Unit And Measurement

Rivision Questions

1. The numerical value of 1 J in the new system, in which length is measured in 10 meter, mass in 10 kg and time in 1 minute, will be:

(a) 36×10^{-4} (b) 36×10^{-3}

(c) 36×10^{-2} (d) 36×10^{-1}

Ans. *(d) 36×10^{-1}*

Explanation: $n_1 u_1 = n_2 u_2$

$$1 \times \left(\frac{kg \times m^2}{s^2}\right) = n_2 \left(\frac{10\,kg \times (10\,m)^2}{s^2}\right)$$

$$n_2 = \frac{60 \times 60}{10 \times 100} = 3.6 \text{ unit}$$

2. The speed (v) of sound in a gas in given by

$$v = kP^x \rho^y$$

Where k is dimensionless constant, P is pressure, and ρ is the density,

(a) $x = \frac{1}{2}, y = \frac{1}{2}$ (b) $x = -\frac{1}{2}, y = -\frac{1}{2}$

(c) $x = \frac{1}{2}, y = -\frac{1}{2}$ (d) $x = -\frac{1}{2}, y = \frac{1}{2}$

Ans. *(c) $x = \frac{1}{2}, y = -\frac{1}{2}$*

Explanation: Let velocity be represented in terms of pressure and density as follows:

$\therefore v \propto p^x \times s^y$

Representing each of them in terms of basic physical quantities:

$\Rightarrow LT^{-1} \propto [ML^{-1}T^{-2}]^x \times [ML^{-3}]^y$

$\Rightarrow LT^{-1} \propto [M]^{x+y} \times [L]^{-x-3y} \times [T]^{-2x}$

Comparing exponents of similar terms:

$\therefore \quad -2x = -1$

$\Rightarrow \quad x = \frac{1}{2}$

Again,

$\therefore \quad -x - 3y = 1$

$\Rightarrow \quad y = -\frac{1}{2}$

So, final answer is:

$$x = \frac{1}{2} \text{ and } y = -\frac{1}{2}$$

3. A physical quantity is measured and the result is expressed as nu where, u is the unit used and n is the numerical value. If the result is expressed in various units, then

a) $n \propto$ size of u (b) $n \propto u^2$

(c) $n \propto \sqrt{u}$ (d) $n \propto \frac{1}{u}$

Ans. *(d) $n \propto \frac{1}{u}$*

Explanation: If the unit is increasing then the numerical value is decreasing and if the unit is decreasing then the numerical value is increasing.

Hence, $n \propto \frac{1}{u}$ is the correct option.

4. Which one of the following pair of quantities has the same dimensions?

(a) Force and work done

(b) Momentum and impulse

(c) Pressure and force

(d) Time period and frequency

[Delhi Gov. SQP 2022]

Ans. *(b) Momentum and impulse*

Explanation: Impulse is defined as the change in momentum.

Impulse, $I = \Delta p = mv - mu$

$= [MLT^{-1}]$

Hence, both Impulse and momentum have the same units/dimensions.

5. Which one of the following quantities has dimensions different from the remaining three?

(a) Energy per unit volume

(b) Force per unit area

(c) Product of voltage and charge per unit volume

(d) Angular momentum per unit mass

Ans. *(d) Angular momentum per unit mass*

Explanation: Energy per unit volume,

$$F = \frac{\text{Energy}}{\text{Volume}} = \frac{[ML^2T^{-2}]}{[L^3]}$$

$$= [ML^{-1}T^{-2}]$$

$$\text{Force per unit area} = \frac{[MLT^{-2}]}{[L^2]}$$

$$= [ML^{-1}T^{-2}]$$

Rough Work :

Product of the charge per unit volume and voltage

$$= Q\frac{(IT)V}{\text{volume}} = \frac{[ML^2T^{-2}]}{[L^3]}$$

$$= [ML^{-1}T^{-2}]$$

Angular momentum per unit mass

$$= \frac{[M^1L^2T^{-1}]}{[M]}$$

$$= [L^2T^{-1}]$$

6. The dimension of light year is ....

(a) T **(b) LT^{-1}**

(c) L **(d) T^{-1}**

[Delhi Gov. SQP 2022]

Ans. *(c) L*

Explanation: We know that, Light year is the unit of distance. The dimension of distance is [L].

So, the dimensional formula of Light year is [L].

7. Find the density of the material of the body in correct significant figures if the mass and volume of a body are 4.237 g and 2.5 cm^3 respectively.

(a) 1.6048 g cm^{-3} **(b) 1.69 g cm^{-3}**

(c) 1.7 g cm^{-3} **(d) 1.695 g cm^{-3}**

Ans. *(c) 1.7 g cm^{-3}*

Explanation: The density of a material is given by, $\rho = \frac{m}{V}$.

Where, m is mass and v is volume.

$$\rho = \frac{4.237}{2.5}$$

$$= 1.6948 \text{ g/cm}^3.$$

8. If momentum (P), area (A) and time (T) are taken to be fundamental quantities, then energy has the dimensional formula:

(a) $[P^1 A^{-1} T^1]$ **(b) $[P^2 A^1 T^1]$**

(c) $[P^1 A^{-1/2} T^1]$ **(d) $[P^1 A^{1/2} T^{-1}]$**

[Delhi Gov. QB 2022]

Ans. *(d) $[P^1A^{½}T^{-1}]$*

Explanation: Let energy,

$$E \propto P^a A^b T^c$$

Or $$E = k\, P^a A^b T^c$$

Or $$[ML^2T^{-2}] = [ML^1T^{-1}]^a [L^2]^b [T]^c$$

$$= [M^a L^{a+2b} T^{-a+c}]$$

Hence, $a = 1$, $b = 12$, $c = -c$

Dimensional formula for E is $[P^1 A^{1/2} T^{-1}]$.

9. The dimensional unit of permeability is:

(a) $[M^{-1} LT^{-2} A]$ **(b) $[ML^{-2} T^{-2}A^{-1}]$**

(c) $[MLT^{-2} A^{-2}]$ **(d) $[MLT^{-1} A^{-1}]$**

Ans. *(c) $[MLT^{-2} A^{-2}]$*

Explanation: Units of permeability are equivalent to $\frac{N}{Amp^2}$

Thus, dimensions are $= \frac{M^1L^1T^{-2}}{A^2}$

$$= [M^1LT^{-2}A^{-2}]$$

10. The dimension of $\frac{1}{2}\varepsilon_0E^2$, where, ε_0 is permittivity of free space and E is electric field, is:

(a) $[MLT^{-1}]$ **(b) $[ML^2T^{-2}]$**

(c) $[ML^{-1}L^{-2}]$ **(d) $[ML^2T^{-1}]$**

Ans. *(c) $[ML^{-1}L^{-2}]$*

Explanation: $\frac{1}{2}\varepsilon_0E^2$ = energy density

$$= \frac{\text{Energy}}{\text{Volume}} = [M^1L^{-1}T^{-2}].$$

$[MLT^{-1}]$ is the dimensional unit of impulse. $[ML^2T^{-1}]$ is the dimensional unit of angular momentum and heat and $[ML^2T^{-1}]$ is the dimensional unit of Planck's constant and angular impulse.

11. The number of significant figures in 0.06900 ....

(a) 2 **(b) 4**

(c) 6 **(d) 5**

[Delhi Gov. SQP 2022]

Ans. *(b) 4*

Explanation: As we know, the trailing zero(s) in a number with a decimal point are significant.

So, 0.06900 has FOUR significant figures.

12. The atmospheric pressure is 10^6 dyne/cm^2 in CGS system. What is its value in SI unit?

(a) 10^5 Newton/m^2 **(b) 10^6 Newton/m^2**

(c) 10^4 Newton/m^2 **(d) 10^3 Newton/m^2**

Ans. *(a) 10^5 newton/m^2*

Explanation: 1 dyne = 1 g × 1 cm/s^2

1 dyne = 10^{-3} kg × 10^{-2} m/s^2

[∵ 1 m = 100 cm

And 1 kg = 1000 g]

1 dyne = 10^{-5} N

$$10^6 \text{ dyne/cm}^2 = \frac{10^6 \times 10^{-5}\text{N}}{(10^{-2})^2\text{m}^2}$$

$$= 10^5 \text{ N/m}^2$$

13. The dimensions of kinetic energy is same as that of:

(a) force **(b) pressure**

(c) work **(d) momentum**

Ans. *(c) work*

Explanation:

According to Work – Energy Theorem,

Work = ΔK.E.

Hence, their dimensions will also be equal.

Short Question Answer [2 Marks]

Q1. **If the centripetal force is of the form $m^a v^b r^c$, find the values of a, b and c.**

Ans. Dimensionally,

Force = $(\text{Mass})^a \times (\text{velocity})^b \times (\text{length})^c$

$$[MLT^{-2}] = [M^a L^b T^{-b}] = [M^a L^b T^{-b}]$$

Equating the exponents of similar quantities,

$$a = 1, b + c = 1, -b = -2$$

or, $$a = 1, b = 2, c = -1$$

or, $$F = \frac{mv^2}{r}$$

Q2. **Magnitude of force experienced by an object moving with speed v is given by $F = kv^2$. Find dimensions of k.** **[Delhi Gov. QB 2022]**

Ans. Given relation is, $F = Kv^2$

Taking dimensions of each term, we get

$$[MLT^{-2}] = [K][LT^{-1}]^2$$

$$\Rightarrow \quad [K] = [MLT^{-2}][L^2T^{-2}]$$

$$[K] = [ML^{-1}]$$

Q3. **If the velocity of light c, the constant of gravitation G and Planck's constant h be chosen as fundamental units, find the dimensions of mass, length and time in terms of c, G and h.**

Ans. $[c] = LT^{-1}$, $[G] = M^{-1}L^3T^{-2}$, $[h] = ML^2T^{-1}$

$$\frac{[h][c]}{[g]} = \frac{ML^2T^{-1}LT^{-1}}{M^{-1}L^3T^{-2}} = M^2$$

Hence, $$[M] = h^{\frac{1}{2}} c^{\frac{1}{2}} G^{-\frac{1}{2}}$$

Again, $$\frac{[h]}{[c]} = \frac{ML^2T^{-1}}{LT^{-1}} = ML$$

$$[L] = \frac{h}{c[M]}$$

$$= \frac{h}{ch^{\frac{1}{2}}c^{\frac{1}{2}}G^{-\frac{1}{2}}}$$

$$= h^{-\frac{1}{2}} c^{-\frac{3}{2}} G^{-\frac{1}{2}}$$

As $$[c] = LT^{-1}$$

$$[T] = \frac{[L]}{c} = \frac{h^{\frac{1}{2}}c^{-\frac{3}{2}}G^{\frac{1}{2}}}{c} = h^{\frac{1}{2}} c^{-\frac{5}{2}} G^{\frac{1}{2}}$$

Q4. Deduce the dimensional formula for the following quantities:

(A) Gravitational constant

(B) Young's modulus

(C) Coefficient of viscosity.

[Delhi Gov. QB 2022]

Ans. (A) Universal constant of gravitation G.

$$\text{Gravitational Force} = G\frac{m_1 m_2}{r^2}$$

$$\Rightarrow \quad G = \frac{F \cdot r^2}{m_1 m_2}$$

$$[G] = \frac{[MLT^{-2}][L^2]}{[M][M]}$$

$$[G] = M^{-1}L^3T^{-2}$$

(B) Unit of Young's Modulus = N/m^2

$$= kgm/s^2m^2 = \frac{kg}{ms^2} = ML^{-1}T^{-2}$$

(C) Coefficient of viscosity

$$n = \frac{F}{A(dv/dx)}$$

$$\Rightarrow \quad = \frac{[MLT^{-2}]}{[L^2]\left[\frac{LT^{-1}}{L}\right]}$$

$$[n] = ML^{-1}T^{-1}$$

Q5. **The frequency of vibration of a string depends of on, (i) tension in the string (ii) mass per unit length of string, (iii) vibrating length of the string. Establish dimensionally the relation for frequency.**

[Delhi Gov. QB 2022]

Ans. Expression of frequency is given as follows:

Frequency,

$$f \propto [L^a] \quad \text{...(i)}$$

$$f \propto [F^b] \quad \text{...(ii)}$$

$$f \propto [M^c] \quad \text{...(iii)}$$

Combining eqn (i), (ii) and (iii) we can say:

$$f = [KL^aF^bM^c]$$

where $M = \dfrac{\text{Mass}}{\text{Unit length}}$

L = Length

F = Tension (Force)

Dimension of $f = [T^{-1}]$

Dimension of right side:

Dimension of force,

$$F = [MLT^{-2}]^b = [M^bL^bT^{-2b}]$$

Dimension of mass per unit length

$$= [ML^{-1}]^c$$

$$= [M^cL^{-c}]$$

So, $$[T^{-1}] = [L^a][M^bL^bT^{-2b}][M^cL^{-c}]$$

$$[M^0L^0T^{-1}] = M^{b+c}L^{a+b-c}T^{-2b}$$

Equating the dimensions of both sides, we get

$$b + c = 0, b + c = 0 \quad \text{...(i)}$$

$$-c + a + b = 0 - c + a + b = 0 \quad \text{...(ii)}$$

$$-2b = -1 - 2b = -1 \quad \text{...(iii)}$$

On Solving the equations, we get

$$a = -1, b = \frac{1}{2} \text{ and } c = \frac{-1}{2}$$

$$\therefore \quad f = KL^{-1}F^{\frac{1}{2}}M^{\frac{-1}{2}}$$

Hence, expression of frequency will be as follows:

$$f = \frac{K}{L}\sqrt{\frac{F}{M}}$$

Q.Find the number of significant figures in following numbers

Ans :

0.02380 - 4

23.08 - 4

23.80 - 4

2380 - 3

43.00 - 4

4300 - 2

4.700×10^2 - 4

4.700×10^{-3} - 4

Q.Write any two limitations of dimensional analysis.

1) Dimensional analysis check only the dimensional correctness of an equation, but not the exact correctness.
2) The dimensionless constants cannot be obtained by this method.
3) We cannot deduce a relation, if a physical quantity depends on more than three physical quantities.
4) The method cannot be considered to derive equations involving more than one term
5) A formula containing trigonometric , exponential and logarithmic function can not be derived from it.
6) It does not distinguish between the physical quantities having same dimensions.

Q6.derive the expression for period of oscillations of a simple pendulum using the method of dimensions

Ans:

$$T \propto m^x \; l^y \; g^z$$

$$T = k\, m^x\, l^y\, g^z \longrightarrow (1)$$

Writing the dimensions on both sides,

$$M^0L^0T^1 = M^x\, L^y\, (LT^{-2})^z$$

$$M^0L^0T^1 = M^x\, L^y\, L^z\, T^{-2z}$$

$$M^0L^0T^1 = M^x\, L^{y+z}\, T^{-2z}$$

equating the dimensions on both sides,

$$x = 0$$

$$y + z = 0$$

$$-2z = 1 \qquad z = \frac{-1}{2}$$

$$y + \frac{-1}{2} = 0 \qquad y = \frac{1}{2}$$

$$\boxed{T = k m^0\, l^{1/2} g^{-1/2}}$$

$$T = k\frac{l^{1/2}}{g^{1/2}}$$

Q7. In the given equatio x= a + bt + ct ² , find the dimensions of a,b and c. (where x is in meters and t in seconds)

Ans:

$$x = a + bt + ct^2$$
$$[x] = [a] = [bt] = [ct^2]$$

$$[a] = [x]$$
$$[a] = L$$

$$[bt] = [x]$$
$$[b] \times T = L$$
$$[b] = \frac{L}{T}$$
$$[b] = LT^{-1}$$

$$[ct^2] = [x]$$
$$[c] \times T^2 = L$$
$$[c] = \frac{L}{T^2}$$
$$[c] = LT^{-2}$$

Basic Electrical Quantities

Physical Quantity	Symbol	Dimensional Formula
Electric charge	Q	$[IT]$
Electric current	I	$[I]$
Potential difference (Voltage)	V	$[ML^2T^{-3}I^{-1}]$
Electric potential energy	U	$[ML^2T^{-2}]$
Electric field	E	$[MLT^{-3}I^{-1}]$
Electric flux	Φ	$[ML^3T^{-3}I^{-1}]$
Electric dipole moment	p	$[ITL]$

Magnetic Quantities

Physical Quantity	Symbol	Dimensional Formula
Magnetic field	B	$[MT^{-2}I^{-1}]$
Magnetic flux	Φ	$[ML^2T^{-2}I^{-1}]$
Magnetic permeability	μ	$[MLT^{-2}I^{-2}]$
Magnetic susceptibility	χ	Dimensionless
Magnetic dipole moment	M	$[IL^2]$

◆ Resistance, Capacitance & Related

Physical Quantity	Symbol	Dimensional Formula
Resistance	R	$[ML^2T^{-3}I^{-2}]$
Resistivity	ρ	$[ML^3T^{-3}I^{-2}]$
Conductance	G	$[M^{-1}L^{-2}T^3I^2]$
Capacitance	C	$[M^{-1}L^{-2}T^4I^2]$
Electric power	P	$[ML^2T^{-3}]$

◆ Electromagnetic & Derived Quantities

Physical Quantity	Symbol	Dimensional Formula
Inductance	L	$[ML^2T^{-2}I^{-2}]$
EMF	ε	$[ML^2T^{-3}I^{-1}]$
Current density	J	$[IL^{-2}]$
Drift velocity	v	$[LT^{-1}]$
Permittivity of free space	ε_0	$[M^{-1}L^{-3}T^4I^2]$