Delta l)^2$$

or $l'^2 = l^2 + \Delta l^2 + 2l\Delta l$

we neglect Δl^2 as Δl is very small.

Also, $l^2 = A$ and $l'^2 = A'$ [area of square]

$$A' = A + 2l(\alpha l \Delta T) \quad [\Delta l = \alpha l \Delta T]$$

$$A' - A = 2l^2 \alpha \Delta T$$

$$\text{or } \Delta A = 2A \alpha \Delta T$$

$$\text{or } \frac{\Delta A}{A \Delta T} = 2\alpha, \quad \text{here } \frac{\Delta A}{A \Delta T} = \beta$$

$$\text{or } \boxed{\beta = 2\alpha}$$

(i) Relation between α and Γ

consider a cube of side l which is increased by Δl on increasing the temperature by ΔT and becomes l' .

$$l' = l + \Delta l$$

$$l'^3 = (l + \Delta l)^3 \quad [\text{taking cube}]$$

$$\text{or } l'^3 = l^3 + \Delta l^3 + 3l \cdot \Delta l^2 + 3l^2 \Delta l$$

$$[(a+b)^3 = a^3 + b^3 + 3ab^2 + 3a^2b]$$

neglect Δl^2 and Δl^3 as Δl is very small, then

$$l'^3 = l^3 + 3l^2 \Delta l$$

$$V - V_0 = 3l^2 (\alpha l \Delta T) \quad [\because l^3 = V \text{ and } \Delta l = \alpha l \Delta T]$$

$$\text{or } \Delta V = 3l^3 \alpha \Delta T$$

$$\text{or } \Delta V = 3V \alpha \Delta T \quad [\because l^3 = V]$$

$$\text{or } \frac{\Delta V}{V \Delta T} = 3\alpha \quad \text{here } \frac{\Delta V}{V \Delta T} = \Gamma$$

$$\text{or } \boxed{\Gamma = 3\alpha}$$

(ii) Relation b/w α , β and Γ

We have

$$\beta = 2\alpha \Rightarrow \alpha = \frac{\beta}{2} \quad - (i)$$

$$\text{and } \Gamma = 3\alpha \Rightarrow \alpha = \frac{\Gamma}{3} \quad - (ii)$$

From (i) and (ii)

$$\alpha = \frac{\beta}{2} = \frac{r}{3}$$

or $6\alpha = 3\beta = 2r$ [multiply by 6]

the ratio of $\alpha : \beta : r = 1 : 2 : 3$

Q. What is conduction? Find the coefficient of thermal conductivity.

A. Conduction: Conduction is the process of flow of heat from one point to another through a substance without the actual transfer of the particle.

Coefficient of thermal conductivity (K)

Consider a cube of side 'x' and area of each face is 'A'. θ_1 and θ_2 are temperature of the two ends of cube and ($\theta_1 > \theta_2$).

The flow of heat depends upon-

$$Q \propto A \frac{d\theta}{dx} \times t$$

or $Q = K A \frac{d\theta}{dx} \times t$

[here $\frac{d\theta}{dx}$ is temp. gradient
(change in temp. per unit distance)]

K is constant and called coefficient of thermal conductivity.

If $A = 1 \text{ m}^2$, $x = 1 \text{ m}$, $t = 1 \text{ sec}$, $\Delta\theta = 1^\circ \text{C}$ then $Q = K$

Thus, the coefficient of thermal conductivity is defined as the amount of heat required to flow through a solid having unit area of cross-section per unit time when temperature gradient is unity.

Mathematically,

$$Q = \frac{K A \Delta T \cdot t}{L}$$

$\Delta T \rightarrow$ temp. difference, $L \rightarrow$ Thickness of material, $t \rightarrow$ time
Rate of heat flow (H)

$$H = \frac{Q}{t} = \frac{K A \Delta T}{L} = K A \frac{\Delta \theta}{dx}$$

SI unit of $K \rightarrow J S^{-1} m^{-1} K^{-1}$ or $W m^{-1} K^{-1}$

Q. State Newton's law of cooling. Derive mathematical expression for Newton's law of cooling.

A. Newton's law of cooling: The rate of heat loss of a body is directly proportional to the temperature difference between the body and the surrounding medium provided small temperature difference.

Mathematically, rate of heat loss

$$\frac{dQ}{dt} \propto (T - T_s)$$

$T \rightarrow$ temp. of body

$T_s \rightarrow$ temp. of surroundings

$$\text{or } \frac{dQ}{dt} = -k(T - T_s)$$

here k is proportionality constant

Derivation

From definition

$$\frac{dQ}{dt} = -k(T - T_s) \quad \text{--- (1)}$$

$$\text{By } dQ = m s dT$$

$$\text{or } \frac{dQ}{dt} = m s \frac{dT}{dt} \quad \text{--- (2)}$$

$m \rightarrow$ mass of the body
 $s \rightarrow$ specific heat
 $dT \rightarrow$ small change in temp.

from eqⁿ (1) and (2)

$$ms \frac{dT}{dt} = -K(T-T_s)$$

$$\text{or } \frac{dT}{(T-T_s)} = \frac{-K}{ms} dt$$

put $\frac{K}{ms} = K$, then

$$\text{or } \frac{dT}{(T-T_s)} = -K dt$$

on integrating both the sides

$$\int \frac{dT}{(T-T_s)} = -\int K dt$$

$$\log_e(T-T_s) = -Kt + C$$

$$\text{or } (T-T_s) = e^{-Kt+C}$$

$C \rightarrow$ integration constant

Taking exponentials

$$T-T_s = C e^{-Kt}$$

At $t=0$ temperature is T_0 , then $T_0-T_s = C e^0$

$$\text{or } T_0-T_s = C$$

$$\text{so } \boxed{T-T_s = (T_0-T_s) e^{-Kt}}$$

This eqⁿ shows that temperature of body decreases exponentially with time

* For numericals use

$$\frac{T_1-T_2}{t} = K \left[\frac{T_1+T_2}{2} - T_s \right]$$

$T_1 \rightarrow$ initial temp.

$T_2 \rightarrow$ temp. after time t

$T_s \rightarrow$ temp. of surroundings

$K \rightarrow$ constant

$$\ln x = \log_e x \\ = 2.303 \log_{10} x$$

Q. Derive relation between C_p and C_v .
OR

Derive Mayer's formula.

A. Consider 1 mole of an ideal gas. Gas is heated to raise the temperature by ΔT .

By first law of thermodynamics

$$\Delta Q = \Delta U + P\Delta V \quad [\Delta W = P\Delta V]$$

Let the heat ΔQ is absorbed at constant volume, then

$$\Delta V = 0 \text{ and we get}$$

$$\Delta Q = \Delta U$$

$$\text{and } \Delta Q = C_v \Delta T \quad [\text{for 1 mole of gas } n=1]$$

$$\text{so } \Delta U = C_v \Delta T \quad \text{--- (I)}$$

Now at constant pressure.

$$\Delta Q = \Delta U + P\Delta V$$

$$C_p \Delta T = \Delta U + P\Delta V \quad \text{--- (II)}$$

Internal energy is same in both the cases because it depends on temperature only.

Thus from eqⁿ (I) and (II), we get

$$C_p \Delta T = C_v \Delta T + P\Delta V$$

$$\text{put } P\Delta V = R\Delta T \quad [\text{Ideal gas eqⁿ}]$$

$$C_p \Delta T = C_v \Delta T + R\Delta T$$

$$\text{OR } C_p = C_v + R$$

$$\text{OR } \boxed{C_p - C_v = R}$$

Proved

Q. Derive an expression for work done by a gas in an isothermal reversible expansion.

A. Work done in isothermal expansion:

Consider n mole of an ideal gas contained in a conducting wall cylinder with movable piston as shown in fig.

Now work done by the gas

$$dW = PA dx$$

$$= PdV$$

Suppose gas expands isothermally from V_1 to V_2 . The total work done is given by:

$$W_{iso} = \int_{V_1}^{V_2} PdV$$

by $PV = nRT$, $P = \frac{nRT}{V}$, then

$$\begin{aligned} W_{iso} &= \int_{V_1}^{V_2} \frac{nRT}{V} dV \\ &= nRT \int_{V_1}^{V_2} \frac{1}{V} dV \end{aligned}$$

$$= nRT [\log_e V]_{V_1}^{V_2}$$

$$\left[\int \frac{1}{x} dx = \log_e x \right]$$

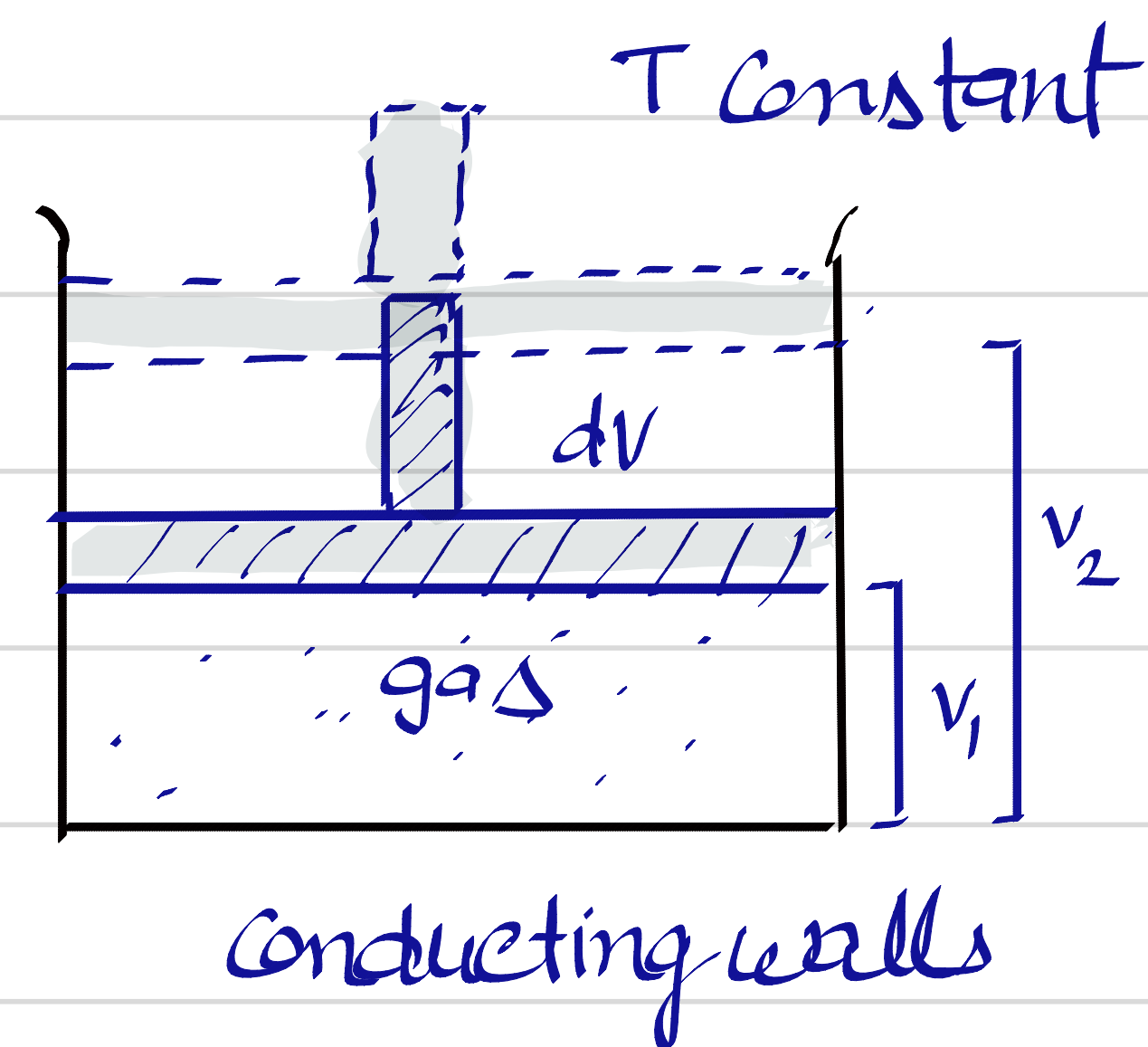
$$= nRT [\log_e V_2 - \log_e V_1]$$

$$W_{iso} = nRT \log_e \frac{V_2}{V_1}$$

$$[\log m - \log n = \log \frac{m}{n}]$$

$$W_{iso} = 2.303 nRT \log_{10} \frac{V_2}{V_1}$$

$$\log_e x = 2.303 \log_{10} x$$



* For isothermal process $PV = \text{constant}$

* By $\Delta Q = \Delta U + \Delta W$, $\Delta U = 0$ as T is constant. i.e.

$\Delta Q = \Delta W$ [Internal energy depends on temp. only]

Q. Derive expression for work done by a gas during adiabatic process.

A Work done in adiabatic expansion:

Consider n moles of an ideal gas contained in an insulating wall cylinder.

Let the piston moves up through a small distance dx , the work done by the gas

$$W_{\text{adia}} = \int_{V_1}^{V_2} P dV$$

For adiabatic change

$$PV^{\gamma} = K \quad \text{or} \quad P = \frac{K}{V^{\gamma}} = KV^{-\gamma}$$

Then $W = \int_{V_1}^{V_2} KV^{-\gamma} dV$

$$= \frac{K}{-\gamma+1} \left[V^{-\gamma+1} \right]_{V_1}^{V_2} \quad \left[\int x^n dx = \frac{x^{n+1}}{n+1} \right]$$

$$= \frac{K}{1-\gamma} \left[V_2^{-\gamma+1} - V_1^{-\gamma+1} \right]$$

$$= \frac{K}{1-\gamma} \left[V_2^{-\gamma} \cdot V_2 - V_1^{-\gamma} \cdot V_1 \right]$$

$$= \frac{1}{1-\gamma} \left[KV_2^{-\gamma} \cdot V_2 - KV_1^{-\gamma} \cdot V_1 \right]$$

but $P_1 V_1^{\gamma} = P_2 V_2^{\gamma} = K$ so $KV_1^{-\gamma} = P_1$ and $KV_2^{-\gamma} = P_2$
then

$$W = \frac{1}{1-\gamma} \left[P_2 V_2 - P_1 V_1 \right]$$

by $PV = nRT$

$$W = \frac{1}{1-\gamma} \left[nRT_2 - nRT_1 \right]$$

i.e

$$W = \frac{nR}{1-\gamma} [T_2 - T_1] = \frac{nR}{\gamma-1} [T_1 - T_2]$$

Q. Find expression for net work done by a Carnot engine.

A. Carnot Engine: A Carnot engine is a theoretical engine that operates on the Carnot cycle.

In Carnot engine cycle

Step 1 Isothermal Expansion

Work done by the gas

$$Q_1 = W_1 = 2.303 \log \frac{V_2}{V_1} \quad - (1)$$

$Q_1 \rightarrow$ heat gained

Step 2. Adiabatic expansion

The work done

$$W_2 = \frac{nR}{\gamma-1} [T_1 - T_2] \quad - (2)$$

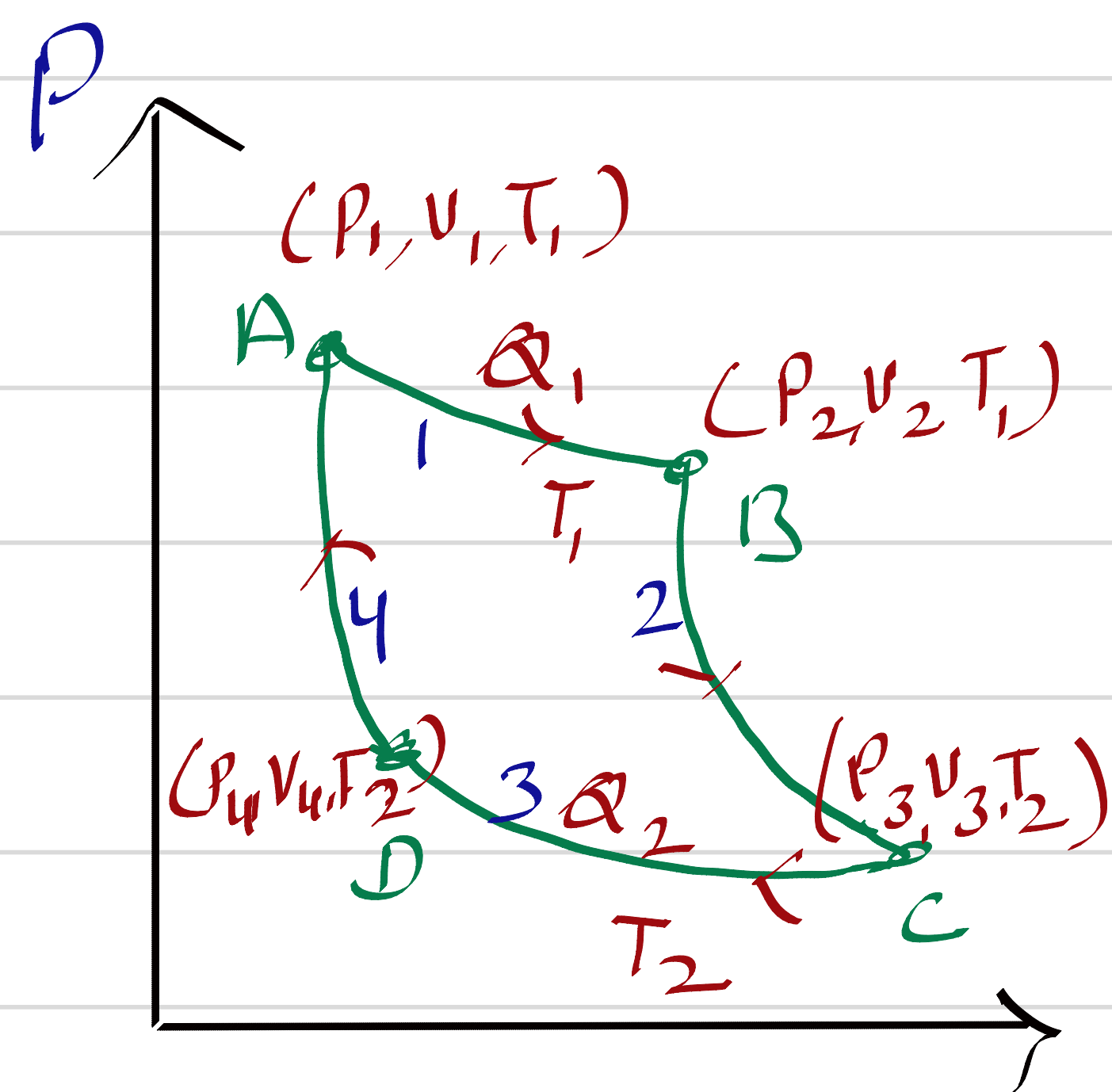
Step 3. Isothermal compression

$$Q_2 = W_3 = 2.303 \log \frac{V_3}{V_4}$$

$Q_2 =$ heat lost

Step 4 - Adiabatic compression

$$W_4 = \frac{nR}{\gamma-1} [T_1 - T_2] \quad - (4)$$



Net work done

$$W_{\text{net}} = W_1 + W_2 - (W_3 + W_4) \quad [-ve \text{ sign} \rightarrow \text{for compression}]$$

$$W_{\text{net}} = W_1 - W_3 \quad [\because W_2 = W_4]$$

or

$$W_{\text{net}} = Q_1 - Q_2$$

or

$$W_{\text{net}} = nRT_1 \log e \frac{V_2}{V_1} - nRT_2 \log e \frac{V_3}{V_4}$$

Efficiency of Carnot engine (η)

$$\eta = \frac{W}{Q_1}$$

$$\eta = \frac{Q_1 - Q_2}{Q_1}$$

$$= 1 - \frac{Q_2}{Q_1}$$

For A to B (Isothermal process)

$$P_1 V_1 = P_2 V_2 \quad \text{--- (1)}$$

For B to C (Adiabatic process)

$$P_2 V_2^r = P_3 V_3^r \quad \text{--- (2)}$$

Again for C to D (Isothermal process)

$$P_3 V_3 = P_4 V_4 \quad \text{--- (3)}$$

Finally D to A (Adiabatic process)

$$P_4 V_4^r = P_1 V_1^r \quad \text{--- (4)}$$

On multiplying (1), (2), (3) and (4)

$$P_1 V_1 P_2 V_2^r P_3 V_3 P_4 V_4^r = P_2 V_2 P_3 V_3^r P_4 V_4 P_1 V_1^r$$

$$\text{or } V_1 V_2^r V_3 V_4^r = V_2 V_3^r V_4 V_1^r$$

$$\text{or } \frac{V_2^r}{V_2} \frac{V_4^r}{V_4} = \frac{V_3^r}{V_3} \frac{V_1^r}{V_1}$$

$$\text{or } V_2^{r-1} \cdot V_4^{r-1} = V_3^{r-1} V_1^{r-1}$$

$$\text{or } (V_2 V_4)^{r-1} = (V_3 V_1)^{r-1}$$

$$\text{or } V_2 V_4 = V_3 V_1$$

$$\text{or } \frac{V_2}{V_1} = \frac{V_3}{V_4} \quad \text{--- (5)}$$

$$\text{Now } \frac{Q_2}{Q_1} = \frac{R T_2 \log_e V_3/V_4}{R T_1 \log_e V_2/V_1} = \frac{T_2 \log_e V_3/V_4}{T_1 \log_e V_2/V_1} \quad [\text{from eq (5)}]$$

$$\text{or } \frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

$$\text{Thus, } \boxed{\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}}$$

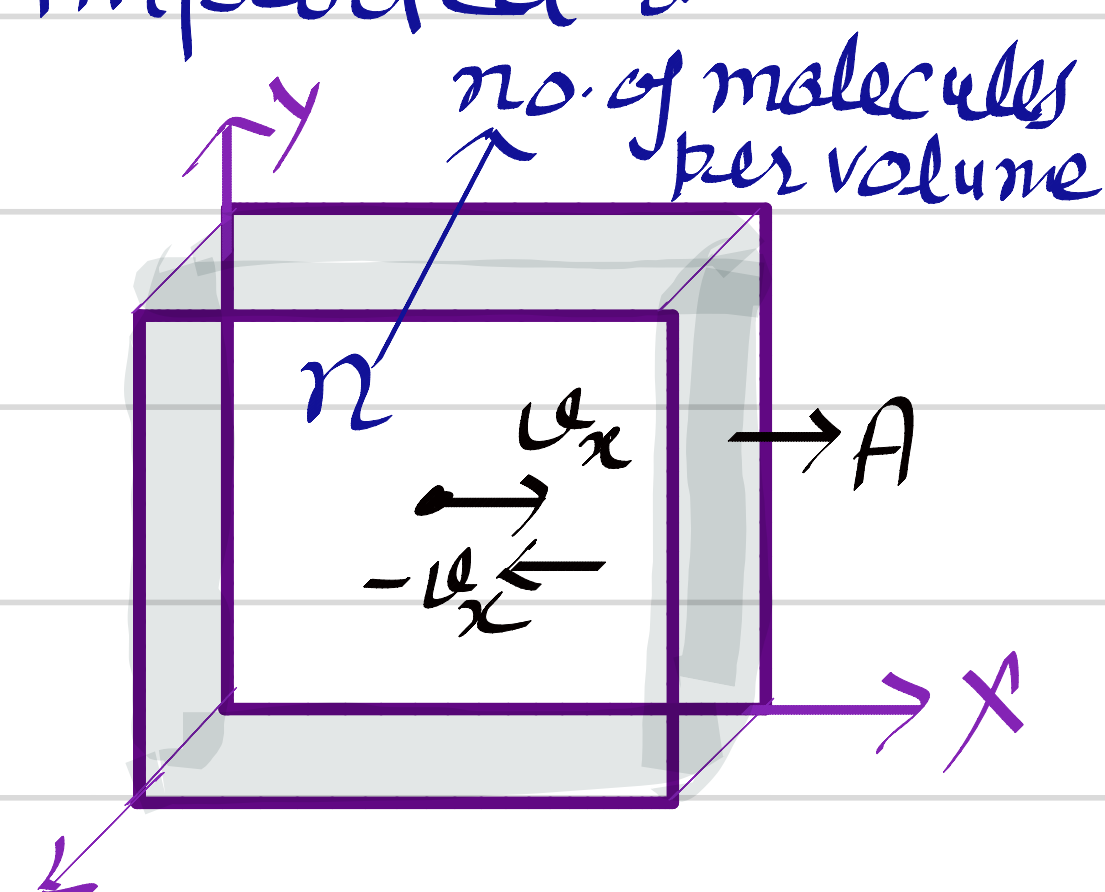
* η depends on the temperatures of source and sink.

Q. Derive expression for pressure exerted by a gas.

A. Consider an ideal gas enclosed in a cubical vessel. Let a molecule moving with velocity (u_x, u_y, u_z) hits the wall perpendicular to the axis of area A and rebound with the same velocity (on the basis of kinetic theory).
The change in momentum of the molecule = $-mu_x - mu_x$
 $= -2mu_x$

By the conservation of momentum, momentum imparted to the wall = $-(-2mu_x) = 2mu_x$

Number of molecules hitting the wall of area A in time $\Delta t = \frac{1}{2} A u_x \Delta t \times n$



Total momentum transferred to the wall

$$\Delta p = 2mu_x \times \frac{1}{2} A u_x \Delta t \times n$$

$$\Delta p = mn u_x^2 A \Delta t$$

Now force on wall,

$$F = \frac{\Delta p}{\Delta t} = \frac{mn u_x^2 A \Delta t}{\Delta t}$$

$$F = mn u_x^2 A$$

$$\text{Pressure on the wall } P = \frac{\text{Force}}{\text{Area}} = \frac{mn u_x^2 A}{A}$$

$$\text{or } P = mn u_x^2$$

Replace u_x^2 by $\bar{u}_x^2$ ($\bar{u}_x$ is average velocity)

$$P = mn \bar{u}_x^2$$

As the gas is isotropic, $\bar{u}_x^2 = \bar{u}_y^2 = \bar{u}_z^2 = \frac{1}{3} \bar{u}^2$

Hence

$$P = \frac{1}{3} mn \bar{u}^2$$

Pressure per unit volume

$$P = \frac{1}{3} \rho \bar{u}^2$$

$$\left[P = \frac{M}{V} = \frac{mn}{1} = mn \right]$$

Q. (a) Define root mean square velocity of gas molecules. Give its relation with temperature.

(b) State Boyle's law and Charles' law of gases from the relation $P = \frac{1}{3} \rho U_{rms}^2$

A (a) Root mean square velocity (U_{rms})

It is defined as the square root of the average of the squares of the individual velocities of the gas molecules.

$$\text{i.e. } U_{rms} = \sqrt{\frac{U_1^2 + U_2^2 + \dots + U_n^2}{n}} = \sqrt{\overline{U^2}}$$

$$U_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M}}$$

$$\therefore U_{rms} \propto \sqrt{T}$$

(b) By

$$P = \frac{1}{3} \rho U_{rms}^2$$

$$\text{or } P = \frac{1}{3} \frac{M}{V} U_{rms}^2$$

$$\text{or } PV = \frac{1}{3} M U_{rms}^2$$

here M is constant and $U^2 \propto T$

then if T is constant, $PV = \text{constant}$

or $P \propto \frac{1}{V}$ Boyle's law

$$\text{Again, } P = \frac{1}{3} \frac{M}{V} U_{rms}^2$$

$$V = \frac{1}{3} \frac{M}{P} U_{rms}^2$$

M is fixed and $U_{rms}^2 \propto T$

If P is kept constant, then

$V \propto T$ Charles' law

* Ideal gas eqⁿ $PV = nRT$

Acc. Boyle's law, $P \propto \frac{1}{V}$ and acc. to Charles' law $V \propto T$

i.e. $\frac{PV}{T} = \text{constant} \Rightarrow PV = RT$, for n mole $PV = nRT$

Q. Calculate the average kinetic energy for one molecule of gas at constant volume.

A. Pressure exerted by one mole of gas

$$P = \frac{1}{3} \rho U_{rms}^2$$

$$= \frac{1}{3} \frac{M}{V} U_{rms}^2$$

$$\text{or } PV = \frac{1}{3} M U_{rms}^2$$

but $PV = RT$, so

$$\frac{1}{3} M U_{rms}^2 = RT$$

$$\text{or } M U_{rms}^2 = 3RT$$

Now average kinetic energy

$$\frac{1}{2} M U_{rms}^2 = \frac{3}{2} RT$$

(K.E)_{av} for 1 mole

$$\frac{1}{2} M U_{rms}^2 = \frac{3}{2} \frac{R}{N} T = \frac{3}{2} K_B T \quad [K_B \rightarrow \text{Boltzmann constant}]$$

$$\text{so } \boxed{\text{Total K.E for 1 mole} = \frac{3}{2} RT}$$

$$\text{and } \boxed{\text{average K.E per molecule} = \frac{3}{2} K_B T}$$

Q. Show that pressure exerted by the gas is two-third of average kinetic energy per unit volume of the gas molecules.

A. According to K.T.G

$$P = \frac{1}{3} \rho U^2$$

Average K.E translation per unit volume

$$E = \frac{1}{2} \rho U^2$$

$$\text{so, } \frac{P}{E} = \frac{\frac{1}{3} \rho U^2}{\frac{1}{2} \rho U^2} = \frac{2}{3}$$

$$\text{or } P = \frac{2}{3} E$$

$$P = \frac{2}{3} \times \text{Average K.E per unit volume}$$

Q. What is law of equipartition of energy. Derive C_p , C_v and γ .

A. Law of equipartition of energy

In equilibrium the total energy is equally distributed in all possible energy modes, with each mode having an average energy equals to $\frac{1}{2} k_B T$.

K.E of a single molecule is

$$E_t = \frac{1}{2} m u_x^2 + \frac{1}{2} m u_y^2 + \frac{1}{2} m u_z^2$$

In thermal equilibrium, the average energy

$$\langle E_t \rangle = \langle \frac{1}{2} m u_x^2 \rangle + \langle \frac{1}{2} m u_y^2 \rangle + \langle \frac{1}{2} m u_z^2 \rangle = \frac{3}{2} k_B T$$

Here,

$$\langle \frac{1}{2} m u_x^2 \rangle = \frac{1}{2} k_B T$$

$$\langle \frac{1}{2} m u_y^2 \rangle = \frac{1}{2} k_B T$$

$$\langle \frac{1}{2} m u_z^2 \rangle = \frac{1}{2} k_B T$$

[$\langle \dots \rangle \rightarrow$ for average]

Derivation of C_p , C_v and γ

(1) Monoatomic gas

For monoatomic gas degree of freedom $f=3$

according to law of equipartition of energy, average energy for each DOF = $\frac{1}{2} k_B T$

so for $f=3$

total internal energy of 1 mole, $U = \frac{3}{2} k_B T \times N_A$

$$\gamma = \frac{C_p}{C_v}$$

$$U = \frac{3}{2} RT \quad \left[\because K_B N_A = R \right]$$

$$C_V = \frac{dU}{dT} = \frac{3}{2} R \Rightarrow \boxed{C_V = \frac{3}{2} R}$$

For an ideal gas, $C_P = C_V + R$

$$C_P = \frac{3}{2} R + R$$

$$\boxed{C_P = \frac{5}{2} R}$$

$$\text{Now } \gamma = \frac{C_P}{C_V} = \frac{\frac{5}{2} R}{\frac{3}{2} R}$$

$$\text{i.e. } \boxed{\gamma = \frac{5}{3}}$$

(11) Diatomic gases

$$U = \frac{5}{2} K_B T \times N_A$$

[$f = 5$ for diatomic gas]

$$U = \frac{5}{2} RT$$

$$C_V = \frac{dU}{dT} = \frac{5}{2} R \Rightarrow \boxed{C_V = \frac{5}{2} R}$$

$$C_P = C_V + R = \frac{5}{2} R + R$$

$$\boxed{C_P = \frac{7}{2} R}$$

$$\gamma = \frac{C_P}{C_V} = \frac{\frac{7}{2} R}{\frac{5}{2} R}$$

$$\text{or } \boxed{\gamma = \frac{7}{5}}$$

* If it is not rigid, $f = 6$, then

$$C_V = \frac{7}{2} R, \quad C_P = \frac{9}{2} R \quad \text{so} \quad \gamma = \frac{9}{7}$$

Polyatomic gas

$$U = \left(\frac{3}{2} k_B T + \frac{3}{2} k_B T + f k_B T \right) N_A$$

i.e. $C_V = \frac{dU}{dT} = (3+f)R$ $[k_B N_A = R]$

so $C_P = (3+f)R + R = (4+f)R$

hence $\gamma = \frac{C_P}{C_V} = \frac{4+f}{3+f}$

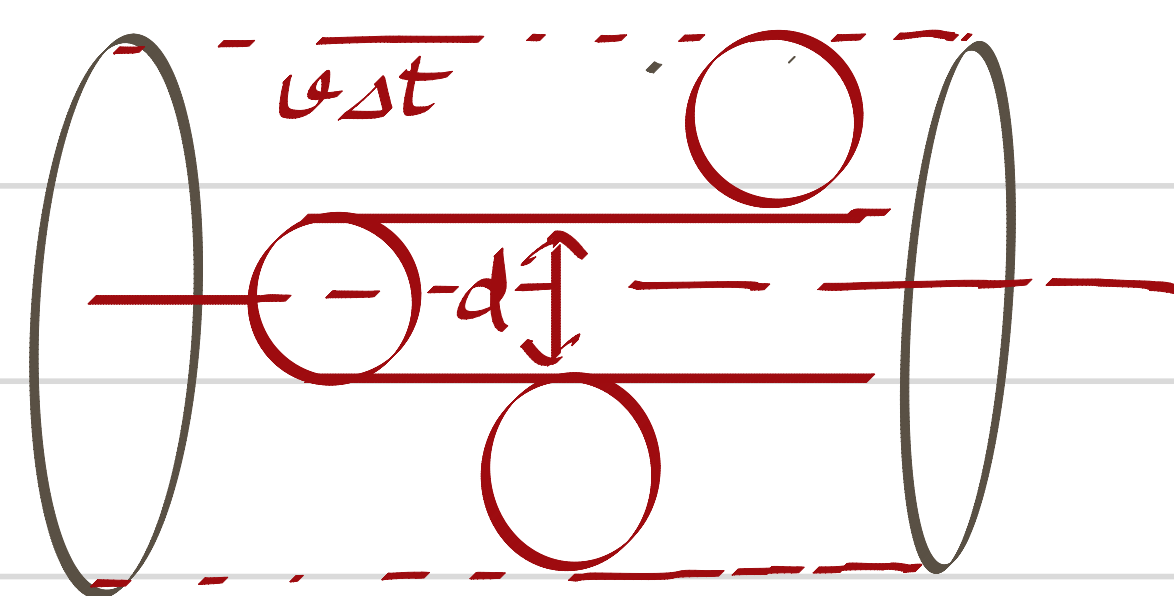
Q. Define mean free path of a gas molecule. Write expression for it.

A. Mean free path: It is the average distance travelled by the molecule between its two successive collisions with other molecules.

mean free path $\bar{\lambda} = \frac{\lambda_1 + \lambda_2 + \dots + \lambda_n}{\text{Total no. of coll}^n}$

For 1 molecule, $\bar{\lambda} = \frac{\bar{v} \Delta t}{\pi d^2 v \Delta t n} = \frac{1}{n \pi d^2}$

$n \rightarrow$ no. density
 $d \rightarrow$ diameter



In this derivation we assume other molecules are at rest, which is not possible. Then actual result

is

$$\bar{\lambda} = \frac{1}{\sqrt{2} n \pi d^2}$$

[The av. relative speed is b/w two molecules is $\sqrt{2}$ times the av. speed of a single molecule]

For 1 mole

$$PV = RT \Rightarrow P = \frac{RT}{V} = n k_B T$$

$$n = \frac{P}{k_B T}$$

then

$$\bar{\lambda} = \frac{k_B T}{\sqrt{2} \pi d^2 P}$$

Q. Explain the difference between (a) periodic and non-periodic motion (b) Harmonic and non harmonic oscillations.
(c) What is simple harmonic motion (SHM)? State the conditions for (SHM) and derived differential equation for SHM.

A. (a) Periodic Motion: A motion that repeats itself at regular time intervals.

e.g motion of pendulum, motion of planets.

Periodic functions: These functions which represent periodic motion. i.e

$$f(t) = f(t+T) = f(T+2T)$$

* Sine and cosine functions are periodic functions.

i.e $f(x) = \sin x$ or $f(x) = \cos x$ are periodic functions.

Non periodic functions: A motion that does not repeats itself at regular time intervals.

e.g motion of a car on road.

(b) Harmonic Oscillations: Oscillations which can be expressed in terms of single harmonic functions.

i.e Sine function or cosine function.

$$y = a \sin \omega t \quad \text{and} \quad x = a \cos \omega t$$

Non harmonic oscillations: Oscillations which can not be expressed in terms of single harmonic function. i.e

$$y = a \sin \omega t + b \sin 2\omega t$$

Simple Harmonic Motion: SHM is a type of periodic motion in which particle moves to and fro repeatedly about a mean position under a restoring force which directly proportional to the displacement from the mean position and directed towards it.

Conditions for SHM:

- (i) The acceleration is always directed towards the mean position
- (ii) The acceleration is proportional to the displacement
 $a \propto -x$

Differential Equation for SHM

From Newton's second law

$$F = ma$$

and $F = -Kx$

i.e. $ma = -Kx$

$$a = \frac{d^2x}{dt^2}$$

$$m \cdot \frac{d^2x}{dt^2} = -Kx$$

$$\text{or } \frac{d^2x}{dt^2} = -\frac{K}{m}x$$

$$\therefore \frac{d^2x}{dt^2} = -\frac{K}{m}x$$

Let $\frac{K}{m} = \omega^2$, then

$$\frac{d^2x}{dt^2} = -\omega^2x$$

$$\text{or } \boxed{\frac{d^2x}{dt^2} + \omega^2x = 0}$$

by $\frac{d^2x}{dt^2} = -\omega^2x$

$$\text{or } \boxed{a = -\omega^2x}$$

Time period

$$\text{By } \omega^2 = \frac{K}{m}$$

$$\left(\frac{2\pi}{T}\right)^2 = \frac{K}{m}$$

$$\frac{2\pi}{T} = \sqrt{\frac{m}{K}}$$

$$\text{or } T = 2\pi \sqrt{\frac{m}{K}}$$

* General Equation $y = A \sin(\omega t + \phi)$ / $x = A \sin(\omega t + \phi)$

Q. Derive the expression for velocity and acceleration of a particle in SHM. Write their maximum values. Discuss the phase difference between them.

A. Velocity in SHM

The displacement in SHM is given by

$$x = A \cos(\omega t + \phi)$$

$\phi \rightarrow$ initial phase

Let at $t=0$, $\phi=0$, then

$$x = A \cos \omega t$$

now velocity

$$v = \frac{dx}{dt}$$

$$= \frac{d}{dt} (A \cos \omega t)$$

$$= -A \sin \omega t \frac{d}{dt} (\omega t)$$

$$v = -A\omega \sin \omega t$$

$$\text{put } \sin \omega t = \sqrt{1 - \cos^2 \omega t}$$

$$v = -A\omega \sqrt{1 - \cos^2 \omega t}$$

by $x = A \cos \omega t$, $\cos \omega t = \frac{x}{A}$
 then $v = -A\omega \sqrt{1 - (x/A)^2}$

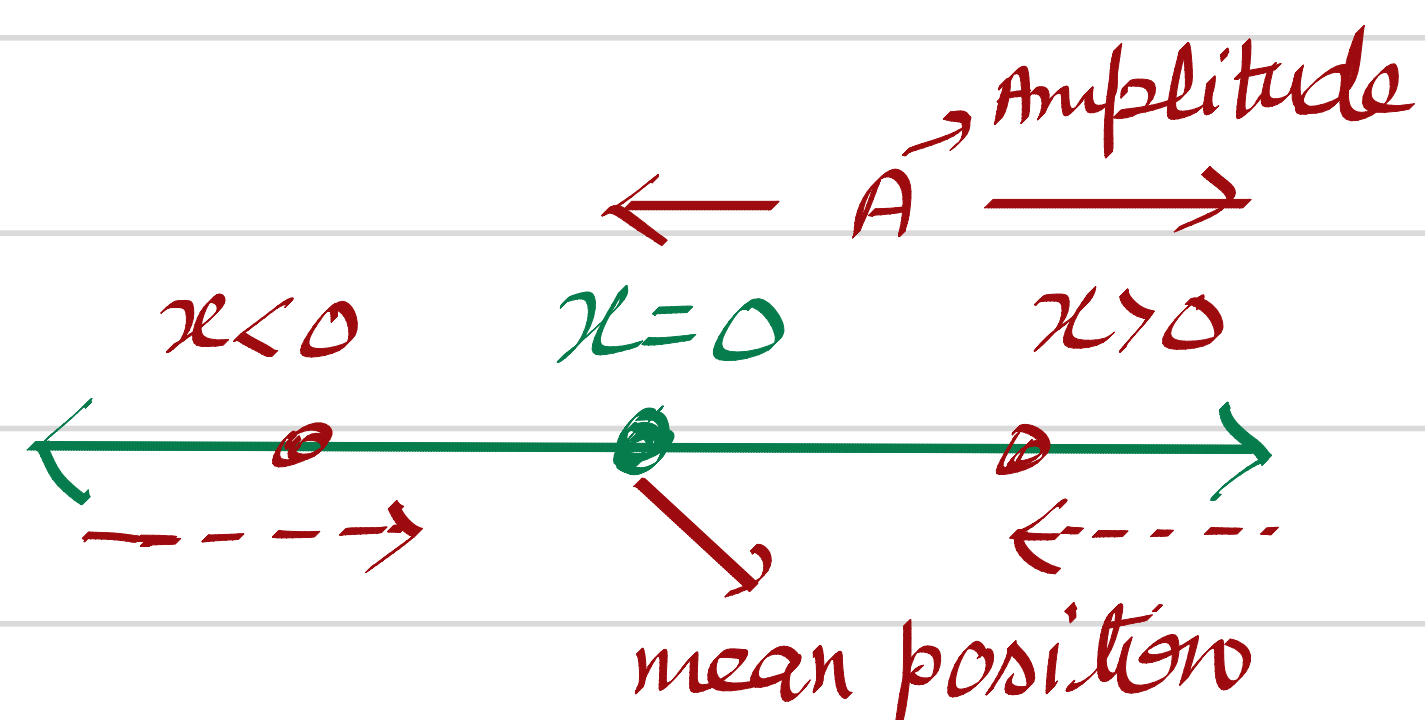
$$\text{or } v = -A\omega \sqrt{\frac{A^2 - x^2}{A^2}}$$

$$\text{or } v = -\omega \sqrt{A^2 - x^2}$$

* $v_{\max} = A\omega$ $[v \text{ is max}^m \text{ at } x=0]$

At mean position v is maximum.

$x \rightarrow$ displacement at t
 $A \rightarrow$ Amplitude
 $\omega \rightarrow$ Angular frequency
 $\omega = 2\pi \nu = \frac{2\pi}{T}$
 $\phi \rightarrow$ initial phase



$$\frac{d}{dt} (\cos \omega t) = -\omega \sin \omega t$$

Acceleration in SHM

$$a = \frac{dv}{dt}$$

here $v = -A\omega \sin \omega t$

$$a = \frac{d(-A\omega \sin \omega t)}{dt}$$

$$= -A\omega \cos \omega t \frac{d(\omega t)}{dt}$$

$$a = -A\omega^2 \cos \omega t$$

by $x = A \cos \omega t$, $\cos \omega t = \frac{x}{A}$

$$a = -A\omega^2 \cdot \frac{x}{A}$$

$$a = -\omega^2 x$$

$$\frac{d}{dt}(\sin \omega t) = \omega \cos \omega t$$

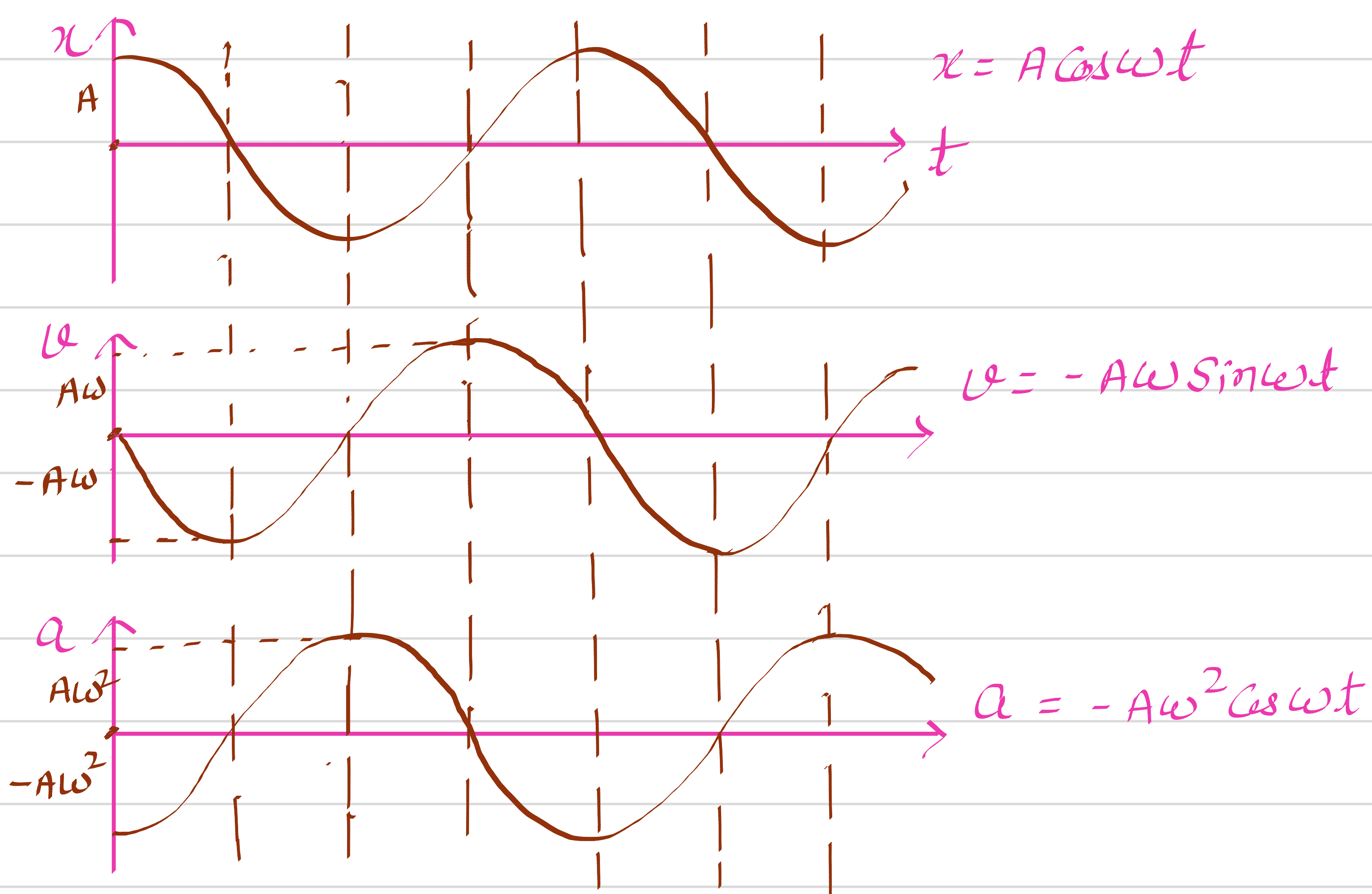
Maximum acceleration

$$a_{\max} = A\omega^2$$

* Acceleration is maximum at the extreme position (at $x=A$)

* Acceleration is zero at mean position (At $x=0$).

Graphs



- Phase difference b/w displacement and velocity -
 $x = A \cos \omega t$, $v = -A\omega \sin \omega t$

$$\text{phase difference} = 90^\circ = \frac{\pi}{2}$$

- Phase difference b/w displacement and acceleration
 $x = A \cos \omega t$, $a = -A\omega^2 \cos \omega t$

$$\text{phase difference} = 180^\circ = \pi$$

- Phase difference b/w velocity and acceleration
 $v = A\omega \sin \omega t$, $a = -A\omega^2 \cos \omega t$

$$\text{phase difference} = 90^\circ = \frac{\pi}{2}$$

* Acceleration a is always opposite to displacement x

Note: These equations can also be taken as:

$$y = A \sin \omega t \quad , \quad v = A\omega \cos \omega t \quad a = -A\omega^2 \sin \omega t$$

Q. Derive expression for the time period of simple pendulum. What is the effect of mass on the time period?

A. Consider a simple pendulum of mass ' m ' suspended from a rigid support with a light, inextensible string of length ' l '.

Suppose at any instant of time ' t ' the bob makes an angle θ with vertical (mean position) as shown in fig.

The restoring force due to gravity

$$F = -mg \sin \theta$$

For small angle θ , $\sin \theta \approx \theta$

$$F = -mg\theta$$

The linear displacement

$$\theta = \frac{x}{l}$$

then,

$$F = -mg \frac{x}{l}$$

$$\text{or } ma = -mg \frac{x}{l}$$

[Newton's 2nd law]

$$\text{or } a = -\left(\frac{g}{l}\right)x \quad \text{--- (1)}$$

$$\text{i.e. } a \propto -x$$

Thus motion of bob is S.H.M.

In S.H.M

$$a = -\omega^2 x$$

from eqn (1)

$$\omega^2 x = \left(\frac{g}{l}\right)x$$

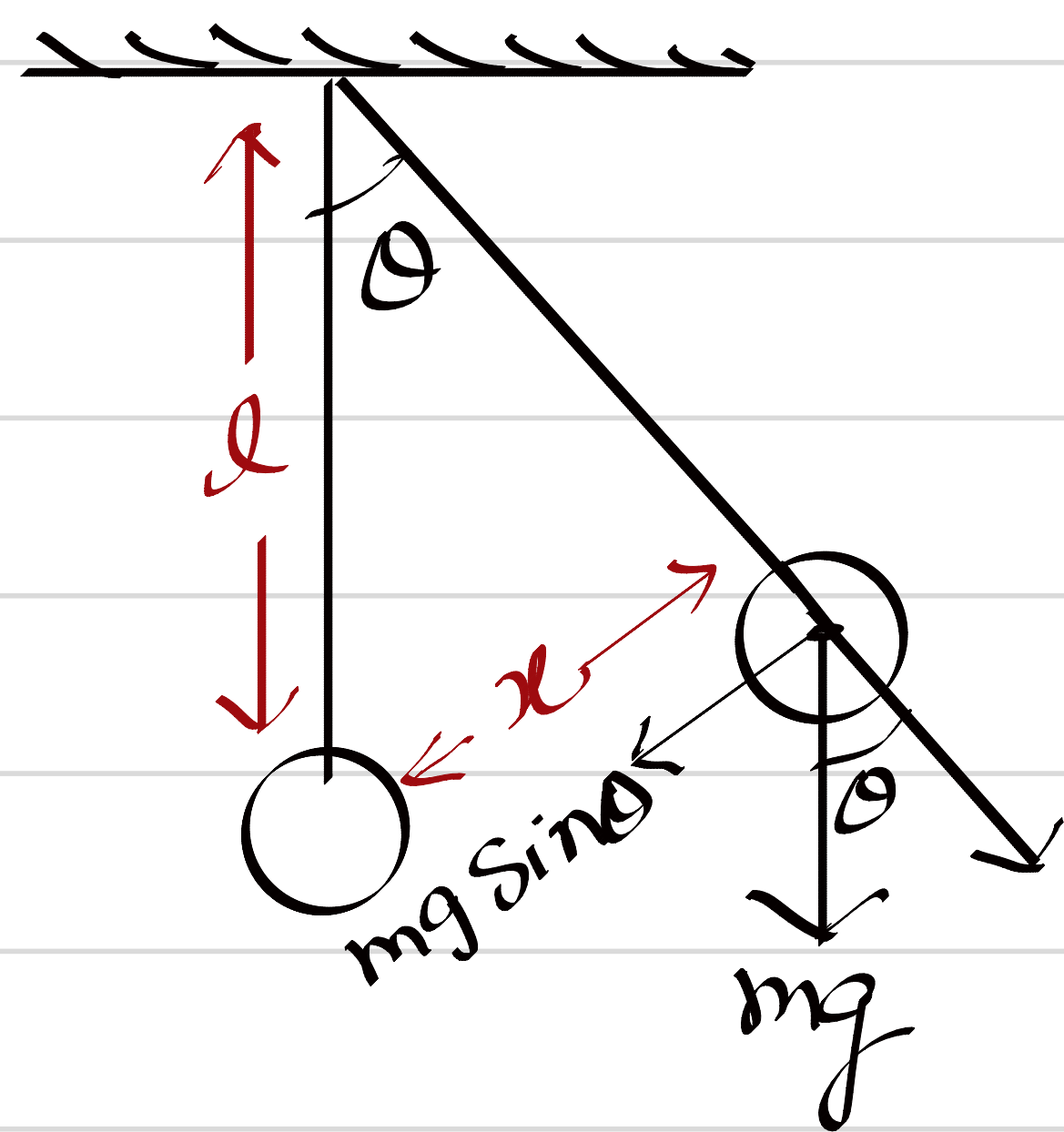
$$\text{or } \omega^2 = \frac{g}{l}$$

$$\omega = \sqrt{\frac{g}{l}}$$

$$\frac{2\pi}{T} = \sqrt{\frac{g}{l}}$$

or

$$T = 2\pi \sqrt{\frac{l}{g}}$$



Formula does not include mass, i.e. Time period is independent of mass of bob.

$T \propto \sqrt{L}$ and $T \propto \frac{1}{\sqrt{g}}$
 * At the centre of earth $g=0$ therefore time period is ∞ there.

Q. Derive expression for the time period of an oscillating spring. How it depends on the mass attached to spring and the spring constant.

A. Consider a mass m attached to a spring with spring constant k . When mass is displaced from its equilibrium position by a distance x .

According to Hooke's law, the restoring force
 $F = -kx$ [-ve sign show opposite dirⁿ of force and displacement]

By Newton's law

$$F = ma = -kx$$

$$\text{or } a = -\left(\frac{k}{m}\right)x \quad \text{---(1)}$$

$$\text{i.e. } a \propto -x$$

Thus eqⁿ represents SHM.

For SHM

$$a = -\omega^2 x$$

from eqⁿ (1)

$$+\omega^2 x = \frac{k}{m} x$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$\frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$

$$\text{or } T = 2\pi \sqrt{\frac{m}{k}}$$

$$* T \propto \sqrt{m}$$

$$* T \propto \frac{1}{\sqrt{k}}$$

Stiffer spring (high k) decreases time period

Q. Derive expression for total energy for a particle undergoing simple harmonic motion (SHM).

A. Kinetic Energy:

$$K = \frac{1}{2} m v^2$$

put $v = -A\omega \sin \omega t$

[If $x = A \cos \omega t$
 $v = -A\omega \sin \omega t$]

$$K = \frac{1}{2} m (-A\omega \sin \omega t)^2$$

$$= \frac{1}{2} m A^2 \omega^2 \sin^2 \omega t$$

$$K = \frac{1}{2} K A^2 \sin^2 \omega t \quad \text{--- (1) } [\because K = m\omega^2]$$

Potential Energy

$$U = \frac{1}{2} K x^2$$

$$= \frac{1}{2} K (A \cos \omega t)^2$$

[$\because x = A \cos \omega t$]

$$U = \frac{1}{2} K A^2 \cos^2 \omega t \quad \text{--- (2)}$$

Total Energy

$$T = K + U$$

$$= \frac{1}{2} K A^2 \sin^2 \omega t + \frac{1}{2} K A^2 \cos^2 \omega t$$

$$T = \frac{1}{2} K A^2$$

[$\because \sin^2 \omega t + \cos^2 \omega t = 1$]

or $T = \frac{1}{2} m \omega^2 A^2$

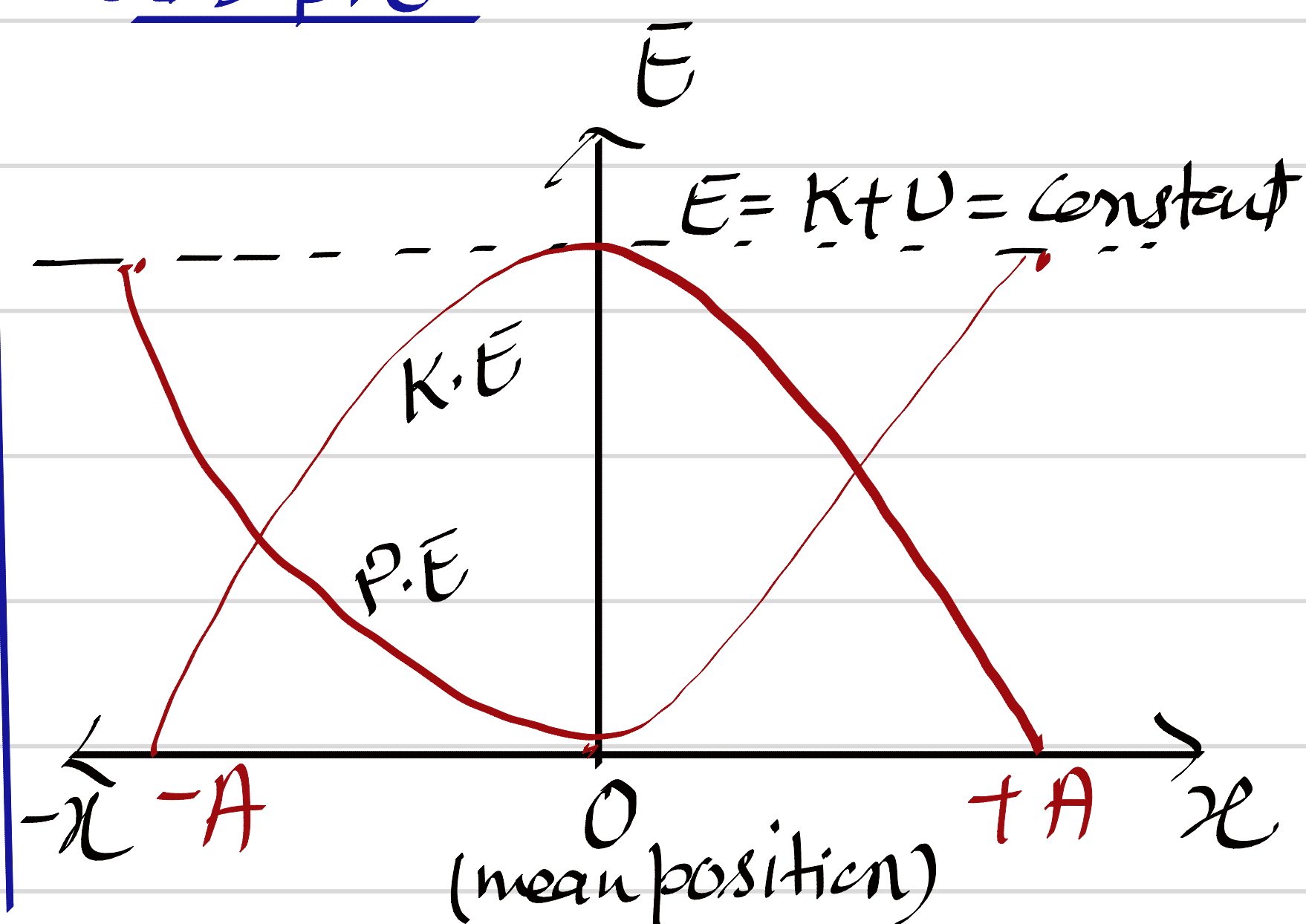
[$K = m\omega^2$]

put $\omega = 2\pi \nu$
 $T = \frac{1}{2} m (2\pi \nu)^2 A^2$

or $T = 2\pi^2 \nu^2 A^2 m$

* Total energy $E \propto m$
 $E \propto \nu^2$
 $E \propto A^2$

Graph



Extra

* Time period of vertical spring oscillations

In equilibrium

$$mg = kx_0 \quad \text{--- (1)}$$

$[x_0 \rightarrow \text{equilibrium extension}]$

$$x_0 = \frac{mg}{k}$$

By Newton's law

$$F = -k(x + x_0)$$

$$\text{Net force } F_{\text{net}} = -k(x + x_0) + mg$$

$$\text{from eqn (1)} \quad mg = kx_0$$

$$F_{\text{net}} = -k(x + x_0) + kx_0$$

$$\text{so } F_{\text{net}} = -kx$$

$$ma = -kx$$

$$a = -\left(\frac{k}{m}\right)x$$

$$-\omega^2 x = -\frac{k}{m}x$$

$$\omega^2 = \frac{k}{m}$$

$$\frac{2\pi}{T} = \sqrt{\frac{k}{m}} \Rightarrow$$

$$T = 2\pi \sqrt{\frac{m}{k}}$$

Q. Derive formula for speed of (a) Transverse wave on stretched string (b) longitudinal wave in a liquid/gas by dimensional method.

A. (a) Speed of transverse wave

$$v \propto T^a \mu^b$$

$$v = kT^a \mu^b \quad \text{--- (1)}$$

$T \rightarrow \text{Tension in string}$

$\mu \rightarrow \text{mass per unit length}$

$$[LT^{-1}] = [MLT^{-2}]^a [ML^{-1}]^b$$

$$[M^0 L^1 T^{-1}] = [M^{a+b} L^{a-b} T^{-2a}]$$

on comparing, $a+b=0$, $a-b=1$ and $-2a=-1$

on solving, $a = \frac{1}{2}$, $b = -\frac{1}{2}$

from eqn (1)

$$v = T^{\frac{1}{2}} \mu^{-\frac{1}{2}}$$

i.e

$$v = \sqrt{\frac{T}{\mu}}$$

(b) Speed of longitudinal wave

$$v \propto B^a \rho^b$$

$B \rightarrow$ Bulk modulus

$$v = K B^a \rho^b \quad (1)$$

$\rho \rightarrow$ density of fluid

$$[LT^{-1}] = [ML^{-1}T^{-2}]^a [ML^{-3}]^b$$

$$[M^0 L^1 T^{-1}] = [M^{a+b} L^{-a-3b} T^{-2a}]$$

on comparing, $a+b=0$, $-a-3b=1$ and $-2a=-1$

on solving $a = \frac{1}{2}$, $b = -\frac{1}{2}$, then from eqⁿ (1)

$$v = \sqrt{\frac{B}{\rho}}$$

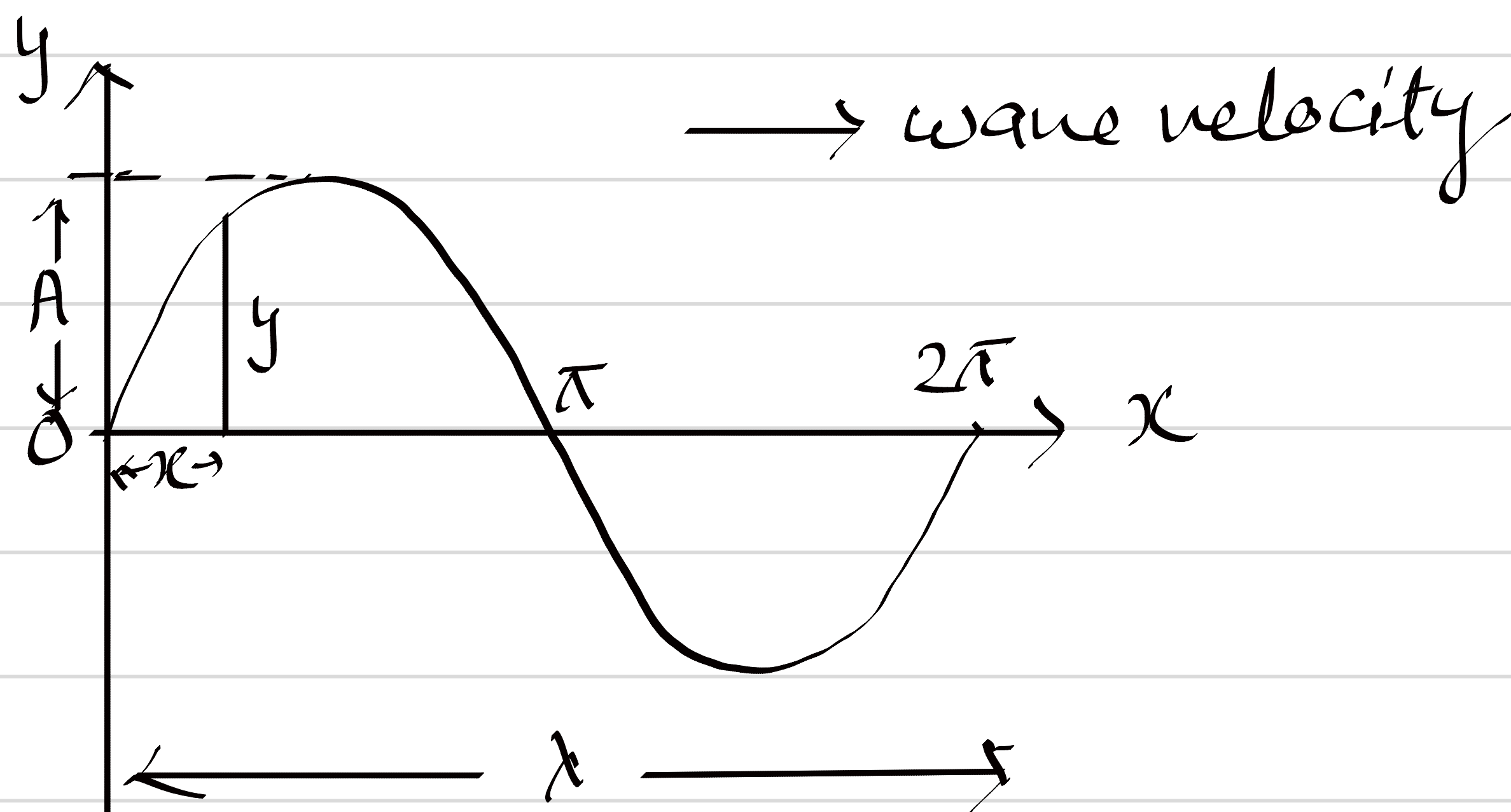
Q What do you understand by progressive waves. Explain it in mathematical form.

A Progressive wave: A wave that travels continuously in a medium, transferring energy from one point to another without permanent displacement of particles.

Types:

(i) Transverse waves $\rightarrow$ Light wave, water waves

(ii) Longitudinal waves $\rightarrow$ Sound waves



Mathematical form: A one dimensional progressive wave travelling in the x direction is given as-

Displacement of the particle at any instant of time t

$$y(x, t) = A \sin(Kx - \omega t + \phi) \text{ [Transverse wave]}$$

$K \rightarrow$ wave number $K = \frac{2\pi}{\lambda}$, $\omega \rightarrow$ angular frequency $\omega = \frac{2\pi}{T}$

$x \rightarrow$ direction of propagation (horizontal)

$y \rightarrow$ direction of particle oscillation (vertical)

and

$$S(x, t) = A \sin(Kx - \omega t + \phi) \quad \phi \rightarrow \text{phase constant}$$

$$\text{put } K = \frac{2\pi}{\lambda} \quad \text{and } \omega = \frac{2\pi}{T}$$

we get

$$y(x, t) = A \sin\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right)$$

$$\text{or } y(x, t) = A \sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$$

$$\text{or } y(x, t) = A \sin \frac{2\pi}{\lambda} (x - vt) \quad \left[\because \frac{\lambda}{T} = v\right]$$

Q. Explain Newton's formula for the speed of sound in a gas. What modification did Laplace introduce to correct it?

A. Newton's formula for the speed of sound in a gas

Newton assumed that sound wave travels through a gas under isothermal conditions.

Then,

$$v = \sqrt{\frac{B}{\rho}}$$

—(1)

$B \rightarrow$ Bulk modulus

$\rho \rightarrow$ density of gas

For isothermal change, $PV = \text{Constant}$

$$PV = \text{constant} \quad [P \rightarrow \text{Pressure}, V \rightarrow \text{Volume}]$$

$$P dV + V dP = 0 \quad [\text{on differentiating}]$$

$$P dV = -V dP$$

$$\text{or } P = - \frac{dP}{dV/V}$$

$$\text{but } - \frac{dP}{dV/V} = B \text{ (Bulk modulus)}$$

$$\text{hence } P = B$$

$$\text{then, } \boxed{V = \sqrt{\frac{P}{\rho}}} \quad [\text{from (1)}]$$

At STP,

$$P = 1.013 \times 10^5 \text{ Pa}, \quad \rho = 1.293 \text{ kg/m}^3$$

so

$$V = \sqrt{\frac{1.013 \times 10^5}{1.293}} = 280 \text{ m/s}$$

This value is about 15% less than the experimental value [331 m/s]. Hence Newton's formula is not acceptable.

Laplace Correction:

Laplace pointed out that sound wave travels under adiabatic condition. This is because of following reasons -

- (1) As sound travels temperature increases in the region of compression and decreases in the region of rarefaction.
- (2) Gas is a poor conductor of heat.
- (3) Compressions and rarefactions are formed rapidly therefore it is adiabatic change.

for adiabatic change

$$PV^r = \text{constant} \quad [r = \frac{C_p}{C_v}]$$

on differentiating

$$P r V^{r-1} dV + V^r dP = 0$$

$$r P V^{r-1} dV = -V^r dP$$

$$\text{or } r P = - \frac{V^r dP}{V^{r-1} dV}$$

$$\text{or } r P = - \frac{V^r dP}{V^r \cdot V^{-1} dV}$$

$$\text{or } r P = - \frac{dP}{dV/V}$$

$$\text{but } - \frac{dP}{dV/V} = B$$

$$\text{so } r P = B$$

$$\text{then by } c = \sqrt{\frac{B}{\rho}}$$

$$c = \sqrt{\frac{r P}{\rho}} = \sqrt{r} \sqrt{\frac{P}{\rho}}$$

for air $r = 1.4$, then

$$c = \sqrt{1.4 \times 280}$$

$$= 1.18 \times 280$$

$$= 331.2 \text{ m/s}$$

$$[\because \sqrt{\frac{P}{\rho}} = 280 \text{ m/s}]$$

This value is very close to the experimental value.

Thus Laplace correction gives more accurate result.

Q. Explain the principle of superposition. Derive expression for the resultant displacement when two waves travelling in the same medium superimpose.

A. Principle of Superposition of waves:

When a number of waves travel through a medium simultaneously, the resultant displacement at any point is equal to the algebraic sum of the displacement of individual waves.

For n no. of waves

$$y = y_1 + y_2 + \dots + y_n$$

Let two waves travelling in the same direction

$$y_1 = a \sin(Kx - \omega t)$$

$$\text{and } y_2 = a \sin(Kx - \omega t + \phi) \quad \phi \rightarrow \text{phase diff.}$$

Net displacement

$$\begin{aligned} y &= a \sin(Kx - \omega t) + a \sin(Kx - \omega t + \phi) \\ &= a [\sin(Kx - \omega t) + \sin(Kx - \omega t + \phi)] \end{aligned}$$

$$\left[\because \sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2} \right]$$

$$y = a \left[\frac{2 \sin(Kx - \omega t + Kx - \omega t + \phi)}{2} \cos \frac{\phi}{2} \right] \quad \left[\begin{array}{l} \cos(\phi) \\ = \cos \phi \end{array} \right]$$

$$= 2a \sin(Kx - \omega t + \frac{\phi}{2}) \cos \frac{\phi}{2}$$

$$\text{or } y = 2a \cos \frac{\phi}{2} \sin(Kx - \omega t + \frac{\phi}{2})$$

$$\text{or } \boxed{y = A \sin(Kx - \omega t + \frac{\phi}{2})} \quad \left[A = 2a \cos \frac{\phi}{2} \right]$$

This equation is a harmonic travelling wave equation in the x -axis.

$$\text{Amplitude } A = 2a \cos \frac{\phi}{2}$$

Initial phase difference = $\frac{\phi}{2}$

* If $\phi = 0$, $A = 2a$, then $y = 2a \sin(Kx - \omega t)$ $\rightarrow$ Constructive Interference $\rightarrow y$ increases

* If $\phi = \pi$, $A = 0$, then $y = 0$ $\rightarrow$ Destructive Interference $\rightarrow y$ decreases

Q. Explain Standing (Stationary) wave and normal modes.

Standing Waves: Standing waves are the waves that remain confined within a region and do not appear to move.

They are formed by the superposition of two waves of same frequency, amplitude and speed in opposite directions.

Normal modes are specific patterns of vibration and occurs at a particular frequency called natural frequency.

Formation of standing waves

Consider two waves

$$y_1 = A \sin(Kx - \omega t)$$

$$y_2 = A \sin(Kx + \omega t) \quad [\text{opposite dir}^n]$$

By principle of superposition

$$y = y_1 + y_2$$

$$y = A \sin(Kx - \omega t) + A \sin(Kx + \omega t)$$

$$y = 2A \sin Kx \cos \omega t$$

This equation represents a standing wave equation.

$\sin kx$ determines Nodes and Antinodes
(spatial vibration)

Nodes: Point of zero displacement
(destructive interference)

Condition:

$$\sin kx = 0 \Rightarrow kx = n\pi, \quad n = 0, 1, 2, 3, \dots$$

Since $k = \frac{2\pi}{\lambda}$, we get

$$x = \frac{n\lambda}{2}, \quad n = 0, 1, 2, 3, \dots$$

* The distance between any two consecutive nodes is $\frac{\lambda}{2}$

Antinodes: Points of maximum displacement.
(constructive interference)

Condition:

$$\sin kx = \pm 1$$

$$\text{i.e. } kx = (2n+1) \frac{\pi}{2}, \quad n = 0, 1, 2, 3, \dots$$

$$\text{by } k = \frac{2\pi}{\lambda}$$

$$x = \left(n + \frac{1}{2}\right) \frac{\lambda}{2}, \quad n = 0, 1, 2, 3, \dots$$

* The distance between any two consecutive antinodes = $\frac{\lambda}{2}$

Normal modes of standing waves

For a string fixed at both ends

$$\text{by } x = \frac{n\lambda}{2}, \quad \text{put } x = L, \quad \text{then } [L \rightarrow \text{length of string}]$$

$$\lambda n = \frac{2L}{n} \quad n = 1, 2, 3, \dots$$

Corresponding frequencies

$$\nu = \frac{v}{\lambda} = \frac{v}{2L/n}$$

or $\boxed{\nu = \frac{n v}{2L}} \quad [n=1, 2, 3, \dots]$

→ Fundamental mode or First harmonic
(The lowest possible natural frequency)
for $n=1$

$$\nu_1 = \frac{v}{2L} \quad \left[\nu = \frac{n v}{2L} \right]$$

→ Second harmonic (First overtone)
for $n=2$

$$\nu_2 = 2 \left(\frac{v}{2L} \right) \Rightarrow \nu_2 = 2\nu_1$$

→ Third harmonic (second overtone)
for $n=3$

$$\nu_3 = 3 \left(\frac{v}{2L} \right) \Rightarrow \nu_3 = 3\nu_1$$

Ratio of frequencies: $\nu_1 : \nu_2 : \nu_3 \dots = 1 : 2 : 3 \dots$

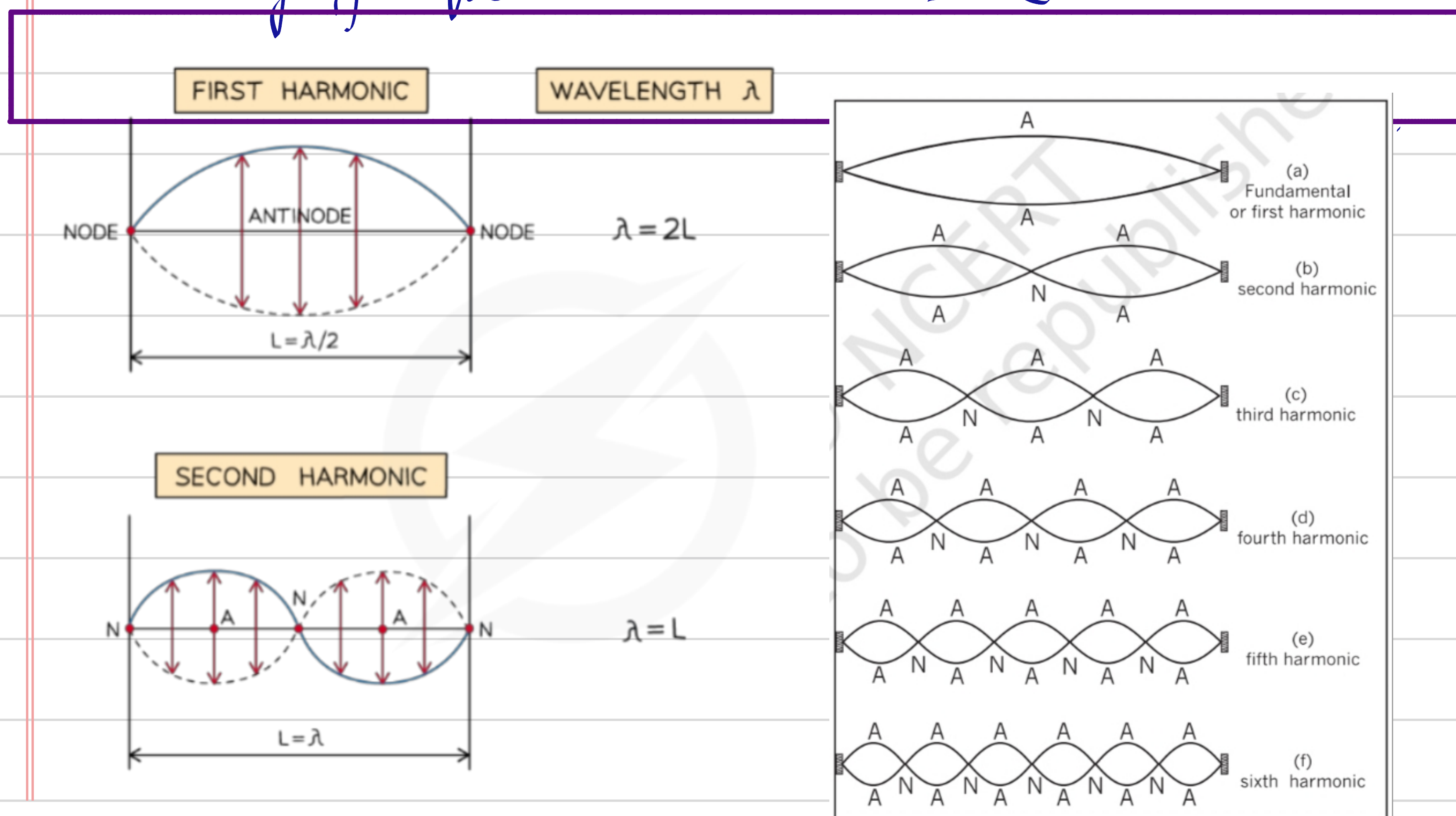
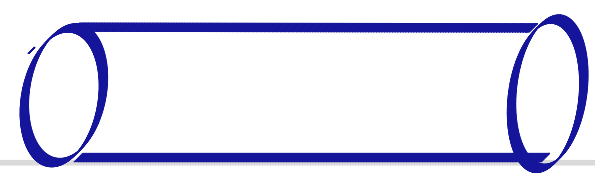


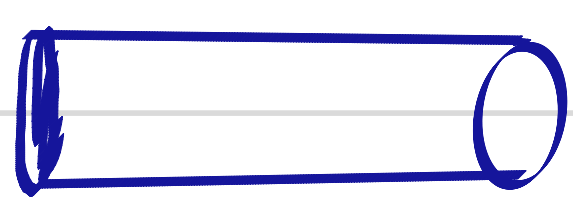
Fig. 14.13 The first six harmonics of vibrations of a stretched string fixed at both ends.

Q. What are the organ pipes. Explain natural frequencies of closed and open organ pipe.

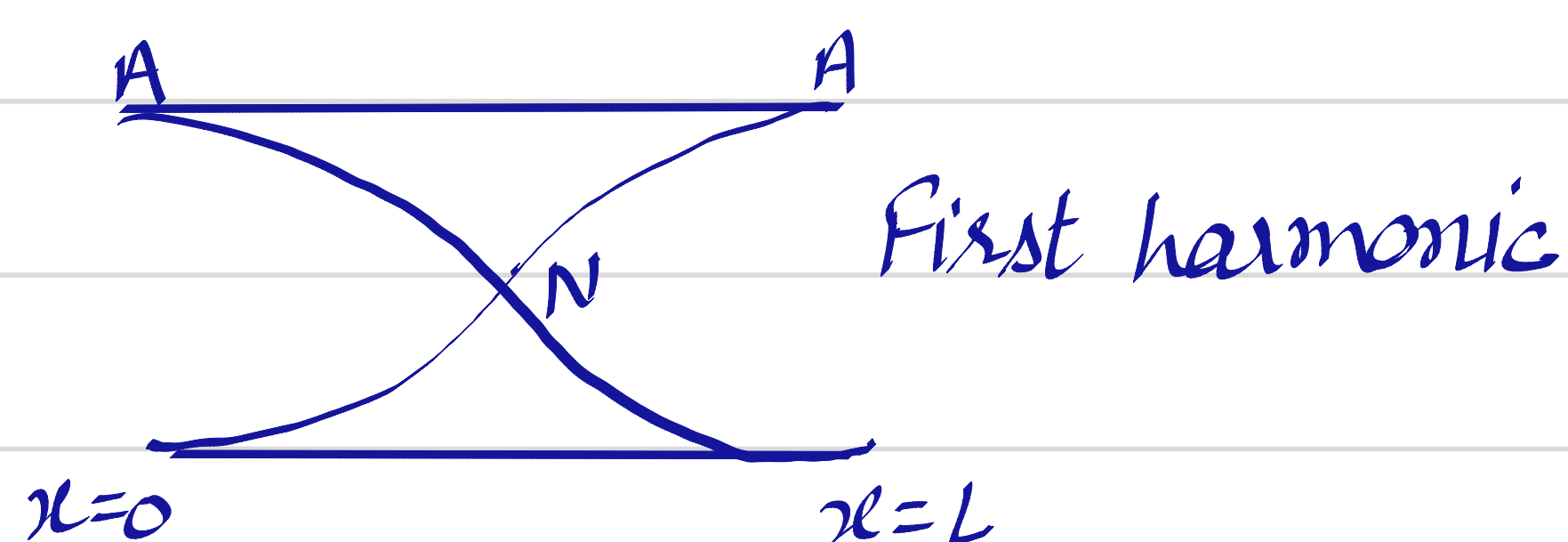
A. Organ pipe: It is the simplest musical instrument that produces sound when air is blown through it.

Two types of organ pipes -

(i) Open organ pipe  open at both ends

(ii) closed organ pipe  one end is closed

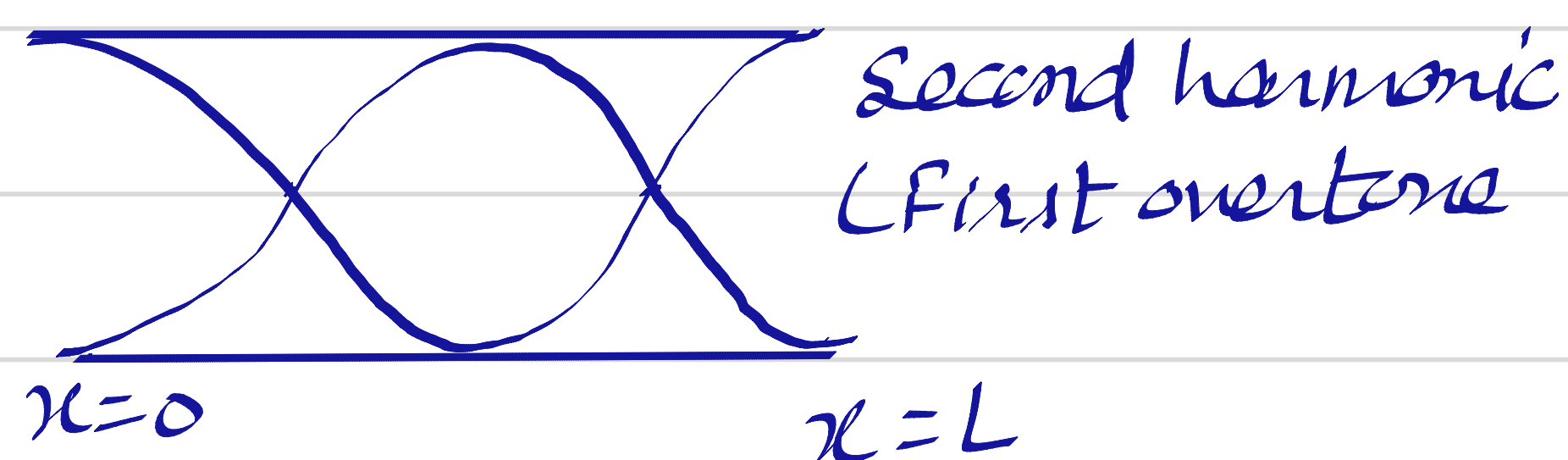
Normal modes of vibration for open organ pipe



$$L = \frac{\lambda_1}{2} \Rightarrow \lambda_1 = 2L$$

$$v_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

$$v_1 = \frac{v}{2L}$$



$$L = \lambda_2$$

$$v_2 = \frac{v}{\lambda_2} = \frac{v}{L}$$

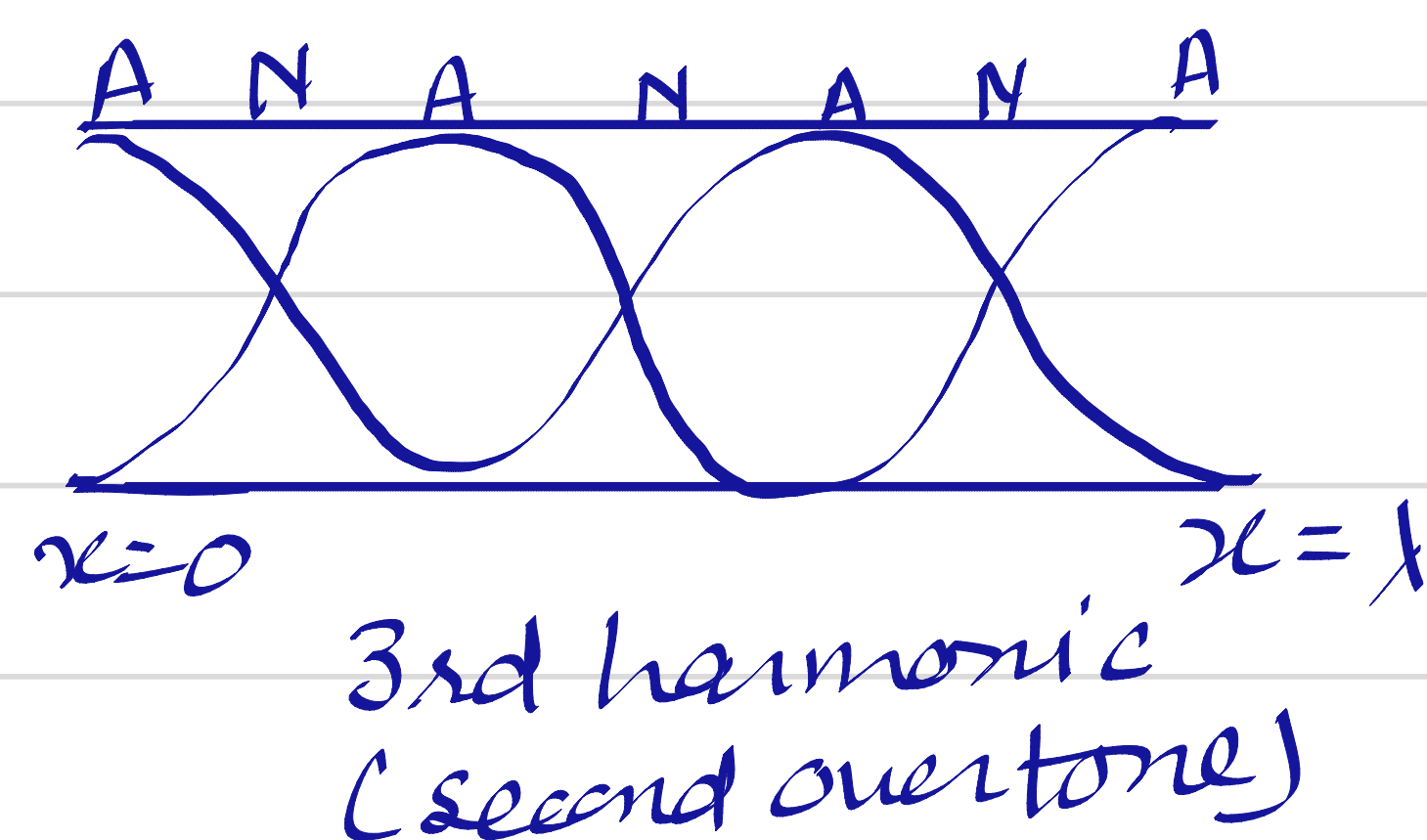
$$v_2 = \frac{2L}{2L} = 2(v_1)$$

Similarly for $n=3$

$$L = \lambda + \frac{\lambda}{2} = \frac{3\lambda}{2} \Rightarrow \lambda_3 = \frac{2L}{3}$$

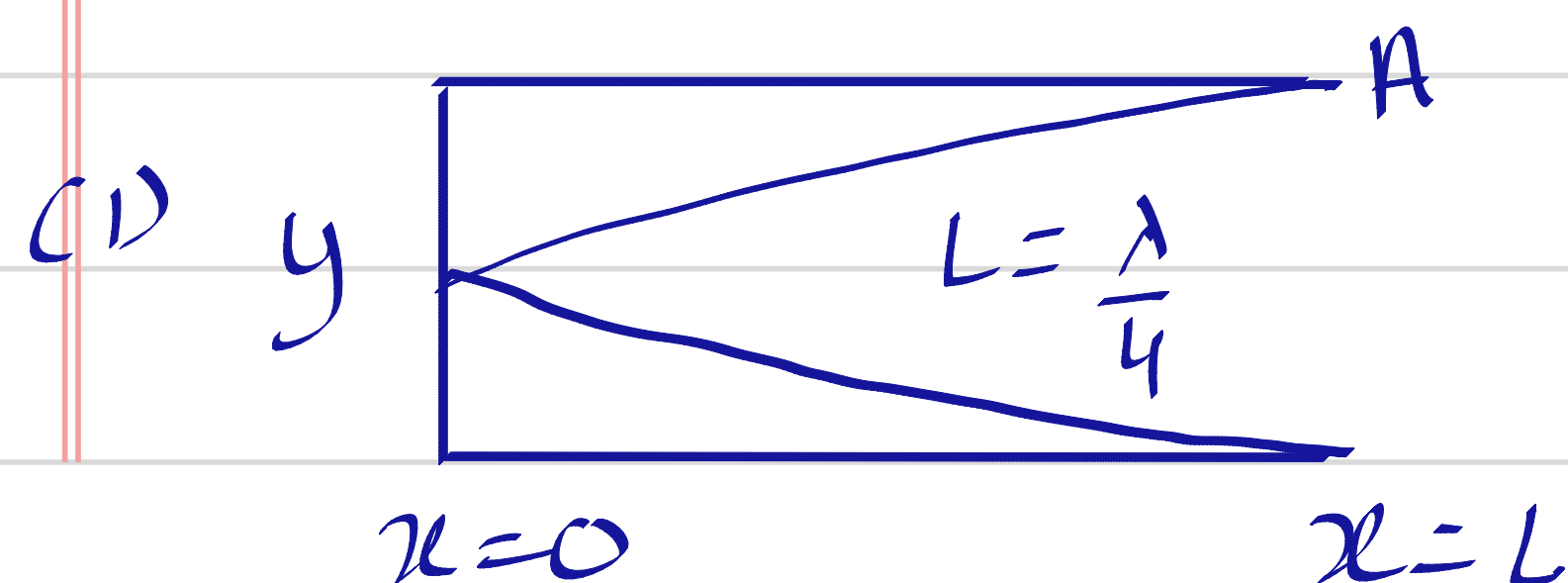
$$v_3 = \frac{v}{\lambda_3} = \frac{v \times 3}{2L}$$

$$v_3 = 3\left(\frac{v}{2L}\right) = 3v_1$$



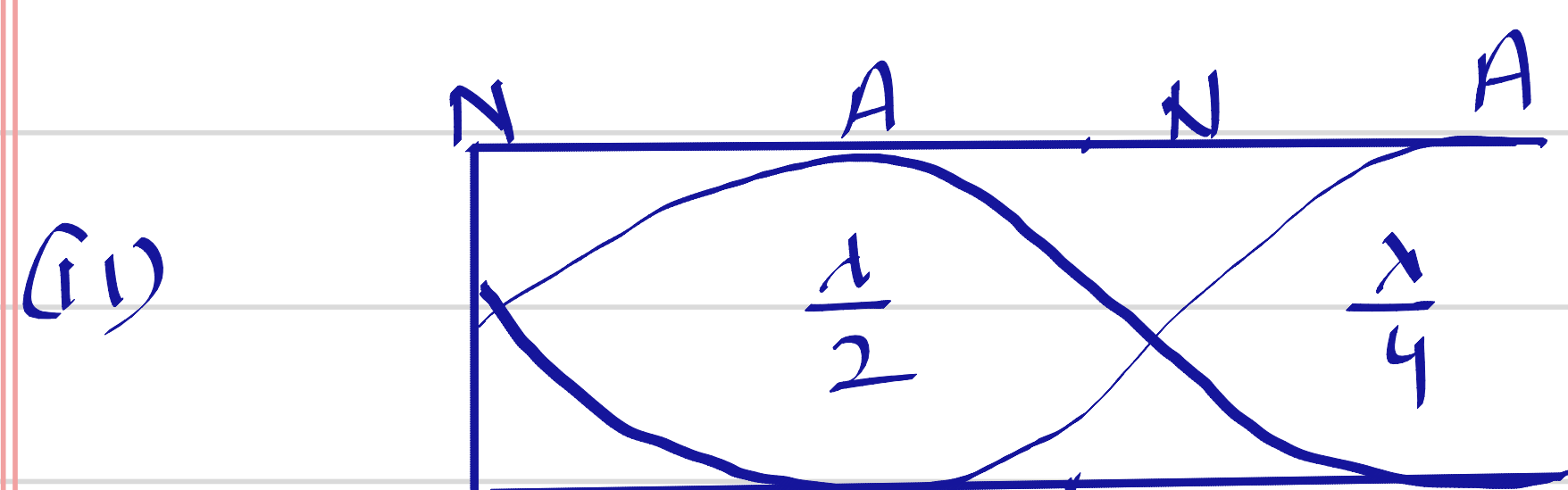
$$v_1 : v_2 : v_3 = 1 : 2 : 3 \quad \text{for open organ pipe}$$

Normal modes of vibrations for closed organ pipe



$$L = \frac{\lambda_1}{4} \Rightarrow \lambda_1 = 4L$$

$$v_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$$



3rd harmonic
(First overtone)

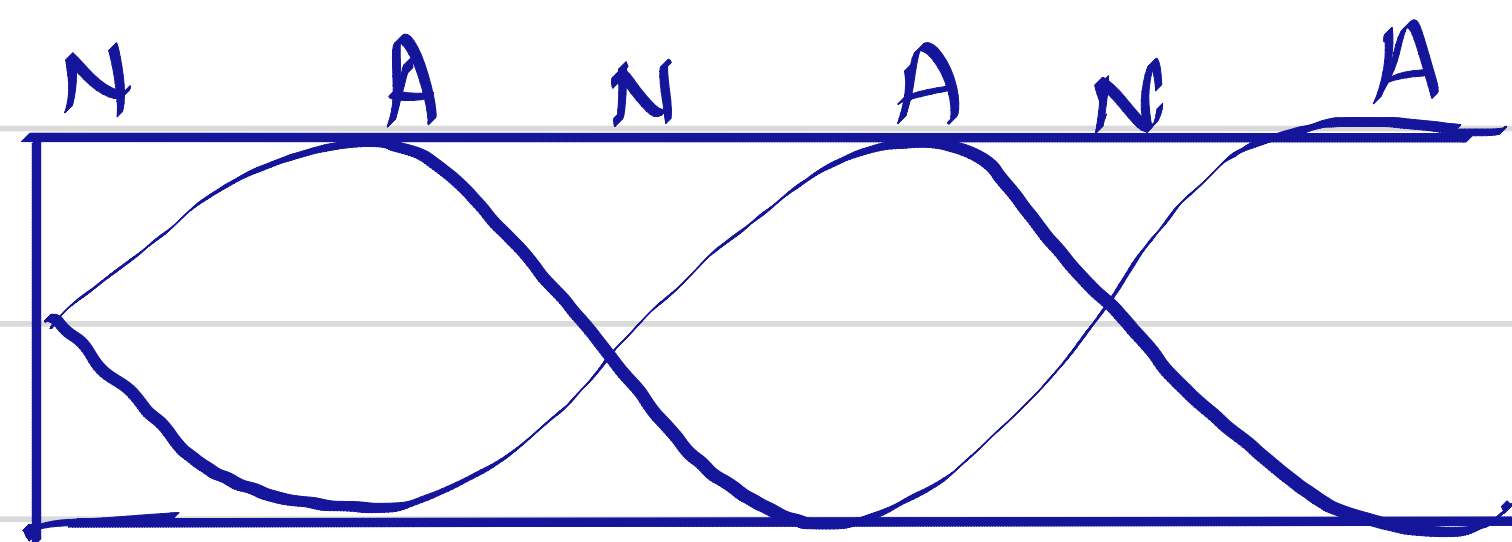
$$L = \frac{\lambda_2}{2} + \frac{\lambda_2}{4}$$

$$L = \frac{3\lambda_2}{4} \Rightarrow \lambda_2 = \frac{4L}{3}$$

$$v_2 = \frac{v}{\lambda_2} = \frac{v}{4L/3}$$

$$v_2 = 3 \left(\frac{v}{4L} \right) = 3v_1$$

(ii)



5th harmonic
(second overtone)

$$L = \lambda_3 + \frac{\lambda_3}{4} = \frac{5\lambda_3}{4}$$

$$\text{or } \lambda_3 = \frac{4L}{5}$$

$$v_3 = \frac{v}{\lambda_3} = \frac{v}{4L/5}$$

$$\text{or } v_3 = 5 \left(\frac{v}{4L} \right) = 5v_1$$

Ration $v_1 : v_2 : v_3 = 1 : 3 : 5 - -$

• Odd harmonics.

Q. What is meant by beats. What are its applications?

A. The periodic rise and fall (waxing and waning) in the intensity of sound due to the superposition of two sound waves slightly different in frequencies travelling in the same direction is called beats.

Phenomenon of beats can be used -

(i) To determine unknown frequency of tuning fork.

(ii) In tuning of musical instruments.

Mathematically,

consider two harmonic sound waves of nearly equal frequencies ω_1 and ω_2 , fixed at $x=0$. Assuming phase difference $\frac{\phi}{2}$ for equal amplitudes

$$S = S_1 + S_2$$

$$= a (\cos \omega_1 t + \cos \omega_2 t)$$

using $\cos A + \cos B$

$$S_1 + S_2 = 2a \cos \frac{\omega_1 - \omega_2}{2} t \cdot \cos \frac{\omega_1 + \omega_2}{2} t$$

if $|\omega_1 - \omega_2| \ll \omega_1, \omega_2$ and $\omega_a \gg \omega_b$

$$\text{then } S_1 + S_2 = 2a \cos \omega_b t \cos \omega_a t$$

$$\text{where } \omega_b = \frac{\omega_1 - \omega_2}{2} \quad \text{and} \quad \omega_a = \frac{\omega_1 + \omega_2}{2}$$

The resultant intensity of wave, waxes and wanes with a frequency $2\omega_b = \omega_1 - \omega_2$ and $\omega = 2\omega_a$

The beat frequency

$$\nu_{\text{beat}} = \nu_1 - \nu_2$$

