Differential Eqn

① An Eqn having → Independent variable (x)
→ dependent variable (Y)
→ differential coefficient. $\left(\frac{dy}{dx}, \frac{d^2y}{dx^2}, \frac{d^3y}{dx^3} \dots\right)$

eg- $x^3 \cdot \frac{d^2y}{dx^2} + y \cdot \frac{dy}{dx} + 4 = 0$

$\frac{dy}{dx} = \cos x$; $x^3 \frac{d^2y}{dx^2} = \sin y$, $\frac{dy}{dx} = 4$

$\frac{dy}{dx} = a$ (× No diff. eqn)
↳ arbitrary constant -

② Ordr / Degree

① ordr of diff. Eqn = Ordr of Highest drivative.

② dugree of D.E = dugree of Highest drivative, after eqn has been expresed in form free from Radicals & fractions

$\left(\frac{d^2y}{dx^2}\right)^2 - 2y \cdot \frac{dy}{dx} + (\sin x) = 0$

* ↳ ordr = 2
dugree = 2

$\frac{dy}{dx}$ → ordr = 1
$\frac{d^2y}{dx^2}$ → ordr = 2
$\frac{d^3y}{dx^3}$ → 3

Q $\frac{d^3y}{dx^3} - \frac{y}{\sin x} = \left(\frac{dy}{dx}\right)^4$
↳ O = 3 ⇒ D = 1.

Q $\frac{\left(\frac{dy}{dx}\right)}{x} = \frac{y}{\left(\frac{dy}{dx}\right)}$ ⇒ O = 1, D = 2

Q $\frac{d^2y}{dx^2} + \sqrt{\frac{dy}{dx}} + x = 0$ ⇒ $\frac{dy}{dx} = \left(\frac{d^2y}{dx^2} + x\right)^2$

⇒ $\frac{dy}{dx} = \left(\frac{d^2y}{dx^2}\right)^2 + x^2 + 2x\frac{d^2y}{dx^2}$

D = 2
O = 2

Q $\sqrt{1+x^2}\, dy - \sqrt{1-y^2}\, dx = 0$

$\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$ ⇒ O = 1, D = 1

Note ③ ① D.E having $e^{\frac{dy}{dx}}$, $\ln\left(\frac{dy}{dx}\right)$, $\sin\left(\frac{dy}{dx}\right)$, $\cos\left(\frac{dy}{dx}\right)$ have **dugree undifined**

② D.E must have free from with Radical sign ⇒ ^ it should not have fractional dugree

diff. coffi'

③ Meaning of ORder

$\frac{d^4y}{dx^4} = 0$? (why)

$\frac{d^3y}{dx^3} = 1$?

$\frac{d^2y}{dx^2} = x$ ← diff.

Int. $\frac{dy}{dx} = \frac{x^2}{2} + C_1$ ← diff.

Int. $y = \frac{x^3}{6} + C_1x + C_2$

$\frac{d^3y}{dx^3} = C_1$

$\frac{d^2y}{dx^2} = C_1x + C_2$

$\frac{dy}{dx} = C_1\frac{x^2}{2} + C_2x + C_3$

$y = C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3x + C_4$

$\Longrightarrow X \Longleftarrow$

So It means there is limit of differentiation to do so that we can easily get Parent eqn → (Basic)

So ⇒

① Meaning of ordur ⇒ = No. of arbitrary Const. present in Basic eqn.

② Meaning of dugree ⇒ = No. of families of Curve involved in Basic eqn.

⇒ Counting No. of Arb. Const —

① $y = ax^2 + bx + c$

$\frac{y}{c} = \frac{a}{c}x^2 + \frac{b}{c} + 1 \Rightarrow$ 3 arb Const.

3 arb.

② $ax^2 + bx + c = 0$

$\frac{a}{c}x^2 + \frac{b}{c}x + 1 = 0$

2 arb. Const

Q ordur of $y = Cx - C^2 - C^3$ If thy are connected & Related we count them "1".

Arb. Const = 1

Ordur = No. of Arb. Const = 1

Q ordur of $y^2 = 2Cx(1+\sqrt{C})$

$\boxed{0 = 1}$

Q $y = (C_1+C_2)e^{x+C_3} + C_4e^{x+C_5}$ $0 = ?$

$= (C_1+C_2)e^{C_3}\cdot e^x + C_4\cdot e^{C_5}e^x$

$= k e^x + \lambda e^x$

$= (k+d)e^x$

$= p\cdot e^x$

→ Ordur = 1

④ Formation of D.E :

① Count No. of arbitrary const = ORdr.

② Differentiate as many times as the No. of arbi. const. init.

③* Eliminate Arbitrary Constant.

Q $y = Ae^x + Be^{-x}$ form D.E ?

↳ Arb. Const. = 2.

Ordr = 2 ⇒ differentiate 2 times

$\frac{dy}{dx} = Ae^x - Be^{-x}$

$\frac{d^2y}{dx^2} = Ae^x + Be^{-x}$

$\boxed{\frac{d^2y}{dx^2} = y}$ Arbitrary constant Not allowed in diff. E.

Q D.E of $Ax^2 + By^2 = C$.

↳ Arb. Const = 2 = Ordr.

diff. 2 times →

$2Ax + 2By\frac{dy}{dx} = 0$ ⟶ make gap b/w (derivatives)

$Ax + By \cdot \frac{dy}{dx} = 0$

↳ $-\frac{A}{B} = \frac{y}{x}\frac{dy}{dx}$ (Help from Previous eqn)

$A + B\left(y\cdot\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right) = 0 \Rightarrow \left(y\cdot\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right) = -\frac{A}{B}$

$\Rightarrow \left(y\cdot\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right) = \frac{y}{x}\cdot\frac{dy}{dx}$.

Now D.E ⟶

Q $y = cx - c^2 - c^3$ form D.E

↳ arb. Const = 1

$\frac{dy}{dx} = c$ Now put c

So $y = x\frac{dy}{dx} - \left(\frac{dy}{dx}\right)^2 - \left(\frac{dy}{dx}\right)^3$

↳ O = 1

D = 3

Q if Soln of $\frac{2+\sin x}{(1+y)} \cdot \frac{dy}{dx} = -\cos x$ Combe Rep. by $\boxed{y = y(x)}$ $y = f(x)$

JEE & $y(0) = 1$, Hun $y(\pi)$

$f(0) = 1$ $\quad f(\pi) \to .$

↳ $x=0, y=1$

$$\int \frac{dy}{1+y} = \int -\frac{\cos x}{2+\sin x} \cdot dx$$

↳ t

$$\ln(1+y) = -\ln(2+\sin x) + \ln c$$

$$\ln((1+y)(2+\sin x) = \ln c \Rightarrow (1+y)(2+\sin x) = c \longrightarrow y = y(x).$$

$\Rightarrow \boxed{y(0)=1}$ $x=0, y=1$

$2(2+0) = c \Rightarrow \boxed{c=4}$

$\Rightarrow (1+y)(2+\sin x) = 4.$

$y(\pi) = ?$ ↳ $x=\pi$ $\Rightarrow (1+y)(2+\sin\pi) = 4$

$\Rightarrow (1+y) = 2 \Rightarrow \boxed{y=1}$

$\boxed{y(\pi)=1}$

JEE

Q if $y' = y+1$ & $y(0)=1$

find $y(\ln 2) = ?$

$$\frac{dy}{dx} = y+1 \Rightarrow \int \frac{dy}{y+1} = \int dx$$

$\Rightarrow$ ① $\ln(y+1) = x+c \longrightarrow y = (y(x)$

$\Rightarrow$ ② $\boxed{y(0)=1}$ $x=0, y=1$

$\Rightarrow \ln 2 = c \Rightarrow$

$\Rightarrow \ln(y+1) = x + \ln 2 \Rightarrow$ Now $y(\ln 2) = ?$ ↳ $x = \ln 2$

$\Rightarrow \ln(y+1) = \ln 2 + \ln 2$

$\ln(y+1) = \ln 4$

$y+1 = 4 \Rightarrow \boxed{y=3}$

Q find $f(x)$ $\{f(x)\geq 0,\ f(0)=0\ ;\ f(1)=1\}$ bounding a curvature trapezoid with base $[0,x]$ such that ABB $y=f(x)$, Co-chis & var. ordinate is directly proportion to $(f(x))^{n+1}$ — $n\in N$ —

$$\int_0^x f(x)\,dx = K(f(x))^{n+1}$$

$$A=\int_0^x f(x)\,dx \propto f(x)^{n+1}$$

$$f(x) \not\equiv = K(n+1)\, f(x)^n \cdot f'(x)$$

$$\int f(x)^{n-1} f'(x) = \int \frac{dx}{K(n+1)} \Rightarrow \frac{1}{n}(f(x))^n = \frac{x}{K(n+1)} + C$$

$\boxed{f(0)=0}$

$\Rightarrow \quad 0 = 0 + C \Rightarrow \boxed{C=0}$

$\boxed{f(1)=1}$

$\Rightarrow \quad \frac{1}{n} = \frac{1}{K(n+1)} + 0$

$\Rightarrow \quad \boxed{K=\frac{n}{n+1}}$

So $\frac{1}{n}(f(x))^n = \frac{x}{\frac{n}{(n+1)}(n+1)} \Rightarrow (f(x))^n = x$

$\Rightarrow \quad \boxed{f(x)=x^{1/n}}$

Q A Curve P.T. (2,3) Satisfying

$$\int_0^x t\cdot y(t)\,dt = x^2 y(x) \qquad (x>0) \text{ is? —}$$

$\hookrightarrow y=y(x)$

$$\int_0^x t\cdot(y(t)\,dt = x^2 y.$$

diff $\longrightarrow$

$$x\cdot y = x^2\cdot\frac{dy}{dx} + y\cdot 2x$$

$$x^2\frac{dy}{dx} = -xy$$

$$-\int\frac{dx}{x} = \int\frac{dy}{y}$$

$$-\ln x = \ln y + \ln c$$

$\boxed{xy=c}$ $\quad c=6 \Rightarrow \boxed{xy=6}$

② Reducible to V.S. Method ⇒ (Var. Separable)

① **When to use ?**

When x & y can not be separated. —

② **Form ?**

$$\frac{dy}{dx} = f(ax+by+c)$$

↳ Linear eqⁿ.

③ **Method**

take Linear fⁿ $= t$

$ax+by+c = t$

differentiate & Remove $\frac{dy}{dx}$ ⇒ See example –

Q Solve $\frac{dy}{dx} = (4x+y+1)^2$

↳ Linear eqⁿ ⇒ Hint

$4x+y+1 = t$

$4 + \frac{dy}{dx} = \frac{dt}{dx} \Rightarrow \frac{dy}{dx} = \frac{dt}{dx} - 4 = (t^2)$

$\frac{dt}{dx} = (t^2+4)$

$\int \frac{dt}{t^2+4} = \int dx$

$\frac{1}{2}\tan^{-1}(t/2) = x + C$

$\frac{1}{2}\tan^{-1}\left(\frac{4x+y+1}{2}\right) = x + C.$

Q Solve $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$

$x+y = t$

$1 + \frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dt}{dx} - 1 = \sin t + \cos t$

$\frac{dt}{dx} = 1 + \sin t + \cos t$

$\int \frac{dt}{1+\sin t+\cos t} = \int dx$

$\frac{1}{2}\int \frac{\sec^2 t/2}{1+\tan t/2} = \int dx$

$\int \frac{dt}{1+\frac{2\tan t/2}{1+\tan^2 t/2} + \frac{1-\tan^2 t/2}{1+\tan^2(t/2)}} = x + C$

$\frac{2}{2}\int \frac{du}{u} = \int dx$

$1+\tan t/2 = u$

$\sec^2 t/2 = 2du$

$\int \frac{\sec^2 t/2\, dt}{1+\tan^2 t/2 + 2\tan t/2 + 1 - \tan^2 t/2} = x + C$

$\ln u = x + C$

$\ln(1+\tan t/2) = x + C.$

(4th) **Linear Diff Eqn (LDE)...**

(1) ↳ in which dependent var. (y) & diff. Coffi. occur only in 1st degree & Not multiplied together.

$$P_n\frac{d^n y}{dx^n} + P_{n-1}\frac{d^{n-1}y}{dx^{n-1}} - - - - - - - - P_1\frac{dy}{dx} + P_0 y = x$$

(degree 1, degree 1, Power 1, Power 1)

↳ only f^n of x or Constant —

(2) · f^n $\sin y$, $\cos y$, $\log y$, e^y, $y\cdot\frac{dy}{dx}$ ⇒ Non Linear —
↳ bcz degree of $y \to \infty$

Homogeneous L.D.E. ⇒

$$\left(\frac{d^3y}{dx^3}\right)^2 + x^2\left(\frac{d^2y}{dx^2}\right)^2 + 3\left(\frac{dy}{dx}\right)^2 = 3y^2$$

1st ⇒ $\boxed{\frac{dy}{dx} + Py = Q}$ * P & Q are Constants or f^n of x →.

(1) I.F. = Integrating factor ⇒ we use Integrating factor to multiply So that Both Side of $\frac{dy}{dx} + Py = Q$ So that It Can be Integrated ⟶.

$$\boxed{I.F. = e^{\int P\cdot dx}}$$

$$\Rightarrow e^{\int P\cdot dx}\cdot\frac{dy}{dx} + e^{\int P\cdot dx}\cdot P\cdot y = Q\cdot e^{\int P dx}$$

$$\Rightarrow \frac{d}{dx}\left(e^{\int P dx}\cdot y\right) = Q\cdot e^{\int P\cdot dx}$$

$$\Rightarrow \boxed{y\cdot e^{\int P\cdot dx} = \int Q\cdot e^{\int P\cdot dx} dx}$$

$$\Rightarrow (2) \rightarrow \boxed{y(I.F.) = \int Q\cdot(I.F.)\,dx}$$

2nd ⇒ $\frac{dx}{dy} + Px = y$

(1) $I.F. = e^{\int P\cdot dy}$

(2) $x\cdot(IF) = \int Q\cdot IF.$

(3rd)

$\frac{dt}{dx} + Pt = Q$

$I.F = e^{\int P dt}$

$t\cdot I.F = \int Q(I.F)$

4th

Bernoullies DE ←

$$\frac{dy}{dx} + P(Y) = Q(Y^n)$$

Taug Y

① Y^n से divide ⇒ $\frac{1}{Y^n}\cdot\frac{dy}{dx} + \frac{P}{Y^{n-1}} = Q.$

② Y के साथ वाले को term = t
Then diff & use form 3"

$$\frac{1}{(1-n)}\cdot\frac{dt}{dx} + P\cdot t = Q$$

Let $\frac{1}{Y^{n-1}} = t$

$Y^{-n+1} = t$

$(1-n)\, Y^{-n}\frac{dy}{dx} = \frac{dt}{dx}$

$\frac{1}{Y^n}\cdot\frac{dy}{dx} = \left(\frac{1}{1-n}\right)\frac{dt}{dx}$

(II) ⇒ B ⇒

Eqn Rducible to linear form ⇒

$$f'(Y)\cdot\frac{dy}{dx} + f(Y)\,P = Q$$

Put $f(Y) = t$

$f'(Y)\frac{dy}{dx} = \frac{dt}{dx}$

$$\frac{dt}{dx} + tP = Q$$

$I.F = e^{\int P dx}$

Soln → $\boxed{t\cdot(I.F.) = \int (I.F.)\,Q\cdot dx}$

Clairaut's form -

$Y = P\cdot x + f(P)$

Replace P by Constant "C"

where $\boxed{P = \frac{dy}{dx} = C}$ ⇒ Soln → $\boxed{Y = x\cdot C + f(C)}$

eg - $Y = \frac{dy}{dx}\cdot x + \sin^{-1}\left(\left(\frac{dy}{dx}\right)^2 + 3\right)$ Soln ⇒ $Y = xC + \sin^{-1}(C+3)$.

Q $2xy - x^2\frac{dy}{dx} = 1$

$x^2\frac{dy}{dx} - 2xy = -1$

$\frac{dy}{dx} - \frac{2}{x}y = \frac{-1}{x^2}$ $\Rightarrow$ ① $I.f = e^{-\int\frac{2}{x}dx} = e^{-2\ln x} = e^{\ln x^{-2}} = \frac{1}{x^2}$

$\Rightarrow$ ② $y\cdot\frac{1}{x^2} = +\int\frac{-1}{x^2}\times\frac{1}{x^2}$

$= -\int x^{-4}\,dx$

$y\cdot\frac{1}{x^2} = \frac{x^{-3}}{3}$

Bernoulli Problem →

Q $x\cdot\frac{dy}{dx} + y = x^3y^6$.

two y & इस में रखक की degree > 1

$\frac{dy}{dx} + \frac{y}{x} = x^2y^6$

$\frac{1}{y^6}\cdot\frac{dy}{dx} + \frac{1}{xy^5} = x^2$ $\Rightarrow$ $\frac{1}{y^5} = t \Rightarrow y^{-5} = t$

$\Rightarrow -5y^{-6}\frac{dy}{dx} = \frac{dt}{dx}$

$\frac{1}{y^6}\cdot\frac{dy}{dx} = \frac{1}{-5}\frac{dt}{dx}$

$-\frac{1}{5}\cdot\frac{dt}{dx} + \frac{t}{x} = x^2$

$\longrightarrow$ $\frac{dt}{dx} - \frac{5t}{x} = -5x^2$

① $I.f. = e^{\int -5/x\,dx} = e^{-5\ln x} = \frac{1}{x^5}$

② $t\cdot\frac{1}{x^5} = \int -5x^2\cdot\frac{1}{x^5}\cdot dx.$

Q $\cos x\,dy = (y\sin x - y^2)\,dx$

$\frac{dy}{dx} - y\tan x = -y^2\sec x.$ $\longrightarrow$ BDE

$\searrow$ y^2 से divide →

$-\frac{1}{y^2}\frac{dy}{dx} + \frac{1}{y}\cdot\tan x = \sec x.$

$\Rightarrow \frac{1}{y} = t$

$\frac{dt}{dx} + t\cdot\tan x = \sec x$ $\Rightarrow -\frac{1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}$

① $I.f. = e^{\int\tan x\,dx} = e^{\ln|\sec x|}$

$= |\sec x|$

② $t\cdot\sec x = \int \sec x\cdot\sec x\,dx$

Mixture problem

Q A tank contain 100 Litre lime Soln (चूना) in which 50 gm of Salt dissolve. lime containing 2 gram/Litre of Salt Runs into the tank at Rate 5 litre/min & flows out of tank at Rate of 4 litre/min →

form diff. eqn & Solve it & find Expression for amount of Salt Present at time "t" →

① 2 gm/Litre, 5 Litre/min } 10 gm/min Salt Input

② 1 Min → (5−4) litre

t min → t litre (Inside).

Soln ⇒ Let y is amount of Salt at time "t" →

$\frac{dy}{dt}$ = (Rate of Salt arrives) − (Rate of Salt dipart).

$= 10 - \left(\frac{Y}{V(t)}\right) \cdot \text{time}$

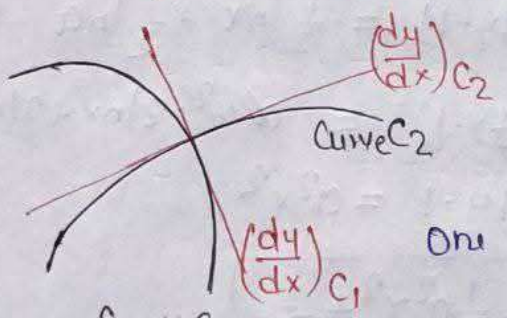
Y → Concentration, V(t) → Volume (100 Litre initially & after time t some Litre more added)

$=$

$$\boxed{\frac{dy}{dt} = 10 - \frac{Y}{100+t} \cdot t}$$

$$\frac{dy}{dt} + Y \cdot \frac{t}{100+t} = 10.$$

Orthogonal Trajectory –



find $\frac{dy}{dx}$ only interms of x & y

$$\boxed{\left(\frac{dy}{dx}\right)_{C_1} \times \left(\frac{dy}{dx}\right)_{C_2} = -1}$$

⇒ what we are going to do →

One Curve will be given (C_1) & we have to find Curve (C_2) that is orthogonal to C_1 & to find that path is called orthogonal Trajectory →

Q find O.T of $y = ax^2$

① $\frac{dy}{dx} = 2ax = 2x\left(\frac{y}{x^2}\right)$

$\left(\frac{dy}{dx}\right)_{C_1} = \frac{2y}{x}$

② $\left(\frac{dy}{dx}\right)_C \cdot \left(\frac{dy}{dx}\right)_{C_2} = -1$

$\left(\frac{dy}{dx}\right)_{C_2} = \frac{-x}{2y}$

$2y\,dy = -x\,dx$

$y^2 = \frac{-x^2}{2} + C$ ⇒ This is curve that is orthogonal to C_1

⇒ Eqⁿ Reducible to Homogeneous form

Eqⁿ Form $\frac{dy}{dx} = \frac{ax+by+c}{Ax+By+C}$ or $= \frac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2}$ $\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Can be Reduced to Homogeneous form →

By taking $x = X+h$, $y = Y+K$ such that $a_1h+b_1k+c_1=0$, $a_2h+b_2k+c_2=0$ } Solve & find (h, k)

Qu Solve $x\frac{dy}{dx} = y + 2\sqrt{y^2-x^2}$ $\Rightarrow \frac{dy}{dx} = \frac{y}{x} + 2\sqrt{\left(\frac{y}{x}\right)^2 - 1}$

↳ $\frac{y}{x} = z$ & Solve

$\Rightarrow y+\sqrt{y^2-x^2} = Cx^3$ Ans

Q Solve -

$(2x-y+1)dx + (2y-x+1)dy = 0$

$\frac{dy}{dx} = -\frac{(2x-y+1)}{(2y-x+1)}$ del $y = Y+K$ & $x = X+h$.

Then $\frac{dy}{dx} = \frac{dY}{dX} = -\frac{(2X-Y+2h-K+1)}{(2Y-X+2K-h+1)}$

Now $2h-K+1=0$ & $2K-h+1=0$

Solving $h=-1, K=-1$

$\frac{dY}{dX} = -\frac{(2X-Y)}{(2Y-X)}$ → Now Homogeneous → (HDE)

$\frac{dY}{dX} = -\frac{\left(2-\frac{Y}{X}\right)}{\left(2\frac{Y}{X}-1\right)}$ → $\frac{Y}{X} = z$

$\frac{dY}{dX} = z + X\frac{dz}{dX}$

$z + X\frac{dz}{dX} = \frac{-(2-z)}{(2z-1)}$

$X\frac{dz}{dX} = -\left(\frac{2-z}{2z-1} + z\right) = -\left(\frac{2-z+2z^2-z}{2z-1}\right)$

$\int \frac{2z-1}{2z^2-2z+2}dz = -\int \frac{dX}{X}$

$\frac{1}{2}\ln(z^2-z+1) = -\ln X + \ln K \Rightarrow \ln\left(\sqrt{z^2-z+1}\right)\cdot X = \ln K$

$\Rightarrow X\cdot\sqrt{z^2-z+1} = K \Rightarrow \sqrt{(Y^2-XY+X^2)} = K.$

$\Rightarrow (Y+1)^2 - (Y+1)(X+1) + (X+1)^2 = K^2$

Exact differentials Must be Remembered

① $x\,dy + y\,dx = d(xy)$

② $\frac{y\,dx - x\,dy}{y^2} = d\left(\frac{x}{y}\right)$

③ $\frac{dx+dy}{x+y} = d(\ln(x+y))$

④ $\frac{y\,dx - x\,dy}{xy} = d\left(\ln\left(\frac{x}{y}\right)\right)$

⑤ $\frac{y\,dx - x\,dy}{x^2+y^2} = d\left(\tan^{-1}\left(\frac{x}{y}\right)\right)$

⑥ $d\left(-\frac{1}{xy}\right) = \frac{x\,dy + y\,dx}{x^2y^2}$

⑦ $d\left(\frac{e^y}{x}\right) = \frac{xe^y dy - e^y dx}{x^2}$

⑧ $\frac{x\,dx + y\,dy}{\sqrt{x^2+y^2}} = d(\sqrt{x^2+y^2})$

⑨ $\frac{x\,dy - y\,dx}{x^2} = d\left(\frac{y}{x}\right)$

⑩ $\frac{x\,dy + y\,dx}{xy} = d(\ln(xy))$

⑪ $\frac{x\,dy - y\,dx}{xy} = d\left(\ln\left(\frac{y}{x}\right)\right)$

⑫ $\frac{x\,dy - y\,dx}{x^2+y^2} = d\left(\tan^{-1}\frac{y}{x}\right)$

⑬ $\frac{x\,dx + y\,dy}{x^2+y^2} = d\left(\ln\sqrt{x^2+y^2}\right)$

⑭ $d\left(\frac{e^x}{y}\right) = \frac{ye^x dx - e^x dy}{y^2}$

Q. Solve $x\,dx + y\,dy = \frac{x\,dy - y\,dx}{x^2+y^2}$

$$\frac{1}{2}\,d(x^2+y^2) = d\left(\tan^{-1}(y/x)\right) \Rightarrow \frac{1}{2}(x^2+y^2) = \tan^{-1}\left(\frac{y}{x}\right) + C$$

Q Solve $\{(x+1)\left(\frac{y}{x} + \sin y\right)\}dx + (x + \ln x + x\cos y)\,dy = 0$

$\Rightarrow (y\,dx + x\,dy) + \left(\frac{y}{x}dx + \ln x\,dy\right) + (\sin y\,dx + x\cos y\,dy) = 0$

$\Rightarrow d(xy) + d(y\ln x) + d(x\sin y) = 0$ Integrating –

$\Rightarrow xy + y\ln x + x\sin y = C.$

Questions →

① $y\,dx - x\,dy + 3x^2y^2e^{x^3}dx = 0$ $\Rightarrow \frac{x}{y} + e^{x^3} = C$

② $y\,dx + (x + x^2y)\,dy = 0$ $\Rightarrow \frac{-1}{xy} + \log y = C$

③ $(xy^4 + y)\,dx - x\,dy = 0$ $\Rightarrow 3x^4y^3 + 4x^3 = Cy^3$

④ $\frac{dy}{dx} = \frac{2xy}{x^2 - 1 - 2y}$ $\Rightarrow \frac{x^2}{y} = \frac{1}{y} - 2\log y + C.$

Application of 1st ORdr D.E

① Growth and Decay Problems ⇒.

de N(t) denotes amount of Substance or popolation — (either growing or dicaying →

⇒① If we Assume dicay/grow Rate $\left(\frac{dN}{dt}\right)$ is proportional to amount of Substance present then —

$$\frac{dN}{dt} = KN \quad \text{(grow)} \qquad \text{Or} \qquad \frac{dN}{dt} = -KN \quad \text{(dicay)}$$

⇒② Solve D.E for N at any time "t" —

Q if popula^{n} of country doubles in 50yr. then No. of years in which Popl^{n} will be Triple under Assumption that Rate of Increase is Proportional to No. of Inhabitants. is —

(a) 79yr. (b) 78yr.
(c) 77yr (d) 76yr.

let $x \longrightarrow$ pobl^{n} at time t (yrs)

Then— $\frac{dx}{dt} \propto x \Rightarrow \int \frac{dx}{dt} = \int kx \quad \Rightarrow \log x = kt + c$

$$\Rightarrow \quad x = e^{kt+c} \quad \Rightarrow \boxed{x = x_0 e^{kt+c}}$$

now $x_0 =$ popl^{n} at $t=0$

Since it double's in 50yr's — at $\boxed{t=50 \Rightarrow x = 2x_0}$

$$2x_0 = x_0 e^{50k} \Rightarrow 50k = \log 2$$

$$\Rightarrow \boxed{k = \frac{\ln 2}{50}}$$

So $x = x_0 e^{\frac{\ln 2}{50} \cdot t}$

Now $x = 3x_0$ & $t = ?$ $\Rightarrow 3 = e^{\frac{\ln 2}{50} \cdot t}$

$$\Rightarrow \boxed{t = \frac{50 \ln 3}{\ln 2}} = 79\text{yrs}.$$

② Temprature problem

① Newton's Law of Cooling -

"time Rate of change of temperature of body is proportional to temperature difference b/w body & Sourrounding Medium."

→ let T = Temp. of body
Tm = Temp. of medium (Sourrounding)

$-\frac{dT}{dt} \propto (T-Tm) \Rightarrow \boxed{\frac{dT}{dt} = -k(T-Tm)}$

(-) Since Colling.

bcz T > Tm. (Cooling process)
if T < Tm ⇒ ⊕ve

Q A body at a temp. of 50f is placed outdoors where the Temp. is 100f if Rate of change of temp. of body is proportional to temp. difference b/w body & medium. if after 5min the Temp of body is 60f. find

(a) How long it take the body to Reach a temp. of 75f.

(b) & The temp. of body after 20min —

T temp. of body at any time "t"
Tm = 100 (Sourrounding)

$\frac{dT}{dt} = -k(T-Tm) \Rightarrow \frac{dT}{dt} + kT = 100k.$

$\int \frac{dT}{T-Tm} = \int -k\,dt \Rightarrow \ln(T-Tm) = -kt + C$

$\Rightarrow T = Tm + e^{-kt+C}$

$\Rightarrow T = To\,e^{-kt} + 100$

$\boxed{T=50 \text{ at } t=0} \Rightarrow 50 = To + 100 \Rightarrow To = -50$

$\Rightarrow T = -50e^{-kt} + 100$

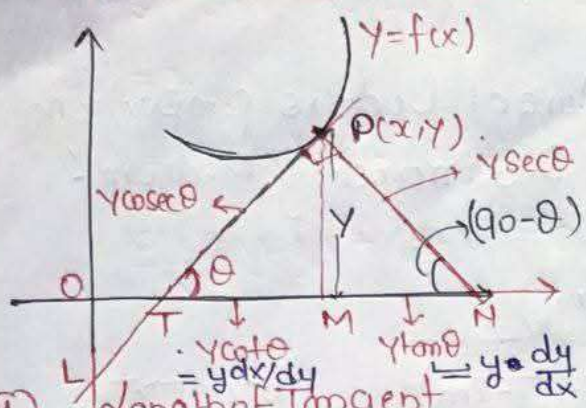
at $\boxed{t=5, T=60} \Rightarrow 60 = -50e^{-5k} + 100 \Rightarrow k = -\frac{1}{5}\ln\left(\frac{40}{50}\right)$

$T = -50e^{\frac{t}{5}\ln\frac{4}{5}} + 100$

(a) $t=?\ T=75 \Rightarrow 75 = -50e^{\frac{t}{5}\ln 4/5} + 100 \Rightarrow t=?$

(b) $T=?$ when $t=20$.

③ Geometrical Applications



$\tan\theta = \frac{dy}{dx}$

① Length of Tangent

PT is difined as Length of tangent

$$PT = |Y \operatorname{cosec}\theta| = |Y\sqrt{1+\cot^2\theta}| = \left|Y\sqrt{\left\{1+\left(\frac{dx}{dy}\right)^2\right\}}\right|$$

$$\left\{\begin{array}{l} \text{In}\triangle PTM \\ \sin\theta = \frac{Y}{PT} \\ PT = Y\operatorname{cosec}\theta \end{array}\right\}$$

② Length of Normal

PN is difined as Length of Normal

$$PN = |Y\sec\theta| = \left|Y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\right|$$

$$\left\{\begin{array}{l} \operatorname{cosec}(90-\theta) = \frac{PN}{Y} \\ PN = Y\sec\theta \end{array}\right\}$$

③ Length of Sub-tangent –

TM difined as Subtangent

$$TM = |Y\cot\theta| = \left|Y\cdot\frac{dx}{dy}\right|$$

④ Length of Sub Normal –

MN is Sub Normal –

$$MN = |Y\tan\theta| = \left|Y\cdot\frac{dy}{dx}\right|$$

⑤ Intercepts made by tangents on the Coordinate Axes → (OT & OL)

Tangent →

$$Y - y = \frac{dy}{dx}(X-x)$$

for $OT = X = x - y\frac{dx}{dy}$ (at $Y=0$)

So $\boxed{OT = x - y\frac{dx}{dy}}$ → Intercept on X-axis

TM → Subtangent

$OL = Y = y - x\frac{dy}{dx}$ (at $X=0$)

$\boxed{OL = y - x\frac{dy}{dx}}$ → Intercept on Y-axis