

Polynomials

1. Degree of a Polynomial

- **Linear polynomial** → Degree 1
- **Quadratic polynomial** → Degree 2
- **Cubic polynomial** → Degree 3

2. Relationship between Zeros and Coefficients

(i) Quadratic Polynomial:

$$ax^2 + bx + c$$

- Sum of zeros: $\alpha + \beta = -\frac{b}{a}$
- Product of zeros: $\alpha\beta = \frac{c}{a}$

(ii) Cubic Polynomial:

$$ax^3 + bx^2 + cx + d$$

- Sum of zeros: $\alpha + \beta + \gamma = -\frac{b}{a}$
- Sum of product of zeros (two at a time):
 $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$
- Product of zeros: $\alpha\beta\gamma = -\frac{d}{a}$

3. Remainder Theorem

If a polynomial $f(x)$ is divided by $(x-a)$

→ Remainder = $f(a)$

3. Factor Theorem

If $f(a) = 0$, then $(x - a)$ is a factor of $f(x)$.

4. Identities related to Polynomials

- $x^2 - (\alpha + \beta)x + \alpha\beta$ → Quadratic formed from given roots
- $x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$ → Cubic formed from given roots

1. Squares (Identities)

- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- $(x + a)(x + b) = x^2 + (a + b)x + ab$
- $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

2. Cubes (Identities)

- $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
- $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
- $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
- $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

3. Special Identities

- $(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$
- $(x + y + z)^3 = x^3 + y^3 + z^3 + 3(x + y)(y + z)(z + x)$
- $(a + b)^2 - (a - b)^2 = 4ab$
- $(a + b)^3 + (a - b)^3 = 2(a^3 + b^3)$
- $(a + b)^3 - (a - b)^3 = 6ab(a + b)$

4. Factorization Identities

- $a^2 - b^2 = (a - b)(a + b)$
- $x^2 + (a + b)x + ab = (x + a)(x + b)$
- $x^2 - (a + b)x + ab = (x - a)(x - b)$

