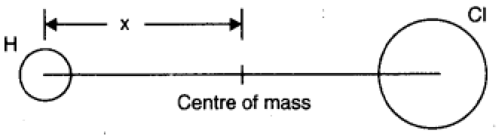
Physics class 11th

System of Particles and Rotational Motion

Q.1 In the HCl molecule, the separation between the nuclei of the two atoms is about 1.27 A (1 A = 10^{-10}m). Find the approximate location of the CM of the molecule, given that a chlorine atom is about 35.5 times as massive as a hydrogen atom and nearly all the mass of an atom is concentrated in its nucleus.

Answer: Let us choose the nucleus of the hydrogen atom as the origin for measuring distance.

Mass of hydrogen atom, m_1 = 1 unit (say)
Since cholorine atom is 35.5 times as massive as hydrogen atom,
mass of cholorine atom, m_2 = 35.5 units



Now, $x_1 = 0$ and $x_2 = 1.27$ Å $= 1.27 \times 10^{-10}$ m

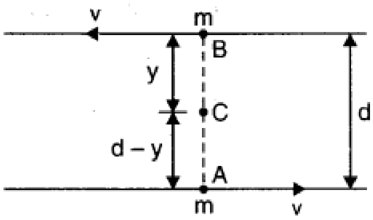
Distance of centre of mass of HCl molecule from the origin is given by

$$X = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{1 \times 0 + 35.5 \times 1.27 \times 10^{-10}}{1 + 35.5} \text{ m}$$

$$= \frac{35.5 \times 1.27}{36.5} \times 10^{-10} \text{ m} = 1.235 \times 10^{-10} \text{m} = 1.235 \text{ Å}$$

Q.2.Two particles, each of mass m and speed v, travel in opposite directions along parallel lines separated by a distance d. Show that the vector angular momentum of the two particle system the same whatever be the point about which the angular momentum is taken.

Answer:



Angular momentum about A,

$$L_A = mv \times 0 + mv \times d$$
$$= mvd$$

Angular momentum about B,

$$L_B = mv \times d + mv \times 0$$
$$= mvd$$

Angular momentum about C,

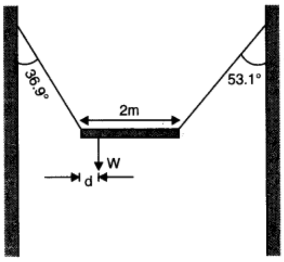
$$L_c = mv \times y + mv \times (d - y) = mvd$$

In all the three cases, the direction of angular momentum is the same.

$$\therefore \quad \vec{L}_A = \vec{L}_B = \vec{L}_C.$$

Q3. **A non-uniform bar of weight W is suspended at rest by two strings of negligible weight as shown in Fig. The angles made by the strings with the vertical are 36.9° and ' 53.2° respectively. The bar is 2 m long. Calculate the distance d of the centre of gravity of the bar from its left end.**

Answer:



As it is clear from Fig.,

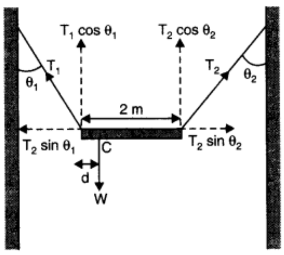
$$\theta_1 = 36.9°, \quad \theta_2 = 53.1°.$$

If T_1, T_2 are the tensions in the two strings, then for equilibrium along the horizontal,

$$T_1 \sin \theta_1 = T_2 \sin \theta_2$$

or
$$\frac{T_1}{T_2} = \frac{\sin \theta_2}{\sin \theta_1} = \frac{\sin 53.1°}{\sin 36.9°}$$

$$= \frac{0.7404}{0.5477} = 1.3523$$



Let d be the distance of centre of gravity C of the bar from the left end.

For rotational equilibrium about C,

$$T_1 \cos \theta_1 \times d = T_2 \cos \theta_2 (2 - d)$$
$$T_1 \cos 36.9° \times d = T_2 \cos 53.1° (2 - d)$$
$$T_1 \times 0.8366\ d = T_2 \times 0.6718\ (2 - d)$$

Put $T_1 = 13523\ T_2$ and solve to get

$$d = 0.745 \text{ m}$$

$$\tau = I_1\alpha_1 = I_2\alpha_2$$

or
$$\frac{\alpha_1}{\alpha_2} = \frac{I_2}{I_1} = \frac{(2/5)MR^2}{MR^2} = \frac{2}{5}$$

or
$$\alpha_2 = \frac{5}{2}\alpha_1.$$

Q4.(a) Find the moment of inertia of a sphere about a tangent to the sphere, given the moment of inertia of the sphere about any of its diameters to be 2 MR^2/5, where M is the mass of the sphere and R is the radius of the sphere.
(b) Given the moment of inertia of a disc of mass M and radius R about any of its diameters to 1 be 1/4 MR^2, find the moment of inertia about an axis normal to the disc passing through a point on its edge.
Answer: (a) Moment of inertia of sphere about any diameter = 2/5 MR^2
Applying theorem of parallel axes, Moment of inertia of sphere about a tangent to the sphere

= 2/5 MR^2 +$M(R)^2$ =7/5 MR^2
(b) We are given, moment of inertia of the disc about any of its diameters = 1/4 MR^2
(i) Using theorem of perpendicular axes, moment of inertia of the disc about an axis passing through its centre and normal to the disc = 2 x 1/4 MR^2 = **1/2 MR^2.**
(ii) Using theorem axes, moment of inertia of the disc passing through a point on its edge and normal to the dies
= 1/2 MR^2+ MR^2 = **3/2 MR^2.**

Q5.A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 rad s^{-1}. The radius of the cylinder is 0.25 m. What is the kinetic energy associated with the rotation of the cylinder? What is the magnitude of angular momentum of the cylinder about its axis?
Answer: M = 20 kg
Angular speed, ω = 100 rad s^{-1};

R = 0.25 m
Moment of inertia of the cylinder about its axis
=1/2 MR^2

= 1/2 x 20 $(0.25)^2$ kg m^2

= 0.625 kg m^2
Rotational kinetic energy,
E_r = 1/2 $I\omega^2$

= 1/2 x 0.625 x $(100)^2$ J = 3125 J
Angular momentum,
L = $I\omega$= 0.625 x 100 J_s= 62.5 J_s.

Q 6.(a)A child stands at the centre of a turntable with his arms outstretched. The turntable is set rotating with an angular speed of 40 rev/min. How much is the angular speed of the child if he folds his hands back and thereby reduces his moment of inertia to 2/5 times the initial value? Assume that the turntable rotates without friction,
(b) Show that the child's new kinetic energy of rotation is more than the initial kinetic energy of rotation. How do you account of this increase in kinetic energy?

Answer: (a) Suppose, initial moment of inertia of the child is I1 Then final moment of inertia,

$$I_2 = \frac{2}{5}I_l$$

Also, $\nu_1 = 40 \text{ rev min}^{-1}$

By using the principle of conservation of angular momentum, we get

$$I_1\omega_1 = I_2\omega_2 \quad \text{or} \quad I_1(2\pi\nu_1) = I_2(2\pi\nu_2)$$

or

$$\nu_2 = \frac{I_1\nu_1}{I_2} = \frac{I_1 \times 40}{\frac{2}{5}\times I_1} = 100 \text{ rev min}^{-1}$$

(b)

$$\frac{\text{Final K.E. of rotation}}{\text{Initial K.E. of rotation}} = \frac{\frac{1}{2}I_2\omega_2^2}{\frac{1}{2}I_1\omega_1^2} = \frac{\frac{1}{2}I_2(2\pi\nu_2)^2}{\frac{1}{2}I_1(2\pi\nu_1)^2} = \frac{I_2\nu_2^2}{I_1\nu_1^2} = \frac{\frac{2}{5}I_1\times(100)^2}{\frac{2}{5}I_1\times(40)^2} = 2.5$$

Clearly, final $(K.E.)_{rot}$ becomes more because the child uses his internal energy when he folds his hands to increase the kinetic energy.

Q7.A rope of negligible mass is wound round a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N? What is the linear acceleration of the rope? Assume that there is no slipping.

Answer: Here, M = 3 kg, R = 40 cm = 0.4 m

Moment of inertia of the hollow cylinder about its axis.

$I = MR^2 = 3(0.4)^2 = 0.48$ kg m^2

Force applied $F = 30$ N

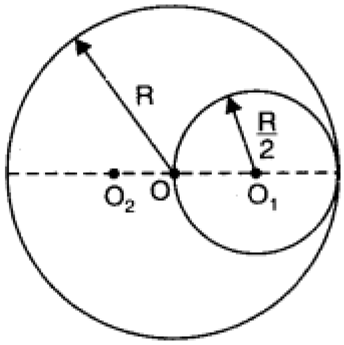
∴ Torque, $\tau = F \times R = 30 \times 0.4 = 12$ N–m.

If α is angular acceleration produced, then from $\tau = I\alpha$

$$\alpha = \frac{\tau}{I} = \frac{12}{0.48} = 25 \text{ rad s}^{-2}$$

Linear acceleration, $a = R\alpha = 0.4 \times 25 = 10 \text{ ms}^{-2}$.

Q8.From a uniform disk of radius R, a circular hole of radius R/2 is cut out. The centre of the hole is at R/2 from the centre of the original disc.Locate the centre of gravity of the resulting flat body.



Answer: Let from a bigger uniform disc of radius R with centre O a smaller circular hole of radius R/2 with its centre at O_1 (where R OO_1 = R/2) is cut out. Let centre of gravity or the centre of mass of remaining flat body be at O_2, where OO_2 = x. If σ be mass per unit area, then mass of whole disc $M_1 = \pi R^2\sigma$ and mass of cut out part

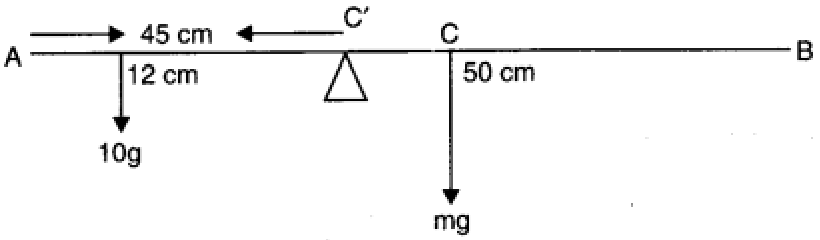
$$M_2 = \pi\left(\frac{R}{2}\right)^2 \sigma = \frac{1}{4}\pi R^2 \sigma = \frac{M_1}{4}$$

$$\therefore \qquad x = \frac{M_1 \times (0) - M_2(OO_1)}{M_1 - M_2} = \frac{0 - \frac{M_1}{4} \times \frac{R}{2}}{M_1 - \frac{M_1}{4}} = -\frac{R}{6}$$

i.e., O_2 is at a distance R/6 from centre of disc on diametrically opposite side to centre of hole.

(MID-Term 2025) **Q9. A metre stick is balanced on a knife edge at its centre. When two coins, each of mass 5g are put one on top of the other at the 12.0 cm mark, the stick is found to be balanced at 45.0 cm. What is the mass of the metre stick?**

Answer: Let m be the mass of the stick concentrated at C, the 50 cm mark, see fig.



For equilibrium about C, the 45 cm mark,

10 g (45 – 12) = mg (50 – 45)

10 g x 33 = mg x 5

=> m = 10 x 33/5

or m = 66 grams.

Q10. A hoop of radius 2 m weighs 100 kg. It rolls along a horizontal floor so that its centre of mass has a speed of 20 cm/s. How much work has to be done to stop it?

Answer: Here, R = 2 m, M = 100 kg

v = 20 cm/s = 0.2 m/s

Total energy of the hoop $=1/2Mv^2 + 1/2I\omega^2$

$=1/2Mv^2 + 1/2(MR^2)\,\omega^2$

$=1/2Mv^2 + 1/2Mv^2 = Mv^2$

Work required to stop the hoop = total energy of the hoop $W = Mv^2 = 100\,(0.2)^2 = 4$ Joule.

Q11. A bullet of mass 10 g and speed 500 m/s is fired into a door and gets embedded exactly at the centre of the door. The door is 1.0 m wide and weighs 12 kg. It is hinged at one end and rotates about a vertical axis practically without friction. Find the angular speed of the door just after the bullet embeds into it.(Hint: The moment of inertia of the door about the vertical axis at one end is $ML^2/3$.)

Answer: Angular momentum imparted by the bullet,

L = mv x r

$=(10 \times 10^{-3}) \times 500 \times 1/2$

=2.5

Also ,$I=ML^2/3$

$=12 \times (1.0)^2/3$

$=4$ kg m^2

Since $L=I\omega$

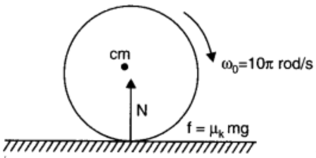
$\omega=L/I$

=2.5/4

=0.625 rad / s

Q12.- A solid disc and a ring, both of radius 10 cm are placed on a horizontal table simultaneously, with initial angular speed equal to 10π rad/s. Which of two will start to roll earlier? The coefficient of kinetic friction is u_k = 0.2.

Answer: When a disc or ring starts rotatory motion on a horizontal surface, initial translational velocity of centre of mass is zero.



The frictional force causes the centre of mass to accelerate linearly but frictional torque causes angular retardation. As force of normal reaction N = mg, hence frictional force f = u_k N = u_k mg.

For linear motion f = u_k . mg = ma ———-(i)

and for rotational motion, $\tau = f \cdot R = \mu_k\, mg \cdot R = -I\alpha$ ...(ii)

Let perfect rolling motion starts at time t, when $v = R\omega$

From (i) $a = \mu_k \cdot g$

$\therefore$ $v = u + at = 0 + \mu_k \cdot g \cdot t$ ...(iii)

From (ii) $\alpha = -\dfrac{\mu_k \cdot mgR}{I} = -\dfrac{\mu_k \cdot mgR}{mK^2} = -\dfrac{\mu_k \cdot gR}{K^2}$

$\therefore$ $\omega = \omega_0 + \alpha t = \omega_0 - \dfrac{\mu_k \cdot gR}{K^2}t$ ...(iv)

Since $v = R\omega$, hence $\mu_k \cdot g \cdot t = R\left[\omega_0 - \mu_k \cdot \dfrac{gR}{K^2}t\right]$

$\Rightarrow$ $t^2 = \dfrac{R\omega_0}{\mu_k \cdot g\left(1+\dfrac{R^2}{K^2}\right)}$

For disc, $K^2 = \dfrac{R^2}{2}$, hence $t = \dfrac{\omega_0 R}{\mu_k \cdot g\left(1+\dfrac{R^2}{R^2/2}\right)} = \dfrac{\omega_0 R}{3\mu_k \cdot g}$

For ring, $K^2 = R^2$, hence $t = \dfrac{\omega_0 R}{\mu_k \cdot g\left(1+\dfrac{R^2}{R^2}\right)} = \dfrac{\omega_0 R}{2\mu_k \cdot g}$

Thus, it is clear that disc will start to roll earlier. The actual time at which disc starts rolling will be

$$t = \frac{\omega_0 R}{2\mu_k \cdot g} = \frac{(10\pi)\times(0.1)}{3\times(0.2)\times 9.8} = 0.538.$$

Q13.A cylinder of mass 10 kg and radius 15 cm is rolling perfectly on a plane of inclination 30°. The coefficient of static friction us = 0.25.
(a) How much is the force of friction acting on the cylinder?
(b) What is the work done against friction during rolling?
(c) If the inclination O of the plane is increased, at what value of 8 does the cylinder begin to skid, and not roll perfectly?
Answer:

(a) $$f = \frac{1}{2} mg \sin\theta$$
$$= \frac{1}{3} \times 10 \times 9.8 \times \sin 30° \text{ N} = \mathbf{16.3\ N.}$$

(b) **No work is done** against friction during rolling.

(c) $$\mu = \frac{1}{3} \tan\theta \quad \text{or} \quad \tan\theta = 3\mu$$
$$\text{tam}\,\theta = 3 \times 0.25 = 0.75$$
$$\theta = \tan^{-1}(0.75)$$
$$= 36.87° = \mathbf{37°.}$$

Mechanical Properties of Fluids

Q1. Toricelli's barometer used mercury. Pascal duplicated it using French wine of density 984 kg m-3. Determine the height of the wine column for normal atmospheric pressure.
Answer:

We know that atmospheric pressure, $P = 1.01 \times 10^5$ Pa.

If we use French wine of density, $\rho = 984$ kg m^{-3}, then height of wine column should be h_m, such that $P = h\rho g$

$$\Rightarrow \quad h_m = \frac{P}{\rho g} = \frac{1.01 \times 10^5}{984 \times 9.8} = 10.47\text{m} \simeq 10.5 \text{ m}$$

Q2. A U tube contains water and methylated spirit separated by mercury. The mercury columns in the two arms are in level with 10.0 cm of water in one arm and 12.5 cm of spirit in the other. What is the relative density of spirit?

Answer: For water column in one arm of U tube,
$h_1 = 10.0$ cm; ρ_1 (density) = 1 g cm^{-3}
For spirit column in other arm of U tube,
$h_2 = 12.5$ cm; ρ_2 =?
As the mercury columns in the two arms of U tube are in level, therefore pressure exerted by each is equal.
Hence **$h_1\rho_1 g = h_2\rho_2 g$**
or $\rho_2 = h_1\rho_1/h_2$
=10 x 1/12.5 = 0.8 g cm^{-3}
Therefore, relative density of spirit = ρ_2/ρ_1 = 0.8/1 **= 0.8 Ans.**

Q3. if 15.0 cm of water and spirit each are further poured into the respective arms of the tube, what is the difference in the levels of mercury in the two arms? (Relative density of mercury = 13.6)
Answer:
Height of the water column, h_1 = 10 + 15 = 25 cm
Height of the spirit column, h_2 = 12.5 + 15 = 27.5 cm
Density of water, ρ_1 = 1 g cm^{-3}
Density of spirit, ρ_2 = 0.8 g cm^{-3}
Density of mercury = 13.6 g cm^{-3}
Let h be the difference between the levels of mercury in the two arms.
Pressure exerted by height h, of the mercury column:
= $h\rho g$
= $h \times 13.6g$ … (i)
Difference between the pressures exerted by water and spirit:
= $\rho_1 h_1 g - \rho_2 h_2 g$
= $g(25 \times 1 - 27.5 \times 0.8)$
= $3g$ … (ii)
Equating equations (i) and (ii), we get:

13.6 hg = 3g

$\boldsymbol{h}$ **= 0.220588 ≈ 0.221 cm**

Hence, the difference between the levels of mercury in the two arms is **0.221 cm.**

Q4.In a test experiment on a model aeroplane in a wind tunnel, the flow speeds on the upper and lower surfaces of the wing are 70 m $s^{-1}$and 63 m s^{-1} respectively. What is the lift on the wing if its area is 2.5 m^2 ? Take the density of air to be 1.3 kg m^{-3}

Answer:

Let v_1, v_2 be the speeds on the upper and lower surfaces of the wing of aeroplane, and P_1 and P_2 be the pressures on upper and lower surfaces of the wing respectively.

Then v_1 = 70 ms^{-1}; v_2 = 63 ms^{-1}; ρ = 1.3 kg m^{-3}.

From Bernoulli's theorem

$$\frac{P_1}{\rho} + gh + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + gh + \frac{1}{2}v_2^2$$

$$\therefore \quad \frac{P_1}{\rho} - \frac{P_2}{\rho} = \frac{1}{2}(v_2^2 - v_1^2)$$

or $$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) = \frac{1}{2}\times 1.3\,[(70)^2 - (63)^2]\ \text{Pa} = 605.15\ \text{Pa}.$$

This difference of pressure provides the lift to the aeroplane. So, lift on the aeroplane

= pressure difference × area of wings

= 605.15 × 2.5 N = 1512.875 N

= 1.51 × 10^3 N.

Q5. **he cylindrical tube of a spare pump has a cross-section of 8.0 cm^2 one end of which has 40 fine holes each of diameter 1.0 mm. If the liquid flow inside the tube is 1.5 m min^{-1}, what is the speed of ejection of the liquid through the holes?**

Answer:

Total cross-sectional area of 40 holes, a_2

$$= 40 \times \frac{22}{7} \times \frac{(1\times 10^{-3})^2}{4} m^2$$

$$= \frac{22}{7} \times 10^{-5} m^2$$

Cross-sectional area of tube, a_1 = 8 × 10^{-4} m^2

Speed inside the tube, v_1= 1.5 m min^{-1} = $\frac{1.5}{60} ms^{-1}$;

Speed of ejection, v_2 = ?

Using $a_2 v_2 = a_1 v_1$,

we get

$$v_2 = \frac{a_1 v_1}{a_2} = \frac{8\times 10^{-4} \times \frac{1.5}{60} \times 7}{22 \times 10^{-5}} ms^{-1} = 0.64\ ms^{-1}.$$

Q6. **What is the pressure inside a drop of mercury of radius 3.0 mm at room temperature? Surface tension of mercury at that temperature (20°C) is 4.65 x 10^{-1} Nm^{-1}. The atmospheric pressure is 1.01 x 10^5 Pa. Also give the excess pressure inside the drop.**

Answer:

$$\text{Excess pressure} = \frac{2\sigma}{R} = \frac{2\times 4.65 \times 10^{-1}}{3\times 10^{-3}} = 310\ \text{Pa}$$

$$\text{Total pressure} = 1.01 \times 10^5 + \frac{2\sigma}{R}$$

$$= 1.01 \times 10^5 + 310 = 1.0131 \times 10^5\ \text{Pa}$$

Q7. (a) What is the largest average velocity of blood flow in an artery of radius 2 x 10^3 m if the flow must remain laminar?

(b) What is the corresponding flow rate? Take viscosity of blood to be 2.084 x 10^{-3} Pa-s. Density of blood is 1.06 x 10^3 kg/m^3 .

Answer:

Here, $r = 2 \times 10^{-3}$ m; $D = 2r = 2 \times 2 \times 10^{-3} = 4 \times 10^{-3}$ m;

$\eta = 2.084 \times 10^{-3}$ Pa–s; $\rho = 1.06 \times 10^3$ kg m^{-3}.

For flow to be laminar, $N_R = 2000$

(*a*) Now, $v_c = \dfrac{N_R \eta}{\rho D} = \dfrac{2000 \times (2.084 \times 10^{-3})}{(1.06 \times 10^3) \times (4 \times 10^{-3})} = 0.98$ m/s.

(*b*) Volume flowing per second $= \pi r^2 v_c = \dfrac{22}{7} \times (2 \times 10^{-3})^2 \times 0.98 = \mathbf{1.23 \times 10^{-5}\ m^3s^{-1}}$.

Q8. A plane is in level flight at constant speed and each of its wings has an area of 25 m^2. If the speed of the air is 180 km/h over the lower wing and 234 km/h over the upper wing surface, determine the plane's mass. (Take air density to be 1 kg/m^3), g = 9.8 m/s^2.

Answer:

Here speed of air over lower wing, $v_1 = 180$ km/h $= 180 \times \dfrac{5}{18} = 50$ ms^{-1}

Speed over the upper wing, $v_2 = 234$ km/h $= 234 \times \dfrac{5}{18} = 65$ ms^{-1}

$\therefore$ Pressure difference, $P_1 - P_2 = \dfrac{1}{2}\rho\,(v_2^2 - v_1^2) = \dfrac{1}{2} \times 1\,(65^2 - 50^2) = 862.5$ Pa

$\therefore$ Net upward force, $F = (P_1 - P_2)A$

This upward force balances the weight of the plane.

$\therefore \quad mg = F = (P_1 - P_2)A \qquad [A = 25 \times 2 = 50 \text{ m}^2]$

$\therefore \quad m = \dfrac{(P_1 - P_2)\,A}{g} = \dfrac{862.5 \times 50}{9.8} = 4400$ N.

Q9. In Millikan's oil drop experiment, what is the terminal speed of an uncharged drop of radius 2.0 x 10^{-5} m and density 1.2 x 10^3 kg m^{-3}. Take the viscosity of air at the temperature of the experiment to be 1.8 x 10^{-5} Pa-s. How much is the viscous force on the drop at that speed? Neglect buoyancy of the drop due to air.

Answer: Here radius of drop, r = 2.0 x 10^{-5} m, density of drop, p = 1.2 x 10^3 kg/m^3,
viscosity of air TI = 1.8 x 10^{-5} Pa-s.

Neglecting upward thrust due to air, we find that terminal speed is

$$v_T = \frac{2}{9}\frac{r^2 \rho g}{\eta} = \frac{2 \times (2.0 \times 10^{-5})^2 \times (1.2 \times 10^3) \times 9.8}{9 \times (1.8 \times 10^{-5})}$$

$$= 5.81 \times 10^{-2} \text{ ms}^{-1} \quad \text{or} \quad 5.81 \text{ cm s}^{-1}$$

Viscous force at this speed,

$$F = 6\pi\eta r v = 6 \times 3.14 \times (1.8 \times 10^{-5}) \times (2.0 \times 10^{-5}) \times (5.81 \times 10^{-2})$$

$$= 3.94 \times 10^{-10} \text{ N}.$$

Q10. **Mercury has an angle of contact equal to 140° with soda-lime glass. A narrow tube of radius 1.0 mm made of this glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside? Surface tension of mercury at the temperature of the experiment is 0.465 Nm^{-2}. Density of mercury = 13.6 x 10 kg m^{-3}**

Answer:

Radius of tube, $r = 1.00$ mm $= 10^{-3}$ m

Surface tension of mercury, $\sigma = 0.465$ Nm^{-1}

Density of mercury, $\sigma = 13.6 \times 10^3$ kg m^{-3}

Angle of contact, $\theta = 140°$

$$\therefore \quad h = \frac{2\sigma\cos\theta}{r\rho g} = \frac{2 \times 0.465 \times \cos 140°}{10^{-3} \times 13.6 \times 10^3 \times 9.8}$$

$$= \frac{2 \times 0.465 \times (-0.7660)}{10^{-3} \times 13.6 \times 10^3 \times 9.8}$$

$$= -5.34 \times 10^{-3} \text{ m} = \mathbf{-5.34\ mm}$$

Negative sign shows that the mercury level is depressed in the tube.

Thermal property of matter

Q1. The-electrical resistance in ohms of a certain thermometer varies with temperature according to the approximate law: R = R0 [1 + α (T – T_0)].
The resistances is 101.6 Ωat the triple-point of water 273.16 K, and 165.5 Ωat the normal melting point of lead (600.5 K). What is the temperature when the resistance is 123.4 Ω ?

Answer: Here, R0 = 101.6 Ω; T0 = 273.16 K Case

(i) R_1= 165.5 Ω; T_1 = 600.5 K, Case

(ii) R_2 = 123.4 , T_2 = ?

Using the relation R = R0[1 + α (T – T0)]

Case (i) 165.5 = 101.6 [1 + α (600.5 – 273.16)]

$$\alpha = \frac{165.5-101.6}{101.6\times(600.5-273.16)} = \frac{63.9}{101.6\times327\times34}$$

Case (*ii*)

$$123.4 = 101.6\ [1 + \alpha\ (T_2 - 273.16)]$$

or

$$123.4 = 101.6\left[1+\frac{63.9}{101.6\times327.34}(T_2-273.16)\right]$$

$$= 101.6 + \frac{63.9}{327.34}\ (T_2 - 273.16)$$

or

$$T_2 = \frac{(123.4-101.6)\times327.34}{63.9}+273.16 = 111.67 + 273.16$$

$$= \mathbf{384.83\ K}$$

Q2. **A brass wire 1.8 m long at 27 °C is held taut with little tension between two rigid supports. If the wire is cooled to a temperature of – 39 °C, what is the tension developed in the wire, if its diameter is 2.0 mm ? Co-efficient of linear expansion of brass = 2.0 x $10^{-5}K^{-1}$; Young's modulus of brass = 0.91 x 10^{11} Pa**

Ans.

Here,

$$l = 1.8\text{ m},$$
$$\Delta t = (-39 - 27)\ °C = -66\ °C$$
$$\alpha = 2.0 \times 10^{-5}\ K^{-1}$$
$$Y = 0.91 \times 10^{11}\ Pa$$
$$A = \frac{\pi D^2}{4} = \frac{22}{7}\times\frac{1}{4}\ (2 \times 10^{-3})^2\ m^2$$

Now,

$$Y = \frac{Fl}{A\Delta l} \Rightarrow \Delta l = \frac{Fl}{AY} \quad \text{or} \quad l\alpha\Delta t = \frac{Fl}{AY}$$

or

$$F = -Y A \alpha \Delta t$$

or

$$F = -0.91 \times 10^{11} \times \frac{22}{7}\times\frac{1}{4}(2 \times 10^{-3})^2 \times 2.0 \times 10^{-5} \times 66\ N$$
$$= -3.77 \times 10^2\ N.$$

Q3. **A brass rod of length 50 cm and diameter 3.0 mm is joined to a steel rod of the same length and diameter. What is the change in length of the combined rod at 250 °C, if the original lengths are at 40.0 °C ? Is there a 'thermal stress' developed at the junction ? The ends of the rod are free to expand (Co-efficient of linear expansion of brass = 2.0 x 10^{-5} °C^{-1}, steel = 1.2 x 10^{-5} °C^{-3}.**

Ans.

Here, $l_{brass} = l_{steel} = 50\ cm$, $d_{brass} = d_{steel} = 3\ mm = 0.3\ cm$, $\Delta l_{brass} = ?$, $\Delta l_{steel} = ?$

$$\Delta T = 250 - 40 = 210\ °C.$$
$$\alpha_{brass} = 2 \times 10^{-5}\ °C^{-1} \quad \text{and} \quad \alpha_{steel} = 1.2 \times 10^{-5}\ °C^{-1}$$

Now

$$\Delta l_{brass} = \alpha_{brass} \times l_{brass} \times \Delta T$$
$$= 2 \times 10^{-5} \times 50 \times 210 = 0.21\ cm$$

Now

$$\Delta l_{steel} = \alpha_{steel} \times l_{steel} \times \Delta T$$
$$= 1.2 \times 10^{-5} \times 50 \times 210$$
$$= 0.126\ cm \approx 0.13\ cm$$

∴ Total change in length, $\Delta l = \Delta l_{brass} + \Delta l_{steel} = 0.21 + 0.13 = 0.34$ cm

Since the rod is not clamped at its ends, no thermal stress is developed at the junction.

Q4.The coefficient of volume expansion of glycerine is 49 x $10^{-5}K^{-1}$. What is the fractional change in its density for a 30 °C rise in temperature ?

Ans.

Here, $\gamma = 49 \times 10^{-5}$ °C^{-1}, $\Delta T = 30$ °C

As $V = V + \Delta V = V(1 + \gamma \Delta T)$

$\therefore$ $V' = V(1 + 49 \times 10^{-5} \times 30) = 1.0147$ V

Since $\rho = \frac{m}{V}$, $\rho'' = \frac{m}{V'} = \frac{m}{1.0147V} = 0.9855\,\rho$

Fractional change in density $= \frac{\rho - \rho'}{\rho}$

$$= \frac{\rho - 0.9855\,\rho}{\rho} = 0.0145.$$

Q5. A 10 kW drilling machine is used to drill a bore in a small aluminium block of mass 8.0 kg. How much is the rise in temperature of the block in 2.5 minutes, assuming 50% of power is used up in heating the machine itself or lost to the surroundings? Specific heat of aluminium = 0.91 J g^{-1} K^{-1} .

Answer: Power = 10 kW = 10^4 W

Mass, m=8.0 kg = 8 x 10^3 g

Rise in temperature, ΔT = ?

Time, t = 2.5 min = 2.5 × 60 = 150 s

Specific heat, C = 0.91 Jg^{-1} K^{-1}

Total energy = Power × Time = $10^4 \times 150$ J

= 15×10^5 J

As 50% of energy is lost,

$\therefore$ Thermal energy available,

$$\Delta Q = \frac{1}{2} \times 15 \times 10^5 = 7.5 \times 10^5 \text{ J}$$

Since $\Delta Q = mc\Delta T$

$$\therefore \quad \Delta T = \frac{\Delta Q}{mc} = \frac{7.5 \times 10^5}{8 \times 10^3 \times 0.91} = 103° \text{ C.}$$

Q6.A body cools from 80 °C to 50°C in 5 minutes. Calculate the time it takes to cool from 60 °C to 30°C. The temperature of the surroundings is 20 °C.

Answer:

According to Newton's law of cooling, the rate of cooling is proportional to the difference in temperature.

Here average of 80 °C and 50 °C = 65 °C

Temperature of surroundings = 20°C

$\therefore$ Difference = 65 – 20 = 45 °C

Under these conditions, the body cools 30°C in time 5 minutes

$$\therefore \quad \frac{\text{Change in temp.}}{\text{Time}} = K\,\Delta\,T$$

or $$\frac{30}{5} = K \times 45° \quad ...(i)$$

The average of 60°C and 30°C is 45°C which is 25°C (45 – 20) above the room temperature and the body cools by 30°C (60 – 30) in a time t (say)

$$\therefore \quad \frac{30}{t} = K \times 25 \quad ...(ii)$$

where K is same for this situation as for the original.

Dividing eqn. (i) by (ii), we get

$$\frac{30/5}{30/t} = \frac{K \times 45}{K \times 25}$$

or $$\frac{t}{5} = \frac{9}{5}$$

$$\Rightarrow \quad t = 9 \text{ min.}$$

Q7.Calculate the heat required to convert 3 kg of ice at –12 °C kept in a calorimeter to steam at 100 °C at atmospheric pressure. Given specific heat capacity of ice = 2100 J kg^{-1} K^{-1}, specific heat capacity of water = 4186 J kg^{-1} K^{-1}, latent heat of fusion of ice = 3.35 × 10^5 J kg^{-1} and latent heat of steam = 2.256 ×10^6 J kg^{-1}.

Answer We have

Mass of the ice, m = 3 kg

specific heat capacity of ice, s_{ice} = 2100 J kg^{-1} K^{-1}

specific heat capacity of water, s_{water} = 4186 J kg^{-1} K^{-1}

latent heat of fusion of ice, $L_{f\,ice}$ = 3.35 × 10^5 J kg^{-1}

latent heat of steam, L_{steam} = 2.256 × 10^6 J kg^{-1}

Now, Q = heat required to convert 3 kg of ice at –12 °C to steam at 100 °C,

Q_1 = heat required to convert ice at –12 °C to ice at 0 °C.

= $m\, s_{ice}\, \Delta T_1$ = (3 kg) (2100 J kg^{-1} K^{-1}) [0–(–12)]°C = 75600 J

Q_2 = heat required to melt ice at 0 °C to water at 0 °C

= $m\, L_{f\,ice}$ = (3 kg) (3.35 × 10^5 J kg^{-1})

= 1005000 J

Q_3 = heat required to convert water at 0 °C to water at 100 °C.

= $m s_w\, \Delta T_2$ = (3kg) (4186J kg^{-1} K^{-1}) (100 °C)

= 1255800 J

Q_4 = heat required to convert water at 100 °C to steam at 100 °C.

= $m\, L_{steam}$ = (3 kg) (2.256 ×10^6 J kg^{-1})

= 6768000 J

Q = $Q_1 + Q_2 + Q_3 + Q_4$

= 75600J + 1005000 J + 1255800 J + 6768000 J

= 9.1×10^6 J

TRY THIS : A pan filled with hot food cools from 94 °C to 86 °C in 2 minutes when the room temperature is at 20 °C. How long will it take to cool from 71 °C to 69 °C? Ans: 42 s

Thermodynamics

Q1 A geyser heats water flowing at the rate of 3.0 litres per minute from 2 7°C to 77°C. If the geyser operates on a gas burner, what is the rate of consumption of the fuel if its heat of combustion is 4.0 x 10^4 J/g?

Answer: Volume of water heated = 3.0 litre per minute Mass of water heated, m = 3000 g per minute Increase in temperature,

$$\Delta T = 77°C - 27°C = 50°C$$

Specific heat of water, $c = 4.2\ Jg^{-1}\ °C^{-1}$

amount of heat used, $Q = mc\,\Delta T$

or $Q = 3000\ g\ min^{-1} \times 4.2\ Jg^{-1}\ °C^{-1} \times 50°C$

$= 63 \times 10^4\ J\ min^{-1}$

$$\text{Rate of combustion of fuel} = \frac{63\times10^4\,J\,min^{-1}}{4.0\times10^4\,Jg^{-1}} = 15.75\ g\ min^{-1}.$$

Q2. What amount of heat must be supplied to 2.0 x 10^{-2} kg of nitrogen (at room temperature) to raise its temperature by 45 °C at constant pressure? (Molecular mass of N_2 = 28; R = 8.3 J mol^{-1} K^{-1}.)

Answer:

Here, mass of gas, $m = 2 \times 10^{-2}$ kg = 20 g

rise in temperature, ΔT = 45°C

Heat required, ΔQ = ?; Molecular mass, M = 28

Number of moles, $n = \frac{m}{M} = \frac{20}{28} = 0.714$

As nitrogen is a diatomic gas, molar specific heat at constant pressure is

$$C_p = \frac{7}{2}R = \frac{7}{2}\times 8.3\ J\ mol^{-1}\ K^{-1}$$

As $\Delta Q = nC_p\,\Delta T$

$\therefore$ $\Delta Q = 0.714 \times \frac{7}{2} \times 8.3 \times 45\ J = 933.4\ J.$

Q3.Explain why

(a) Two bodies at different temperatures T_1 and T_2, if brought in thermal contact do not necessarily settle to the mean temperature $(T_1 + T_2)/2$?

(b) The coolant in a chemical or nuclear plant (i.e., the liquid used to prevent different parts of a plant from getting too hot) should have high specific heat. Comment.

(c) Air pressure in a car tyre increases during driving. Why?

(d) The climate of a harbour town is more temperate (i.e., without extremes of heat and cold) than that of a town in a desert at the same latitude. Why?

Answer: (a) In thermal contact, heat flows from the body at higher temperature to the body at lower temperature till

temperatures become equal. The final temperature can be the mean temperature $(T_1+T_2)/2$ only when thermal capacities of the two bodies are equal.
(b) This is because heat absorbed by a substance is directly proportional to the specific heat of the substance.
(c) When car is driven, some work is being done on types in order to overcome dissipative forces of friction and air resistance etc. This work done is transformed into heat, due to which temperature of the car types increases.
(d) The climate of a harbour town is more temperate (neither too hot nor too cool) due to formation of sea breeze at day time and land breeze at night time

Q4. **A cylinder with a movable piston contains 3 moles of hydrogen at standard temperature and pressure. The walls of the cylinder are made of a heat insulator, and the piston is insulated by having a pile of sand on it. By what factor does the pressure of the gas increase if the gas is compressed to half its original volume?**

Answer:

Here the process is adiabatic compression and $V_2 = \frac{V_1}{2}$, $P_2 = 1$ atm and for hydrogen (a diatomic gas) $\gamma = 1.4$.

$$\because \quad P_1V_1^\gamma = P_2V_2^\gamma, \quad \text{Hence} \quad P_2 = P_1\left(\frac{V_1}{V_2}\right)^\gamma = 1\text{ atm}\left(\frac{V_1}{\frac{V_1}{2}}\right)^{1.4}$$

$$\Rightarrow \quad P_2 = (2)^{1.4}\text{ atm} = 2.64\text{ atm.}$$

Q5. In changing the state of a gas adiabatically from an equilibrium state A to another equilibrium state B, an amount of work equal to 22.3 J is done on the system. If the gas is taken from state A to B via a process in which the net heat absorbed by the system is 9.35 cal, how much is the net work done by the system in the latter case? (Take 1 cal = 4.19 J)

Answer:

Here, when the change is adiabatic, $\Delta Q = 0$, $\Delta W = -22.3$ J

If ΔU is change in internal energy of the system, then

as $\quad \Delta Q = \Delta U + \Delta W$

$0 = \Delta U - 22.3 \quad$ or $\quad \Delta U = 22.3$ J

In the second case, $\Delta Q = 9.35 \text{ cal} = 9.35 \times 4.2\text{J} = 39.3$ J

$\Delta W = ?$

As $\quad \Delta U + \Delta W = \Delta Q$

$\therefore \quad \Delta W = \Delta Q - \Delta U = 39.3 - 22.3 = 17.0$ J.

Q6. A steam engine delivers 5.4 x 10^8 J of work per minute and services 3.6 x 10^9 J of heat per minute from its boiler. What is the efficiency of the engine? How much heat is wasted per minute?

Answer:

Work done per minute, output = 5.4×10^8 J

Heat absorbed per minute, input = 3.6×10^9 J

Efficiency, $\quad \eta = \frac{\text{output}}{\text{input}} = \frac{5.4 \times 10^8}{3.6 \times 10^9} = 0.15$

$\%\eta = 0.15 \times 100 = 15$

Heat energy wasted/minute

= Heat energy absorbed/minute – Useful work done/minute

$= 3.6 \times 10^9 - 5.4 \times 10^8$

$= (3.6 - 0.54) \times 10^9 = 3.06 \times 10^9$ J.

Q7.An electric heater supplies heat to a system at a rate of 100 W. If system performs work at a rate of 75 Joules per second. At what rate is the internal energy increasing?

Answer:

Here $\quad \Delta Q = 100 \text{ W} = 100$ J/s

$\Delta W = 75$ J/s

Since $\quad \Delta Q = \Delta U + \Delta W$

$\therefore$ Change in internal energy, $\Delta U = \Delta Q - \Delta W = 100 - 75 = 25$ J/s.

Q1.Estimate the fraction of molecular volume to the actual volume occupied by oxygen gas at STP.' Take the diameter of an oxygen molecule to be 3 A.

Answer: Diameter of an oxygen molecule, d = 3 A = 3 x 10^{-10} m. Consider one mole of oxygen gas at STP, which contain total NA = 6.023 x 10^{23} molecules.

Actual molecular volume of 6.023×10^{23} oxygen molecules

$$V_{\text{actual}} = \frac{4}{3}\pi r^3 \cdot N_A$$

$$= \frac{4}{3} \times 3.14 \times (1.5)^3 \times 10^{-3} \times 6.02 \times 10^{23} \text{ m}^3$$

$$= 8.51 \times 10^{-6} \text{ m}^3$$

$$= 8.51 \times 10^{-3} \text{ litre} \qquad [\because 1 \text{ m}^3 = 10^3 \text{ litre}]$$

$\therefore$ Molecular volume of one mole of oxygen

$$V_{\text{actual}} = 8.51 \times 10^{-3} \text{ litre}$$

At STP, the volume of one mole of oxygen

$$V_{\text{molar}} = 22.4 \text{ litre}$$

$$\frac{V_{\text{actual}}}{V_{\text{molar}}} = \frac{8.51 \times 10^{-3}}{22.4} = 3.8 \times 10^{-4} \simeq 4 \times 10^{-4}.$$

Q2.**Molar volume is the volume occupied by 1 mol of any (ideal) gas at standard temperature and pressure (STP : 1 atmospheric pressure, 0 °C). Show that it is 22.4 litres.**

Answer:

For one mole of an ideal gas, we have

$$PV = RT \Rightarrow V = \frac{RT}{P}$$

Putting $R = 8.31$ J mol^{-1} K^{-1}, $T = 273K$ and $P = 1$ atmosphere $= 1.013 \times 10^5$ Nm^{-2}

$$\therefore \quad V = \frac{8.31 \times 273}{1.013 \times 10^5} = 0.0224 \text{ m}^3$$

$$= 0.0224 \times 10^6 \text{ cm}^3 = 22400 \text{ ml} \qquad [1 \text{ cm}^3 = 1\text{ml}]$$

Q3. **An air bubble of volume 1.0 cm^3 rises from the bottom of a lake 40 m deep at a temperature of 12°C. To what volume does it grow when it reaches the surface, which is at a temperature of 35 °C.**

Answer:

Volume of the bubble inside, $V_1 = 1.0 \text{ cm}^3 = 1 \times 10^{-6} \text{ m}^3$

Pressure on the bubble, P_1 = Pressure of water + Atmospheric pressure

$$= \rho gh + 1.01 \times 10^5 = 1000 \times 9.8 \times 40 + 1.01 \times 10^5$$

$$= 3.92 \times 10^5 + 1.01 \times 10^5 = 4.93 \times 10^5 \; Pa$$

Temperature, $T_1 = 12\,°C = 273 + 12 = 285$ K

Also, pressure outside the lake, P_2, $= 1.01 \times 10^5$ N m^{-2}

Temperature, $T_2 = 35\,°C = 273 + 35 = 308$ K, volume V_2 =?

Now $$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\therefore \quad V_2 = \frac{P_1V_1}{T_1} \cdot \frac{T_2}{P_2} = \frac{4.93 \times 10^5 \times 1 \times 10^{-6} \times 308}{285 \times 1.01 \times 10^5} = 5.3 \times 10^{-6} \text{ m}^{-3}$$

Q4. **Estimate the total number of air molecules (inclusive of oxygen, nitrogen, water vapour and other constituents) in a room of capacity 25.0 m^3 at a temperature of 27 °C and 1 atm pressure.**

Answer:

Here, volume of room, $V = 25.0$ m^3, temperature, $T = 27\,°C = 300$ K and

Pressure, $P = 1 \text{ atm} = 1.01 \times 10^5$ Pa

According to gas equation,

$$PV = \mu RT = \mu N_A \cdot k_B T$$

Hence, total number of air molecules in the volume of given gas,

$$N = \mu \cdot N_A = \frac{PV}{k_B T}$$

$$\therefore \quad N = \frac{1.01 \times 10^5 \times 25.0}{(1.38 \times 10^{-23}) \times 300} = 6.1 \times 10^{26}.$$

Q5. **Estimate the average thermal energy of a helium atom at (i) room temperature (27 °C), (ii) the temperature on the surface of the Sun (6000 K), (iii) the temperature of 10 million kelvin (the typical core temperature in the case of a star).**

Answer:

(*i*) Here, $T = 27\ ^\circ C = 27 + 273 = 300$ K

$$\text{Average thermal energy} = \frac{3}{2}kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300 = 6.2 \times 10^{-21} \text{ J.}$$

(*ii*) At $T = 6000$ K,

$$\text{Average thermal energy} = \frac{3}{2}kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 6000 = 1.24 \times 10^{-19} \text{ J.}$$

(*iii*) At $T = 10$ million $K = 10^7$ K

$$\text{Average thermal energy} = \frac{3}{2}kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 10^7 = 2.1 \times 10^{-16} \text{ J}$$

Q6. t what temperature is the root mean square speed of an atom in an argon gas cylinder equal to the rms speed of a helium gas atom at -20 °C? (atomic mass of Ar = 39.9 u, of He = 4.0 u).

Answer: Let C and C' be the rms velocity of argon and a helium gas atoms at temperature T K and T K respectively. Here, M = 39.9; M' = 4.0; T =?; T = -20 + 273 = 253 K

Now, $C = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3RT}{39.9}}$ and $C' = \sqrt{\frac{3RT'}{M'}} = \sqrt{\frac{3R \times 253}{4}}$

Since $C = C'$

Therefore, $\sqrt{\frac{3RT}{39.9}} = \sqrt{\frac{3R \times 253}{4}}$

or $T = \frac{39.9 \times 253}{4} = 2523.7$ K.

Oscillation

Q1.A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this balance, when displaced and released, oscillates with a period of 0.6 s. What is the weight of the body?

Answer:

$M = 50\text{kg}, \ y = 20\text{cm} = 0.2 \text{ m}, T = 0.60$ s

$F = ky$ or $Mg = ky$ or $k = \frac{Mg}{0.2} = \frac{50 \times 9.8}{0.2}\text{Nm}^{-1}$

or $K = 2450 \text{ Nm}^{-1}$

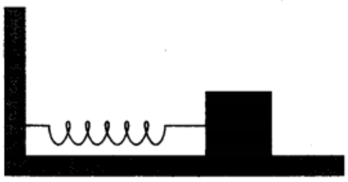
Now, $T = 2\pi\sqrt{\frac{m}{k}}$

or $T^2 = 4\pi^2\frac{m}{k}$ or $m = \frac{T^2 k}{4\pi^2}$

or $m = \frac{0.6 \times 0.6 \times 2460 \times 49}{4 \times 484}\text{kg} = 22.3$ kg

$\Rightarrow$ $mg = 22.3 \times 9.8 \text{ N} = 218.5 \text{ N} = 22.3$ kgf.

Q2. **A spring having with a spring constant 1200 Nm^{-1} is mounted on a horizontal table as shown in Fig. A mass of 3 kg is attached to the free end of the spring. The mass is then pulled sideways to a distance of 2.0 cm and released. Determine (i) the frequency of oscillations, (ii) maximum acceleration of the mass, and (iii) the maximum speed of the mass.**



Answer:

Here, $K = 1200\ Nm^{-1}$; $m = 3.0$ kg, $a = 2.0$ cm $= 0.02$ m

(i) Frequency, $\nu = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\times 3.14}\sqrt{\frac{1200}{3}} = 3.2\ s^{-1}$

(ii) Acceleration, $A = \omega^2\ y = \frac{k}{m}y$

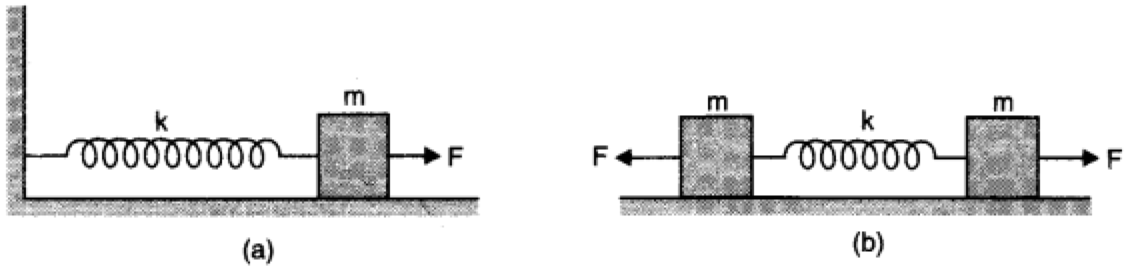
Acceleration will be maximum when y is maximum *i.e.,* $y = a$

$\therefore$ max. acceleration, $A_{max} = \frac{ka}{m} = \frac{1200\times 0.02}{3} = 8\ ms^{-2}$

(iii) Max. speed of the mass will be when it is passing through mean position

$$V_{max} = a\omega = a\sqrt{\frac{k}{m}} = 0.02 \times \sqrt{\frac{1200}{3}} = 0.4\ ms^{-1}$$

Q3. **Figure (a) shows a spring of force constant k clamped rigidly at one end and a mass m attached to its free end. A force F applied at the free end stretches the spring. Figure (b) shows the same spring with both ends free and attached to a mass m at either end. Each end of the spring in Figure – (b) is stretched by the same force F.**



(a) What is the maximum extension of the spring in the two cases?
(b) If the mass in Fig. (a) and the two masses in Fig. (b) are released free, what is the period of oscillation in each case?
Answer:

(a) Let y be the maximum extension produced in the spring in Fig. (a)

Then $F = ky$ (in magnitude) $\therefore\ y = \frac{F}{k}$

If fig. (b), the force on one mass acts as the force of reaction due to the force on the other mass. Therefore, each mass behaves as if it is fixed with respect to the other.

Therefore, $F = ky \Rightarrow y = \frac{F}{k}$

(b) In fig. (a), $F = -ky$

$\Rightarrow$ $ma = -ky \Rightarrow a = -\frac{k}{m}y \quad \therefore\ \omega^2 = \frac{k}{m}$ *i.e.,* $\omega = \sqrt{\frac{k}{m}}$

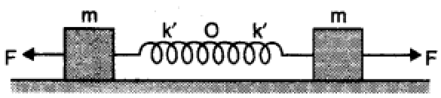
Therefore, period $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

In fig. (b), we may consider that the centre of the system is O and there are two springs each of length $\frac{l}{2}$ attached to the two masses, each m, so that k' is the spring factor of each of the springs.

Then, $K' = 2k$

$\therefore$ $T = 2\pi\sqrt{\frac{m}{k'}}$

$= 2\pi\sqrt{\frac{m}{2k}}$



Q4. **The piston in the cylinder head of a locomotive has a stroke (twice the amplitude) of 1.0 m. If the piston moves with simple harmonic motion with an angular frequency of 200 rev/min, what its maximum speed?**
Answer:

Stroke of piston = 2 times the amplitude

Let A = amplitude, stroke = 1 m

$\therefore \Rightarrow A = \frac{1}{2}$m.

Angular frequency, $\omega = 200$ rad/min.

$V_{max} = ?$

We know that the maximum speed of the block when the amplitude is A,

$V_{max} = \omega A = 200 \times \frac{1}{2} = 100$ m/min.

$= \frac{100}{60} = \frac{5}{3} ms^{-1} = 1.67\ ms^{-1}$.

Q5. **The acceleration due to gravity on the surface of moon is 1.7 ms^{-2}. What is the time period of a simple pendulum on the surface of moon if its time-period on the surface of Earth is 3.5 s? (g on the surface of Earth is 9.8 ms^{-2}.)**

Answer:

Here, $g_m = 1.7 \text{ ms}^{-2}; \quad g_e = 9.8 \text{ ms}^{-2}; \quad T_m = ?; \quad T_e = 3.5 \text{ s}^{-1}$

Since, $T_e = 2\pi\sqrt{\frac{1}{g_e}}$ and $T_m = 2\pi\sqrt{\frac{1}{g_m}}$

$$\therefore \quad \frac{T_m}{T_e} = \sqrt{\frac{g_e}{g_m}} \Rightarrow T_m = T_e = \sqrt{\frac{g_e}{g_m}}$$

$$= 3.5\sqrt{\frac{9.8}{1.7}} = 8.4 \text{ s.}$$

Q6. **A simple pendulum of length l and having a bob of mass M is suspended in a car. The car is moving on a circular track of radius R with a uniform speed v. If the pendulum makes small oscillations in a radial direction about its equilibrium position, what will be its time period?**

Answer: In this case, the bob of the pendulum is under the action of two accelerations.

(*i*) Acceleration due to gravity 'g' acting vertically downwards.

(*ii*) Centripetal acceleration $a_c = \frac{v^2}{R}$ acting along the horizontal direction.

$\therefore$ Effective acceleration, $g' = \sqrt{g^2 + a_c^2}$

or $$g' = \sqrt{g^2 + \frac{v^4}{R^2}}$$

Now time period, $T' = 2\pi\sqrt{\frac{1}{g}} = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + \frac{v^4}{R^2}}}}$.

Q7.A cylindrical piece of cork of density of base area A and height h floats in a liquid of density ρ_1. The cork is depressed slightly and then released. Show that the cork oscillates up and down simple harmonically with a period

$$T = 2\pi\sqrt{\frac{h\rho}{\rho_l g}}$$

where ρ is the density of cork. (Ignore damping due to viscosity of the liquid).

Answer:

Say, initially in equilibrium, y height of cylinder is inside the liquid. Then,
Weight of the cylinder = upthrust due to liquid displaced

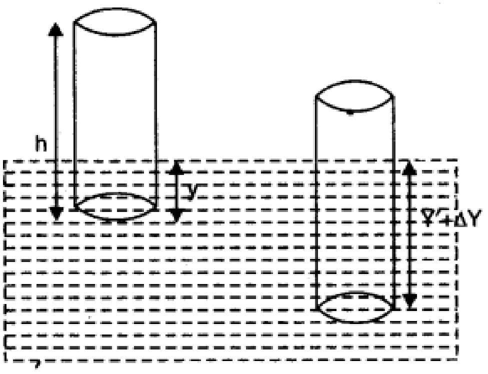
$$\therefore \quad Ah\rho g = Ay\rho_l g$$

When the cork cylinder is depressed slightly by Δy and released, a restoring force, equal to additional upthrust, acts on it. The restoring force is

$$F = A(y + \Delta y)\rho_l g - Ay\rho_l g = A\rho_l g\Delta y$$

$\therefore$ Acceleration, $a = \frac{F}{m} = \frac{A\rho_l g\Delta y}{Ah\rho} = \frac{\rho_l g}{h\rho} . \Delta y$ and the acceleration is directed in a direction opposite to Δy: Obviously, as $a \propto -\Delta y$, the motion of cork cylinder is SHM, whose time period is given by

$$T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}} = 2\pi\sqrt{\frac{\Delta y}{a}} = 2\pi\sqrt{\frac{h\rho}{\rho_l g}}.$$



Q8. **Show that for a particle in linear SHM the average kinetic energy over a period of oscillation equals the average potential energy over the same period.**

Answer: Let the particle executing SHM starts oscillating from its mean position. Then displacement equation is

$$x = A \sin \omega t$$

$\therefore$ Particle velocity, $v = A\omega \cos \omega t$

$\therefore$ Instantaneous K.E., $K = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2 \cos^2 \omega t$

$\therefore$ Average value of K.E. over one complete cycle

$$K_{av} = \frac{1}{T}\int_0^T \frac{1}{2}mA^2\omega^2 \cos^2 \omega t \, dt = \frac{mA^2\omega^2}{2T}\int_0^T \cos^2 \omega t \, dt$$

$$= \frac{mA^2\omega^2}{2T}\int_0^T \frac{(1+\cos 2\omega t)}{2} dt$$

$$= \frac{mA^2\omega^2}{4T}\left[t + \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{mA^2\omega^2}{4T}\left[(T-0) + \left(\frac{\sin 2\omega t - \sin 0}{2\omega}\right)\right]$$

$$= \frac{1}{4}mA^2w^2 \qquad ...(i)$$

Again instantaneous P.E., $U = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2A^2 \sin^2 \omega t$

$\therefore$ Average value of P.E. over one complete cycle

$$U_{av} = \frac{1}{T}\int_0^T \frac{1}{2}m\omega^2A^2 \sin^2 \omega t = \frac{m\omega^2A^2}{2T}\int_0^T \sin^2 \omega t \, dt$$

$$= \frac{m\omega^2A^2}{2T}\int_0^T \frac{(1-\cos 2\omega t)}{2} dt$$

$$= \frac{m\omega^2A^2}{4T}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{m\omega^2A^2}{4T}\left[(T-0) - \frac{(\sin 2\omega t - \sin 0)}{2\omega}\right]$$

$$= \frac{1}{4}m\omega^2A^2 \qquad ...(ii)$$

Simple comparison of (*i*) and (*ii*), shows that

$$K_{av} = U_{av} = \frac{1}{4}m\omega^2A^2$$

Q9. **A body describes simple harmonic motion with an amplitude of 5 cm and a period of 0.2 s. Find the acceleration and velocity of the body when the displacement is (a) 5 cm (b) 3 cm (c) 0 cm**

Answer:

Here, $r = 5$ cm $= 0.05$ m; T = 0.2 s; $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.2} = 10\pi$ rad/s

When displacement is y, then

acceleration, $A = -\omega^2 y$

velocity, $V = \omega\sqrt{r^2 - y^2}$

Case (*a*) When $y = 5$ cm $= 0.05$ m

$A = -(10\pi)^2 \times 0.05 = -5\pi^2$ m/s^2

$V = 10\pi\sqrt{(0.05)^2 - (0.05)^2} = 0.$

Case (*b*) When $y = 3$ cm $= 0.03$ m

$A = -(10\pi)^2 \times 0.03 = -3\pi^2$ m/s^2

$V = 10\pi\sqrt{(0.05)^2 - (0.03)^2} = 10\pi \times 0.04 = 0.4\pi$ m/s

Case (c) When $y = 0$, $A = -(10\pi)^2 \times 0 = 0$

$V = 10\pi\sqrt{(0.05)^2 - 0^2} = 10\pi \times 0.05 = 0.5\pi$ m/s.

Wave

Q1. **A string of mass 2.50 kg is under a tension of 200 N. The length of the stretched string is 20.0 m. If the transverse jerk is struck at one end of the string, how long does the disturbance take to reach the other end?**

Answer:

Tension, $T = 200$ N;

Length, $l = 20.0$ m; Mass, $M = 2.50$ kg

Mass per unit length, $\mu = \frac{2.50}{20.0}$ kg m^{-1} = 0.125 kgm^{-1}

Wave velocity, $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{200\ N}{0.125\ kg\ m^{-1}}}$

or $v = \sqrt{1600}$ ms^{-1} = 40 ms^{-1}

Time, $t = \frac{l}{v} = \frac{20.0}{40} s = \frac{1}{2} s = 0.5$ s.

Q2. A stone dropped from the top of a tower of height 300 m high splashes into the water of a pond near the base of the tower. When is the splash heard at the top given that the speed of sound in air is 340 ms^{-1}? (g = 9.8 ms^{-2})

Answer: Here, h = 300 m, g = 9.8 ms^{-2} and velocity of sound, v = 340 ms^{-1} Let t_1 be the time taken by the stone to reach at the surface of pond.

Then, using $s = ut + \frac{1}{2}at^2 \frac{1}{2}at^2 \Rightarrow h = 0 \times t + \frac{1}{2}gt_1^2$

$\therefore$ $t_1 = \sqrt{\frac{2 \times 300}{9.8}} = 7.82$ s

Also, if t_2 is the time taken by the sound to reach at a height h, then

$$t_2 = \frac{h}{v} = \frac{300}{340} = 0.88 \text{ s}$$

$\therefore$ Total time after which sound of splash is heard = $t_1 + t_2$

= 7.82 + 0.88 = 8.7s.

Q3. A steel wire has a length of 12.0 m and a mass of 2.10 kg. What should be the tension in the wire so that speed of a transverse wave on the wire equals the speed of sound in dry air at 2 0°C = 340 ms^{-1}.

Answer:

Here, $l = 12.0$ m, M = 2.10 kg

$v = 343$ ms^{-1}

Mass per unit length = $\frac{M}{l} = \frac{2.10}{12.0} = 0.175$ kg m^{-1}

As $v = \sqrt{\frac{T}{m}}$

$\therefore$ $T = v^2 . m = (343)^2 \times 0.175 = 2.06 \times 10^4$ N.

Q4. A bat emits ultrasonic sound of frequency 1000 kHz in air. If this sound meets a water surface, what is the wavelength of (a) the reflected sound, (b) the transmitted sound? Speed of sound in air = 340 ms^{-1} and in water = 1486 ms^{-1}.

Answer:

Here, $\nu = 1000 \times 10^3$ Hz = 10^6 Hz, $v_a = 340$ ms^{-1},

$v_w = 1486$ ms^{-1}

Wavelength of reflected sound, λ_a

$$= \frac{v_a}{\nu} = \frac{340}{10^6} \text{m} = 3.4 \times 10^{-4} \text{ m}$$

Wavelength of transmitted sound, λ_ω

$$= \frac{v_w}{\nu} = \frac{1486}{10^6} \text{m} = 1.486 \times 10^{-3} \text{m}$$

Vimp.Q5. A transverse harmonic wave on a string is described by
y(x, t) = 3.0 sin (36 t + 0.018 x + π/4)
where x and y are in cm and t in s. The positive direction of x is from left to right.
(a) Is this a travelling wave or a stationary wave? If it is travelling, what are the speed and direction of its propagation ?
(b) What are its amplitude and frequency?
(c) What is the initial phase at the origin?
(d) What is the least distance between two successive crests in the wave?
Answer:

The given equation is $y(x, t) = 3.0 \sin (36t + 0.018x + \frac{\pi}{4})$, where x and y are in cm and t in s.

(*a*) The equation is the equation of a travelling wave, travelling from right to left (*i.e.*, along $-ve$ direction of x because it is an equation of the type

$$y(x, t) = A \sin (\omega t + kx + \phi)$$

Here, $A = 3.0$ cm, $\omega = 36$ rad s^{-1}, $k = 0.018$ cm and $\phi = \frac{\pi}{4}$.

∴ Speed of wave propagation,

$$v = \frac{\omega}{k} = \frac{36 \text{ rad s}^{-1}}{0.018 \text{ cm}^{-1}} = \frac{36 \text{ rad s}^{-1}}{0.018 \times 10^2 \text{ ms}^{-1}} = 20 \text{ ms}^{-1}$$

(*b*) Amplitude of wave, $A = 3.0$ cm $= 0.03$ m

Frequency of wave $\nu = \frac{\omega}{2\pi} = \frac{36}{2\pi} = 5.7$ Hz

(*c*) Initial phase at the origin, $\phi = \frac{\pi}{4}$

(*d*) Least distance between two successive crests in the wave

$$= \lambda = \frac{2\pi}{k} = \frac{2\pi}{0.018}$$

$$= 349 \text{ cm} = 3.5 \text{ m}$$

Q6. **For the wave described in Exercise 8, plot the displacement (y) versus (t) graphs for x = 0, 2 and 4 cm. What are the shape of these graphs? In which aspects does the oscillatory motion in travelling wave differ from one point to another : amplitude, frequency or phase?**

Answer: The transverse harmonic wave is

$$y(x, t) = 3.0 \sin \left(36t + 0.018x + \frac{\pi}{4}\right)$$

For $x = 0$,

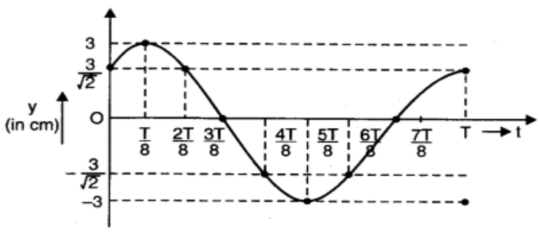
$$y(0, t) = 3 \sin \left(36t + 0 + \frac{\pi}{4}\right) = 3 \sin \left(36t + \frac{\pi}{4}\right) \quad ...(1)$$

Here $\omega = \frac{2\pi}{T} = 36 \Rightarrow T = \frac{2\pi}{36}$

To plot a (*y*) versus (*t*) graph, different values of *y* corresponding to the values of *t* may be tabulated as under (by making use of eqn. (1)).

t	0	$\frac{T}{8}$	$\frac{2T}{8}$	$\frac{3T}{8}$	$\frac{4T}{8}$	$\frac{5T}{8}$	$\frac{6T}{8}$	$\frac{7T}{8}$	T
y	$\frac{3}{\sqrt{2}}$	3	$\frac{3}{\sqrt{2}}$	0	$\frac{-3}{\sqrt{2}}$	-3	$\frac{-3}{\sqrt{2}}$	0	$\frac{3}{\sqrt{2}}$

Using the values of t and y (as in the table), a graph is plotted as under The graph obtained is sinusoidal.



Similar graphs are obtained for y x = 2 cm and x = 4 cm. The (in cm) oscillatory motion in the travelling wave only differs in respect of phase. Amplitude and frequency of oscillatory motion remains the same in all the cases.

Q7. **For the travelling harmonic wave**
y(x, t) = 2.0 cos 2π (10t – 0.0080x + 0.35)
where x and y are in cm and t in s. Calculate the phase difference between oscillatory motion of two points separated by a distance of
(a) 4 m (b) 0.5 m
(c) λ/2 (d) 3λ/4.
Answer:

The given equation can be rewritten as under:

$$y(x, t) = 2.0 \cos [2\pi (10t - 0.0080 x) + 2\pi \times 0.35]$$

or

$$y(x, t) = 2.0 \cos \left[2\pi \times 0.0080 \left(\frac{10t}{0.0080} - x\right) + 0.7\pi\right]$$

Comparing this equation with the standard equation of a travelling harmonic wave,

$$\frac{2\pi}{\lambda} = 2\pi \times 0.0080 \quad \text{or} \quad \lambda = \frac{1}{0.0080} \text{ cm} = 125 \text{ cm}$$

The phase difference between oscillatory motion of two points separated by a distance Δx is given by

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$$

(a) When $\Delta x = 4 \text{ m} = 400 \text{ cm}$, then

$$\Delta\phi = \frac{2\pi}{125}\times 400 = 6.4\,\pi \text{ rad}$$

(b) When $\Delta x = 0.5 \text{ m} = 50 \text{ cm}$, then

$$\Delta\phi = \frac{2\pi}{125}\times 50 = 0.8\,\pi \text{ rad}$$

(c) When $\Delta x = \frac{\lambda}{2} = \frac{125}{2}$ cm, then

$$\Delta\phi = \frac{2\pi}{125}\times\frac{125}{2} = \pi \text{ rad}$$

(d) When $\Delta x = \frac{3\lambda}{4} = \frac{3\times 125}{4}$ cm, then

$$\Delta\phi = \frac{2\pi}{125}\times\frac{3\times 125}{4} = \frac{3\pi}{2}\text{ rad.}$$

Q8.The transverse displacement of a string (clamped at its two ends) is given by y(x, t) = 0.06 sin 2π /3 x cos (120π t)
where x, y are in m and t in s. The length of the string is 1.5 m and its mass is 3 x 10^{-2} kg. Answer the following:
(i) Does the function represent a travelling or a stationary wave?
(ii) Interpret the wave as a superimposition of two waves travelling in opposite directions. What are the wavelength, frequency and speed of propagation of each wave?
(iii) Determine the tension in the string.
Answer: The given equation is
y(x, t) = 0.06 sin 2π/3 x cos 120 πt …(1)
(i)As the equation involves harmonic functions of x and t separately, it represents a stationary wave.

(ii) We know that when a wave pulse

$$y_1 = r \sin \frac{2\pi}{\lambda}(vt - x)$$

travelling along + direction of x-axis is superimposed by the reflected wave

$$y_2 = -r \sin \frac{2\pi}{\lambda}(vt + x)$$

travelling in opposite direction, a stationary wave

$$y = y_1 + y_2 = -2r \sin\frac{2\pi}{\lambda}x \cos\frac{2\pi}{\lambda}vt \text{ is formed.} \quad ...(2)$$

Comparing eqns. (1) and (2), we find that

$$\frac{2\pi}{\lambda} = \frac{2\pi}{3} \Rightarrow \lambda = 3\text{m}$$

Also, $\frac{2\pi}{\lambda}v = 120\,\pi$ or $v = 60\lambda = 60 \times 3 = 180 \text{ ms}^{-1}$

Frequency, $\nu = \frac{v}{\lambda} = \frac{180}{3} = 60$ Hz

Note that both the waves have same wavelength, same frequency and same speed.

(iii) Velocity of transverse waves is

$$v = \sqrt{\frac{T}{m}} \quad \text{or} \quad v^2 = \frac{T}{m}$$

$$T = mv^2, \text{ where } m = \frac{3\times 10^{-2}}{1.5} = 2 \times 10^{-2} \text{ kg/m}$$

$$\therefore \quad T = (180)^2 \times 2 \times 10^{-2} = 648 \text{ N.}$$

Q9. **A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 45 Hz. The mass of the wire is 3.5 x 10^{-2} kg and its linear mass density is 4.0 x 10^{-2} kg m^{-3}. What is (a) the speed of a transverse wave on the string, and (b) the tension in the string?**

Answer:

Here, $n = 45$ Hz, $M = 3.5 \times 10^{-2}$ kg

Mass per unit length = $m = 4.0 \times 10^{-2}$ kg m^{-1}

$$\therefore \quad l = \frac{M}{m} = \frac{3.5\times10^{-2}}{4.0\times10^{-2}} = \frac{7}{8}$$

$$\text{As} \quad \frac{l}{2} = \lambda = \frac{7}{8} \quad \therefore \quad \lambda = \frac{7}{4}\text{m} = 1.75 \text{ m.}$$

(a) The speed of the transverse wave, $v = \nu\lambda = 45 \times 1.75 = 78.75$ m/s

$$\text{(b) As} \quad v = \sqrt{\frac{T}{m}}$$

$$\therefore \quad T = v^2 \times m = (78.75)^2 \times 4.0 \times 10^{-2} = 248.06 \text{ N.}$$

Q10. **A metre-long tube open at one end, with a movable piston at the other end, shows resonance with a fixed frequency source (a turning fork of frequency 340 Hz) when the tube length is 25.5 cm or 79.3 cm. Estimate the speed of sound in air at the temperature of the experiment. The edge effect may be neglected.**

Answer:

Frequency of n^{th} mode of vibration of the closed organ pipe of legnth

$$l_1 = (2n-1)\frac{v}{4l_1}$$

Frequency of $(n+1)^{th}$ mode of vibration of closed pipe of length

$$'l_2' = [2(n+1)-1]\frac{v}{4l_2} = (2n+1)\frac{v}{4l_2}$$

Both the modes are given to resonate with a frequency of 340 Hz.

$$\therefore \quad (2n-1)\frac{v}{4l_1} = (2n+1)\frac{v}{4l_2}$$

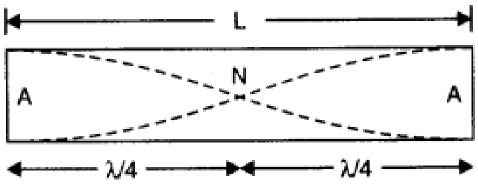
$$\text{or} \quad \frac{2n-1}{2n+2} = \frac{l_1}{l_2} = \frac{25.5}{79.3} = \frac{1}{3}$$

Q11. **A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod is given to be 2.53 k Hz. What is the speed of sound in steel?**

Answer: Here, L = 100 cm = 1m, v = 2.53 k Hz = 2.53 x 10^3 Hz

When the rod is clamped at the middle, then in the fundamental mode of vibration of the rod, a node is formed at the middle and ant mode is formed at each end.

Therefore, as is clear from Fig.



$$L = \frac{\lambda}{4} + \frac{\lambda}{4} = \frac{\lambda}{2}$$

$$\lambda = 2L = 2 \text{ m}$$

$$\text{As} \quad v = \nu\lambda$$

$$\therefore \quad v = 2.53 \times 10^3 \times 2 = \mathbf{5.06 \times 10^3\ ms^{-1}}$$

Q12. **A travelling harmonic wave on a string is described by y (x, t) = 7.5 sin (0.0050x + 12t + π /4)**

(a) what are the displacement and velocity of oscillation of a point at x = 1 cm, and t = 1s? Is this velocity equal to the velocity of wave propagation?

(b) Locate the points of the string which have the same transverse displacement and velocity as the x = 1 cm point at t = 2s, 5s and 11s.

Answer:

The travelling harmonic wave is $y(x, t) = 7.5 \sin(0.0050x + 12t + \pi/4)$

At $x = 1$ cm and $t = 1$ sec,

$$y(1, 1) = 7.5 \sin(0.005 \times 1 + 12 \times 1 \pi/4) = 7.5 \sin(12.005 + \pi/4) \quad ...(i)$$

Now, $\theta = (12.005 + \pi/4)$ radian

$$= \frac{180}{\pi}(12.005 + \pi/4) \text{ degree} = \frac{12.005 \times 180}{\frac{22}{7}} + 45 = 732.55°.$$

$\therefore$ From (i), $y(1, 1) = 7.5 \sin(732.55°) = 7.5 \sin(720 + 12.55°)$

$$= 7.5 \sin 12.55° = 7.5 \times 0.2173 = 1.63 \text{ cm}$$

Velocity of oscillation, $v = \frac{dy}{dt}(1, 1) = \frac{d}{dt}\left[7.5 \sin\left(0.005x + 12 + \frac{\pi}{4}\right)\right]$

$$= 7.5 \times 12 \cos\left[0.005x + 12t + \frac{\pi}{4}\right]$$

At $x = 1\text{cm}$, $t = 1\text{sec}$.

$$v = 7.5 \times 12 \cos(0.005 + 12 + \pi/4) = 90 \cos(732.35°)$$

$$= 90 \cos(720 + 12.55)$$

$$v = 90 \cos(12.55°) = 90 \times 0.9765 = 87.89 \text{ cm/s}.$$

Comparing the given eqn. with the standard form $y(x, t) = t \sin\left[\frac{\pi}{4}(vt + x) + \phi_0\right]$

We get $r = 7.5$ cm, $\frac{2\pi v}{\lambda} = 12$ or $2\pi\nu = 12$

$$\nu = \frac{6}{\pi}$$

$$\frac{2\pi}{\lambda} = 0.005.$$

$$\therefore \quad \lambda = \frac{2\pi}{0.005} = \frac{2 \times 3.14}{0.005} = 1256 \text{ cm} = 12.56 \text{ m}$$

Velocity of wave propagation, $v = \nu\lambda = \frac{6}{\pi} \times 12.56 = \mathbf{24\ m/s}$.

We find that velocity at $x = 1$ cm, $t = 1$ sec is not equal to velocity of wave propagation.

(b) Now, all points which are at a distance of $\pm\lambda, \pm 2\lambda, \pm 3\lambda$ from $x = 1$ cm will have same transverse displacement and velocity. As $\lambda = 12.56$ m, therefore, all points at distances ± 12.6 m, ± 25.2 m, ± 37.8 m.... from $x = 1$ cm will have same displacement and velocity, as at $x = 1$ point $t = 2$ s, 5 s and 11 s.

Q13. Earthquakes generate sound waves inside the earth. Unlike a gas, the earth can experience both transverse (S) and longitudinal (P) sound waves. Typically the speed of S wave is about 4.0 km s^{-1}. A seismograph records P and S waves from an earthquake. The first P wave arrives 4 min before the first S wave. Assuming the waves travel in straight line, at what distance does the earthquake occur?

Answer: Here speed of S wave, $\upsilon_s = 4.0$ km s^{-1} and speed of P wave, $\upsilon_p = 8.0$ km s^{-1}. Time gap between P and S waves reaching the resimograph, t = 40 min = 240 s.

Let distance of earthquake centre = sKm

$$\therefore \quad t = t_s - t_p = \frac{S}{v_s} - \frac{S}{v_p} = \frac{S}{4.0} - \frac{S}{8.0} = \frac{S}{8.0} = 240 \text{ s}$$

or $s = 240 \times 8.0 = 1920$ km.

Q14. **A bat is flitting about in a cave, navigating via ultrasonic beeps. Assume that the sound emission frequency of the bat is 40 kHz. During one fast swoop directly toward a flat wall surface, the bat is moving at 0.03 times the speed of sound in air. What frequency does the bat hear reflected off the wall ?**

Answer: Here, the frequency of sound emitted by the bat, $\upsilon = 40$ kHz. Velocity of bat, $\upsilon_s = 0.03\ \upsilon$, where υ is velocity of sound. Apparent frequency of sound striking the wall

$$\nu' = \frac{v}{v - v_s} \times \nu = \frac{v}{v - 0.03v} \times 40 \text{ kHz}$$

$$= \frac{40}{0.97} \text{kHz}$$

This frequency is reflected by the wall and is received by the bat moving towards the wall.
So $v_s = 0$,

$$v_L = 0.03\ v.$$

$$\nu'' = \frac{(v + v_L)}{v} \times \nu' = \frac{(v + 0.03v)}{v} \left(\frac{40}{0.97}\right)$$

$$= \frac{1.03}{0.97} \times 40 \text{ kHz} = 42.47 \text{ kHz}.$$