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Chapter - Wave Optics

Wave-front

Locus of all the points which are in same phase and are equidistant from the source emitting light is called wave-front.

Wave-front are of three types -

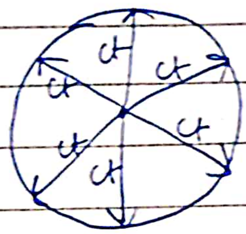
1. Spherical wave-front
2. cylindrical wave-front.
3. Planar-wave front

Spherical wave-front :-

This type of wave-front is formed when source emitting light is point in size.

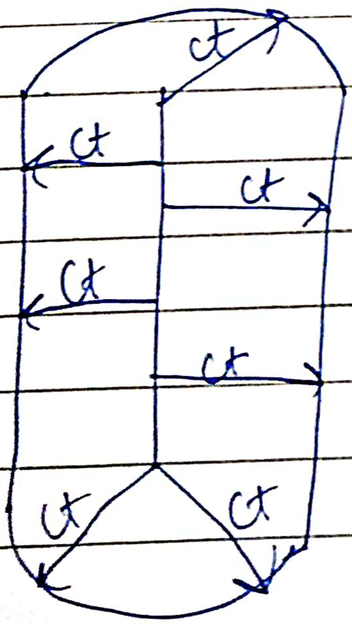
$S = \frac{\text{distance}}{\text{time}}$

distance $= c \times t$.



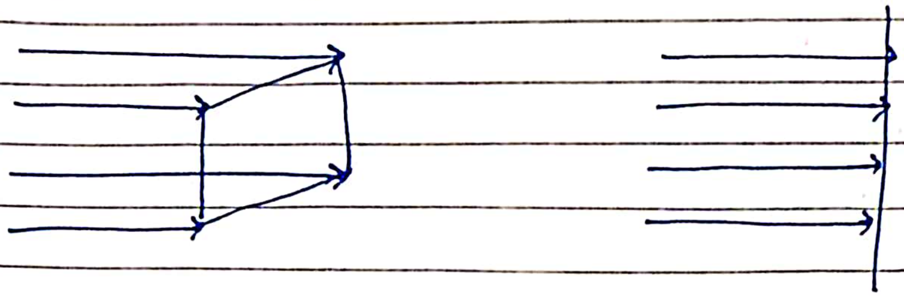
Cylindrical wave-front :-

This type of wave-front is formed when source emitting light is linear in ~~size~~ shape.



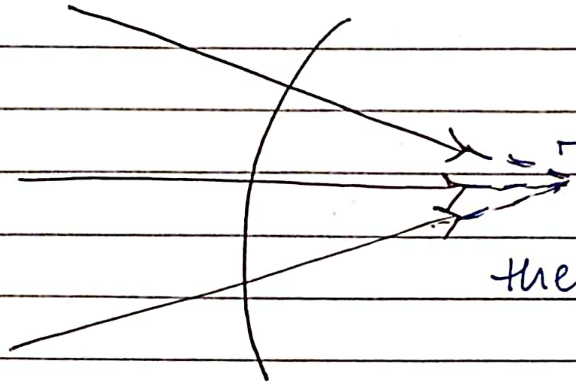
Planar wave-front.

When source emitting light is at infinity; planar wave-front is formed.



Q. Draw wave-front for the following :-

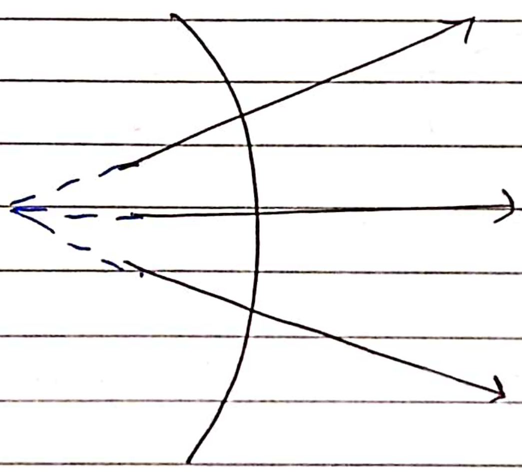
(a)



Converging rays.

→ we will extend it and formed a point and then draw an arc.

(b)



Diverging rays

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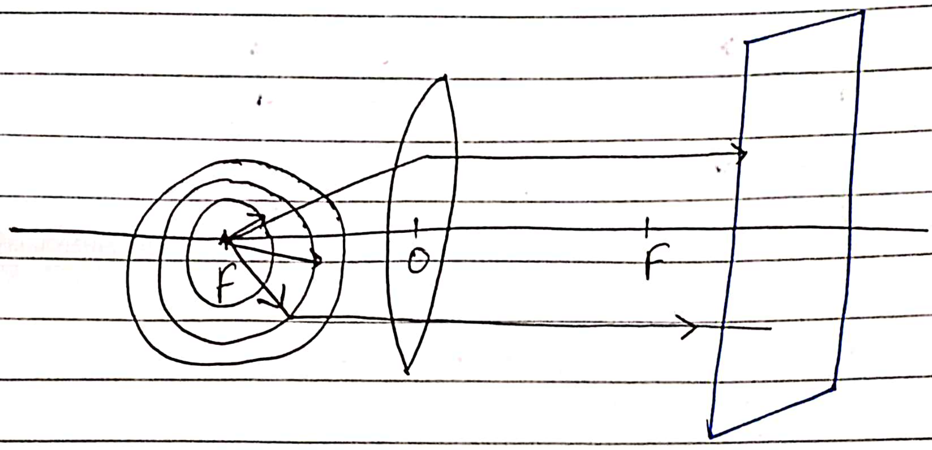
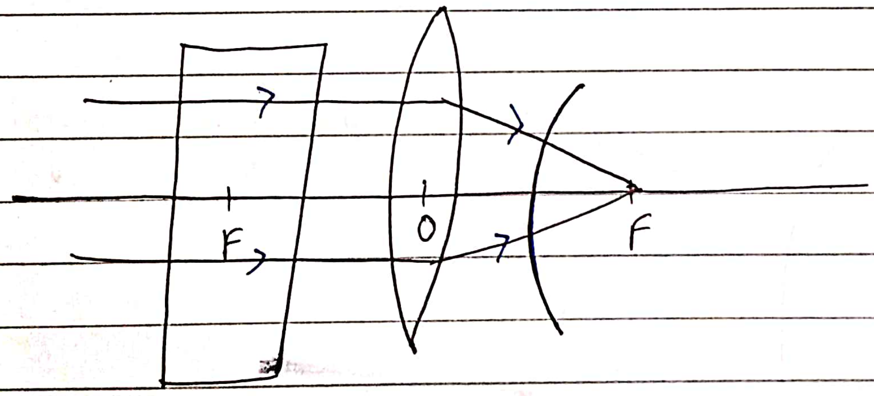
Draw the wave-front of refracted rays if a spherical wave-front is kept at focus of convex lens.

→



The wave-front formed is planar. wave front

Draw wave-front of refracted part when a planar wave front is kept on incident part of convex lens.



The wave front formed is spherical-wave front

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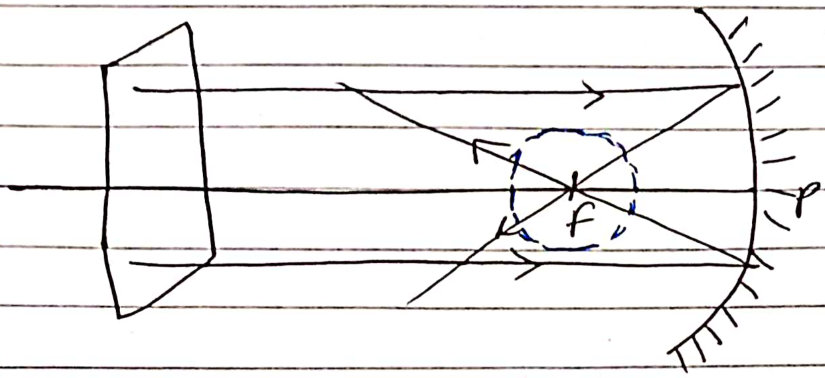
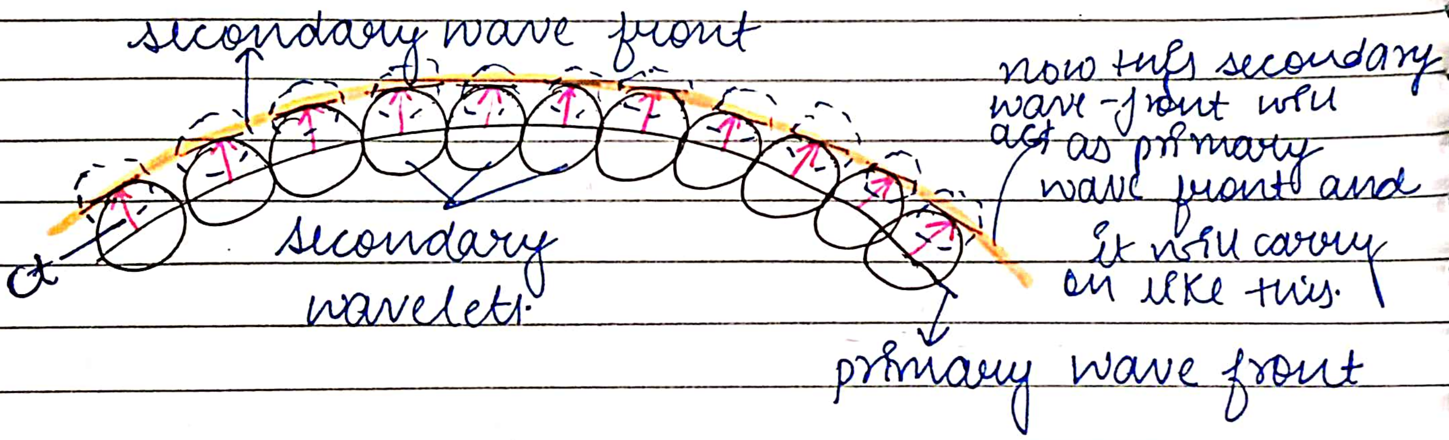
Q. When a planar wave-front is incident on a concave mirror, what type of wave-front is formed after reflection.



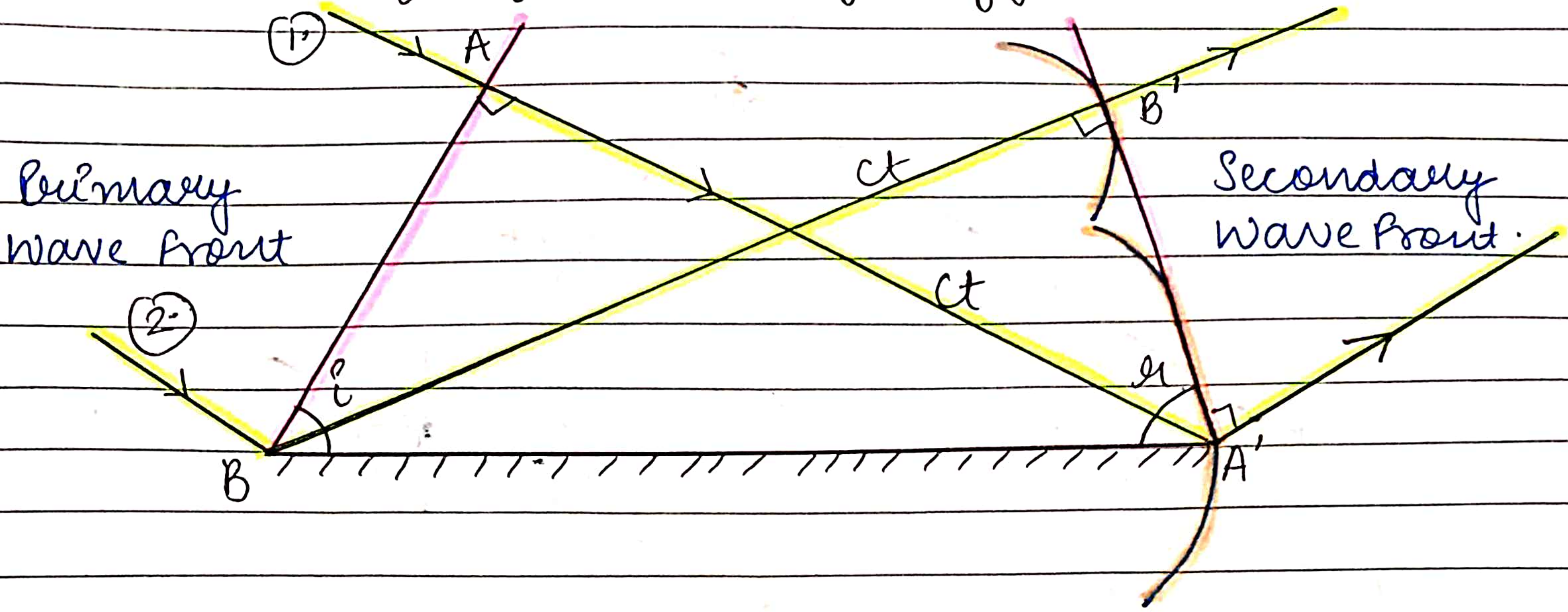
Wavefront formed is spherical-wave-front.

Huygen's Principle

1. Every point on the primary wave front act as a source of secondary wavelets.
2. Envelope of tangents on secondary wavelets give rise to secondary wave-front in forward direction.



Derive laws of reflection using Huygen's principle



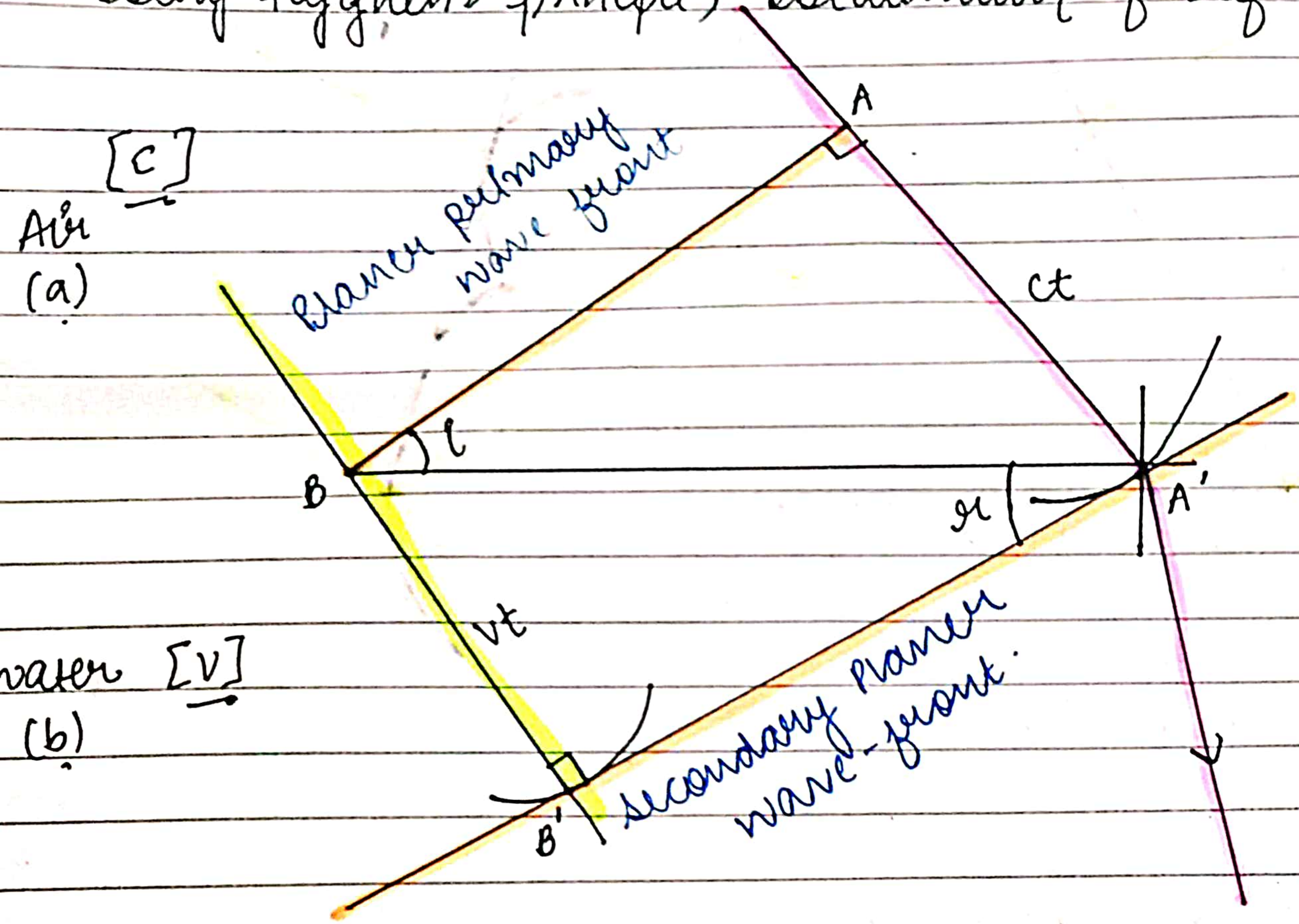
n; $\Delta ABA'$ and $\Delta BB'A'$.

$AA' = BB' = (ct)$.
$BA' = BA' =$ (common)
$\angle A = \angle B' = [90°]$

$\Delta ABA' \cong \Delta BB'A'$ [RHS congruency].

$\therefore \angle i = \angle r$ [C.P.C.T]

Using Huygen's principle; obtain laws of refraction.



In $\Delta ABA'$

$$\sin i = \frac{AA'}{\cancel{BA'}}$$

$$\sin r = \frac{BB'}{\cancel{BA'}}$$

$$\frac{\sin i}{\sin r} = \frac{AA'}{BB'} = \frac{c\cancel{t}}{v\cancel{t}}$$

$$\boxed{\frac{\sin i}{\sin r} = \frac{c}{v} = n}$$

Spiral

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Interferance :-

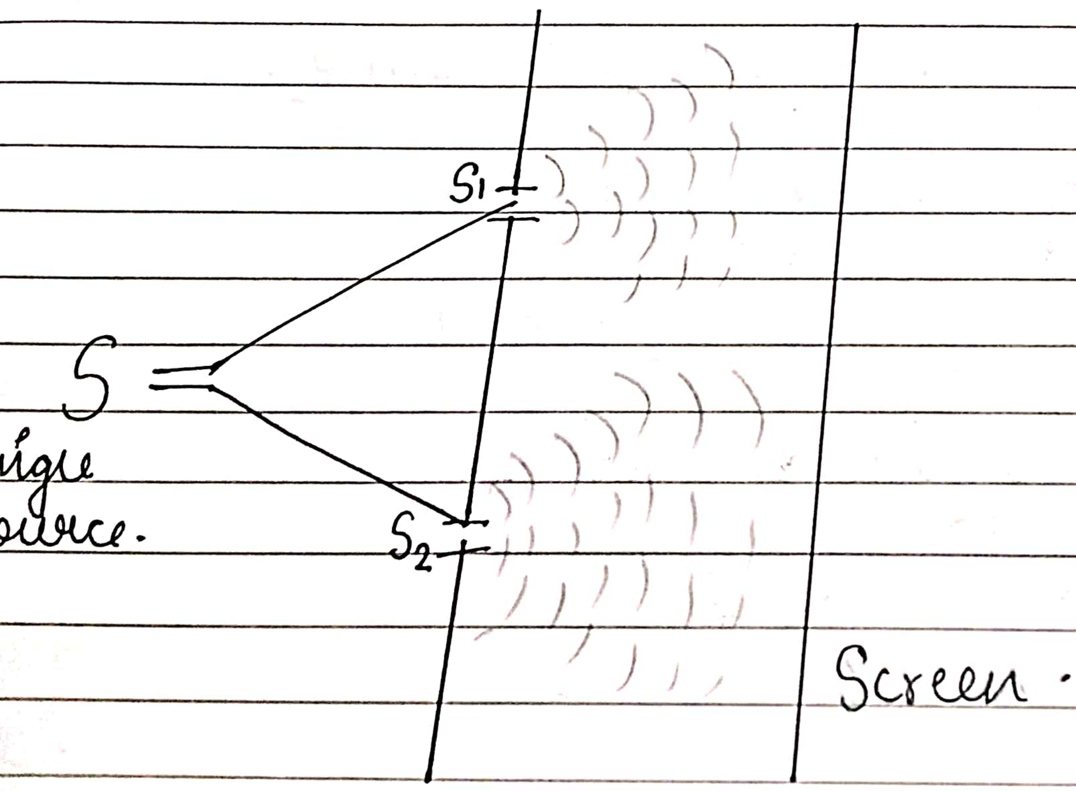
Super position of two waves coming from two sources to redistribute their energy is called interferance.

In order for interferance to occur, the light coming from two sources should have:-

(i) same frequency.
(ii) same wavelength.
(iii) Nearly amplitude.
(iv) Same phase.

Such sources are called coherent sources.

Young's double slit experiment



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Light coming from Source S_1;

$y = a_1 \sin wt \longrightarrow (1)$

II^{nd} Source S_2;

$y_2 = a_2 \sin(wt + \phi) \longrightarrow (2)$

$\phi \rightarrow$ Phase Difference.

Superposition;

$y = y_1 + y_2$.

$y = a_1 \sin wt + a_2 \sin(wt + \phi)$.

$y = a_1 \sin wt + a_2 [\sin wt \cos\phi + \cos wt \sin\phi]$.

$y = a_1 \sin wt + a_2 \sin wt \cos\phi + a_2 \cos wt \sin\phi$.

$y = \sin wt [a_1 + a_2 \cos\phi] + \cos wt [a_2 \sin\phi] — (3)$

let $a_1 + a_2 \cos\phi = A\cos\theta \rightarrow (4)$

$a_2 \sin\phi = A \sin\theta \rightarrow (5)$

Put (4) & (5) in (3);

$y = \sin wt [A\cos\theta] + \cos wt [A\sin\theta]$

$y = A[\sin wt \cos\theta + \cos wt \sin\theta]$

$\boxed{y = A\sin[wt + \theta]} \longrightarrow (6)$

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To find A & θ :-

Squaring and adding (4) & (5);

$$(a_1 + a_2 \cos\phi)^2 + (a_2 \sin\phi)^2 = A^2\cos^2\theta + A^2\sin^2\theta .$$

$$a_1^2 + a_2^2\cos^2\phi + 2a_1a_2\cos\phi + a_2^2\sin^2\phi = A^2 .$$

$$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi} \longrightarrow (7.)$$

Dividing (5) by (4).

$$\tan\theta = \frac{a_2\sin\phi}{a_1 + a_2\cos\phi}$$

$$\theta = \tan^{-1}\left(\frac{a_2\sin\phi}{a_1 + a_2\cos\phi}\right).$$

For $A \rightarrow$ to be max.

$$A \rightarrow A_{max}.$$

$$\underbrace{\cos\phi = +1}$$

$$\phi = 0;\ 2\pi;\ 4\pi \cdots \underbrace{2n\pi,}_{n = 1, 2, 3 \cdots}$$

$$A_{max} = \sqrt{a_1^2 + a_2^2 + 2a_1a_2} .$$

$$A_{max} = \sqrt{(a_1 + a_2)^2}$$

$$\boxed{A_{max} = a_1 + a_2}$$

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For $A \rightarrow A_{min}$

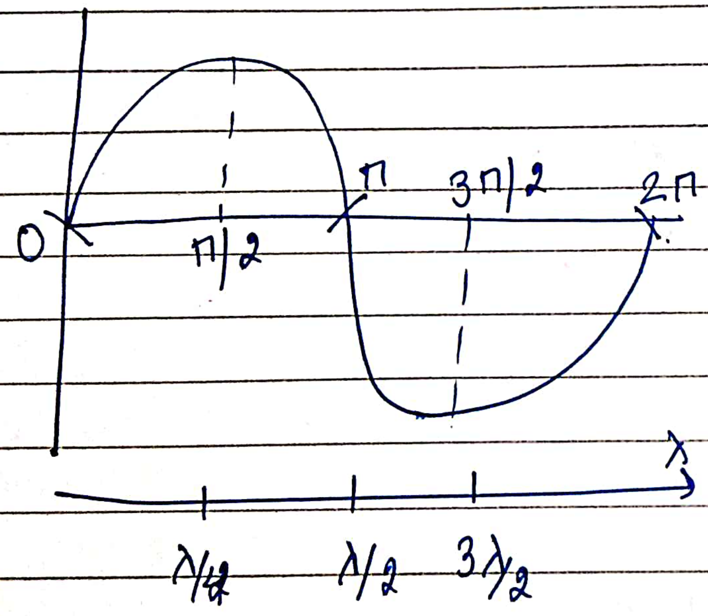
$\cos\phi = -1$

$\phi = \pi ; 3\pi ; 5\pi ; (2n+1)\pi \ldots$

$n = 0, 1, 2, 3 \ldots$

$$\boxed{A_{min} = a_1 - a_2}$$

Relation between Phase Difference (ϕ) (Angle) And Path Difference (x) (Distance).



Angle $\longrightarrow$ Path

$2\pi \longrightarrow \lambda$

$1 \longrightarrow \lambda/2\pi$

$\phi \longrightarrow \dfrac{\lambda}{2\pi}\phi = x.$

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$$\boxed{\phi = \frac{2\pi}{\lambda} x} \longrightarrow (8.)$$

For maxima / constructive interferance / Brightness.

$$\boxed{\phi = 2n\pi} \longrightarrow (9.)$$

comparing (8) & (9)

$$\frac{\cancel{2}\cancel{\pi} x}{\lambda} = \cancel{2} n \cancel{\pi}$$

$$\boxed{x = n\lambda}$$ → Path Difference to maxima.

For minima / Destructive interferance / Darkness.

$$\phi = (2n+1)\pi$$

$$\frac{2\cancel{\pi} x}{\lambda} = (2n+1)\cancel{\pi}$$

$$\boxed{x = (2n+1)\frac{\lambda}{2}}$$

Intensity of pattern (light) seen on screen.

$I \propto (\text{Amplitude})^2$.

I source a_1
II source a_2.

$I_1 \propto a_1^2$
$I_2 \propto a_2^2$.

$$\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2}.$$

$I_{max} \propto A_{max}^2$
$I_{max} \propto (a_1+a_2)^2$.

$I_{min} \propto (a_1-a_2)^2$.

$$\frac{I_{max}}{I_{min}} = \frac{A_{max}^2}{A_{min}^2} = \frac{(a_1+a_2)^2}{(a_1-a_2)^2}.$$

$$A = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi}$$

$$A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi.$$

$$I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi.$$

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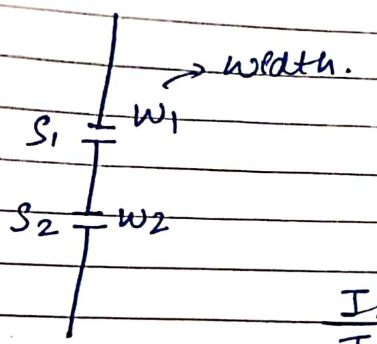

Intensity ∝ width of slit.

$I_1 \propto w_1$

$I_2 \propto w_2$

$$\frac{I_1}{I_2} = \frac{w_1}{w_2} = \frac{a_1^2}{a_2^2}.$$

Q1 Two coherent sources of light have an intensity ratio of 64:25. What is their amplitude ratio.

A1

$$\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2}.$$

$$\frac{\sqrt{64}}{\sqrt{25}} = \frac{a_1}{a_2}$$

$$\frac{8}{5} = \frac{a_1}{a_2}$$

$$\underbrace{a_1 : a_2 = 8:5}$$

Q2 What is the ratio of the width's of the slit if the amplitude ratio is $\sqrt{2} : 1$.

A1

$$\frac{a_1^2}{a_2^2} = \frac{w_1}{w_2}.$$

$$\frac{(\sqrt{2})^3}{(1)^2} = \frac{w_1}{w_2}$$

$w_1 : w_2 = 2:1$

Spiral

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Q3 The two slits in Young's experiment have width in the ratio 1:9. find the ratio of intensity of light at the maxima and the minima in the interferance pattern.

Ans3

$$\frac{I_1}{I_2} = \frac{w_1}{w_2} = \frac{a_1^2}{a_2^2}.$$

$$\frac{1}{9} = \frac{a_1^2}{a_2^2}$$

$$\boxed{\frac{a_1}{a_2} = \frac{1}{3}}$$

$$\frac{I_{max}}{I_{min}} = \frac{(a_1+a_2)^2}{(a_1-a_2)^2} = \frac{(1+3)^2}{(1-3)^2} = \frac{(4)^2}{(-2)^2} = \frac{\cancel{16}}{4} = \frac{4}{1}.$$

$I_{max} : I_{min} = 4:1$.

Q4 The ratio of Intensity at max. & minima in the interferance pattern is 25:9. What will be the ratio of the widths of the slits in Young's experiment.

$$\frac{I_{max}}{I_{min}} = \frac{(a_1+a_2)^2}{(a_1-a_2)^2}.$$

$$\frac{25}{9} = \frac{(a_1+a_2)^2}{(a_1-a_2)^2}$$

$$a_1 + a_2 = 5 \quad \Rightarrow$$

$$a_1 - a_2 = 3$$

$$2a_1 = 2$$

$$\boxed{a_1 = 4}$$

$$\cancel{a_2 = 8} \qquad 4 + a_2 = 5$$

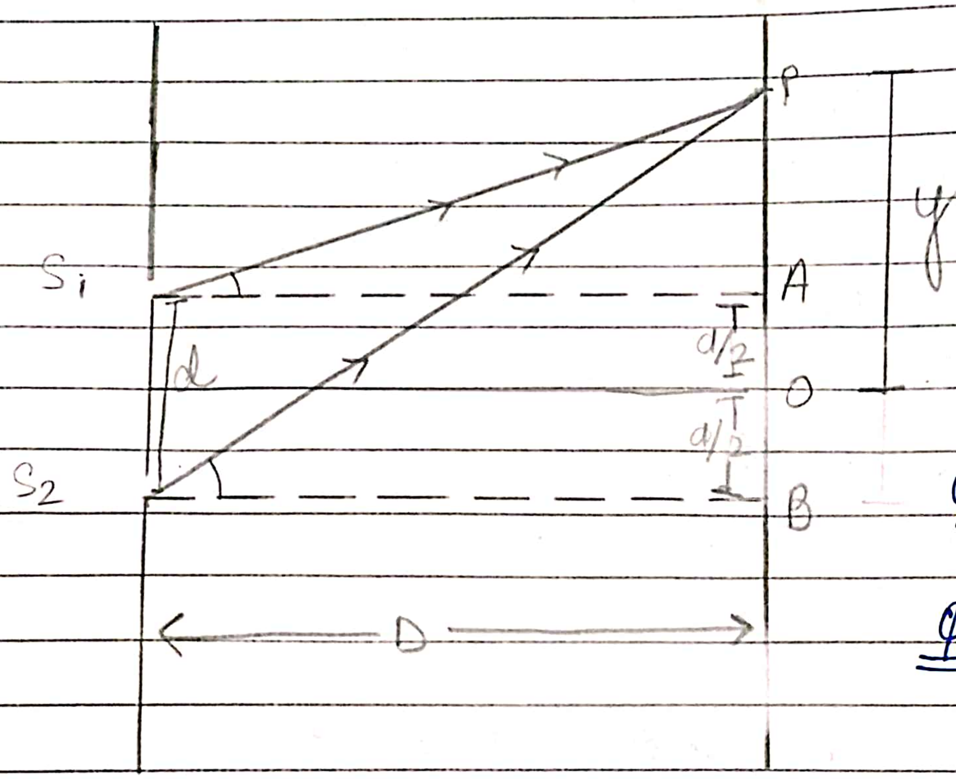
$$\boxed{a_2 = 1}.$$

$$\frac{W_1}{W_2} = \frac{(a_1)^2}{(a_2)^2} = \frac{(4)^2}{(1)^2}$$

$$\text{so } \frac{W_1}{W_2} = \frac{16}{1}$$

$$\underbrace{W_1 : W_2 = 16 : 1}.$$

•) Theory of interference fringes [YDSE].



$\phi = \frac{2\pi(0)}{\lambda}$ $x = 0$

$\phi = 0$

S_1 and S_2 slits.

Path Difference (x);

$$x = S_2P - S_1P \longrightarrow (1)$$

ΔS_2PB;

$$S_2P^2 = S_2B^2 + BP^2.$$

$$S_2P^2 = D^2 + \left(y + \frac{d}{2}\right)^2 \longrightarrow (2)$$

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In ΔS_1PA;

$$S_1P^2 = S_1A^2 + AP^2$$

$$S_1P^2 = D^2 + \left(y - \frac{d}{2}\right)^2 \longrightarrow (3)$$

Subtracting (3) from (2);

$$S_2P^2 - S_1P^2 = \left(y + \frac{d}{2}\right)^2 - \left(y - \frac{d}{2}\right)^2.$$

$$(S_2P - S_1P)(S_2P + S_1P) = 2yd.$$

$$x = \frac{2yd}{S_2P + S_1P}.$$

If angles are small;

$$S_2P \approx S_1P \approx D.$$

$$x = \frac{2yd}{2D}.$$

$$x = \frac{yd}{D} \longrightarrow (4)$$

For maxima / constructive interference / Brightness

$$x = n\lambda \longrightarrow (5)$$

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Comparing (4) & (5);

$$\frac{yd}{D} = n\lambda.$$

$$\boxed{y_n = \frac{n\lambda D}{d}} \rightarrow n^{th} \text{ equation}.$$

I] At $n=0$

$\boxed{y_0 = 0}$ → Central maxima

II] At $n=1$.

$$y_1 = \frac{\lambda D}{d} \rightarrow \text{I maxima}.$$

III] At $n=2$.

$$y_2 = \frac{2\lambda D}{d} \rightarrow \text{II maxima}.$$

IV] At $n=3$

$$y_3 = \frac{3\lambda D}{d} \rightarrow \text{III maxima}.$$

Fringe width (β).

$$\beta = y_n - y_{n-1}$$ } Diff. b/w two consecutive maximas.

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$$\beta = \frac{n\lambda D}{d} - (n-1)\frac{\lambda D}{d}.$$

$$\boxed{\beta = \frac{\lambda D}{d}}$$

for minima;

$$x = (2n+1)\frac{\lambda}{2} \longrightarrow (6.)$$

comparing (4) & (6);

$$\frac{yd}{D} = (2n+1)\frac{\lambda D}{2d} \longrightarrow (7.)$$

case I] At $n = 0$;

$$y_0 = \frac{\lambda D}{2d} \rightarrow \text{I minima.}$$

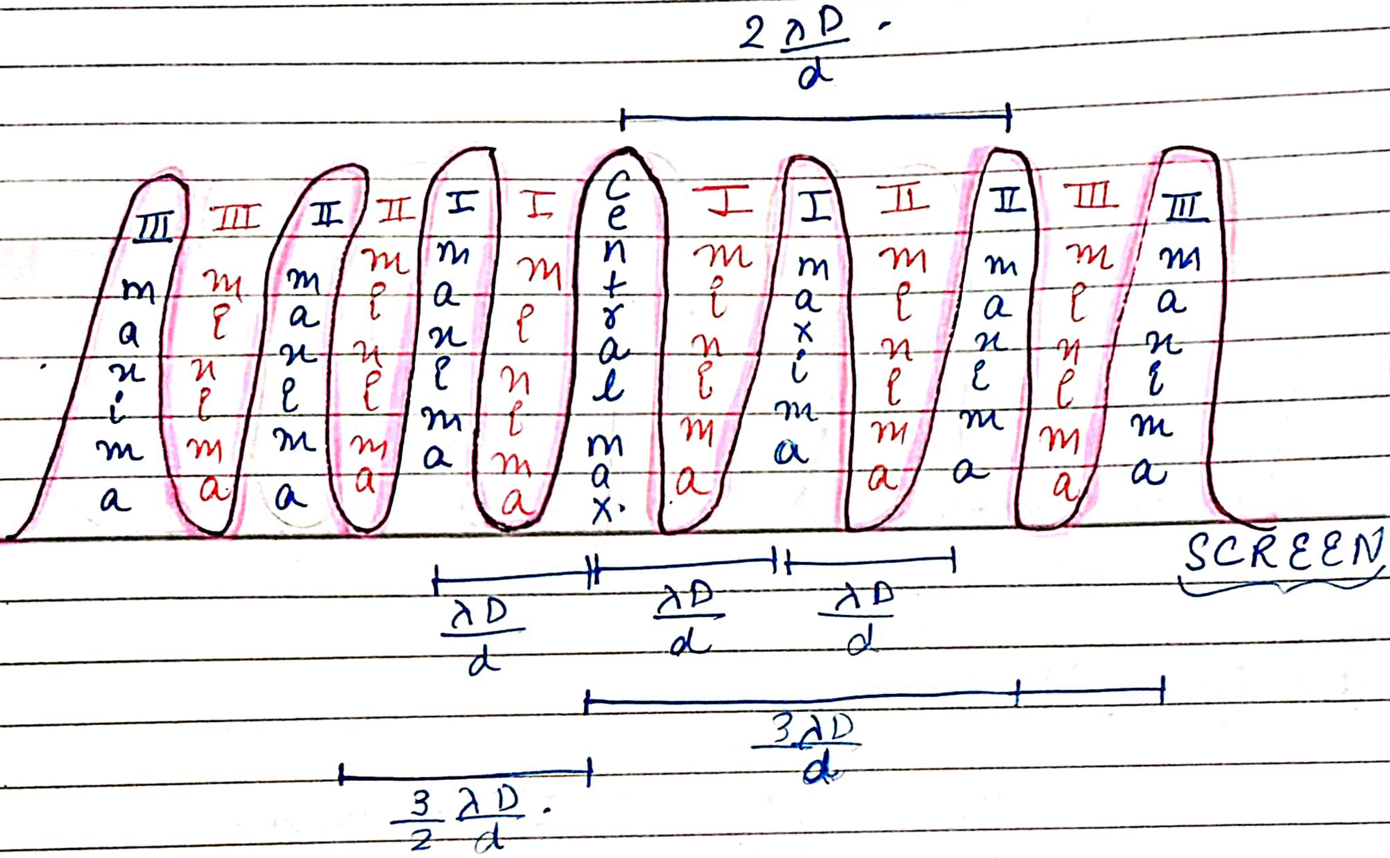
case II] At $n = 1$;

$$y_1 = \frac{3}{2}\frac{\lambda D}{d} \rightarrow \text{II minima.}$$

case III] At $n = 2$;

$$y_2 = \frac{5}{2}\frac{\lambda D}{d} \longrightarrow \text{III minima.}$$

Interference pattern



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For numericals :-

$$\beta = \frac{\lambda D}{d}$$

I] when the apparatus is put in the water;
i.e; D and d will remain same but λ changes;

so;

$$\beta = \frac{\lambda D}{d} \quad ; \quad \beta' = \frac{\lambda' D}{d}.$$

$$\frac{\beta}{\beta'} = \frac{\lambda}{\lambda'} = n \qquad \left\{ n = \frac{c}{v} = \frac{\cancel{\nu}\lambda}{\cancel{\nu}\lambda'} \right\}$$

$$\boxed{\beta' = \frac{\beta}{n}}$$

Acc. to cauchy's;

$$n \propto \frac{1}{\lambda}$$

so; when; we put app. in water it n is greater as compared to air. so; λ will decrease and β will also decrease

when find y between y_{4th} of maxima / Brightness and y_{6th} of minima / dullness.

$$y = y_{4th_{max}} - y_{6th_{min}}.$$

when to find y on both 2nd place.

$$\boxed{y = 4\frac{\lambda D}{d}.}$$

Spiral

Diffraction
Bending of light around a sharp corner is called diffraction.

Conditions:-
Size of slit should be comparable to the size of the wavelenght to the light used.

Types of diffractions:-

1. Fresnel → when distance b/w source and slit and slit and screen is finite.

2. Fraunhoffer → when distance b/w source and slit & screen & slit is infinite.

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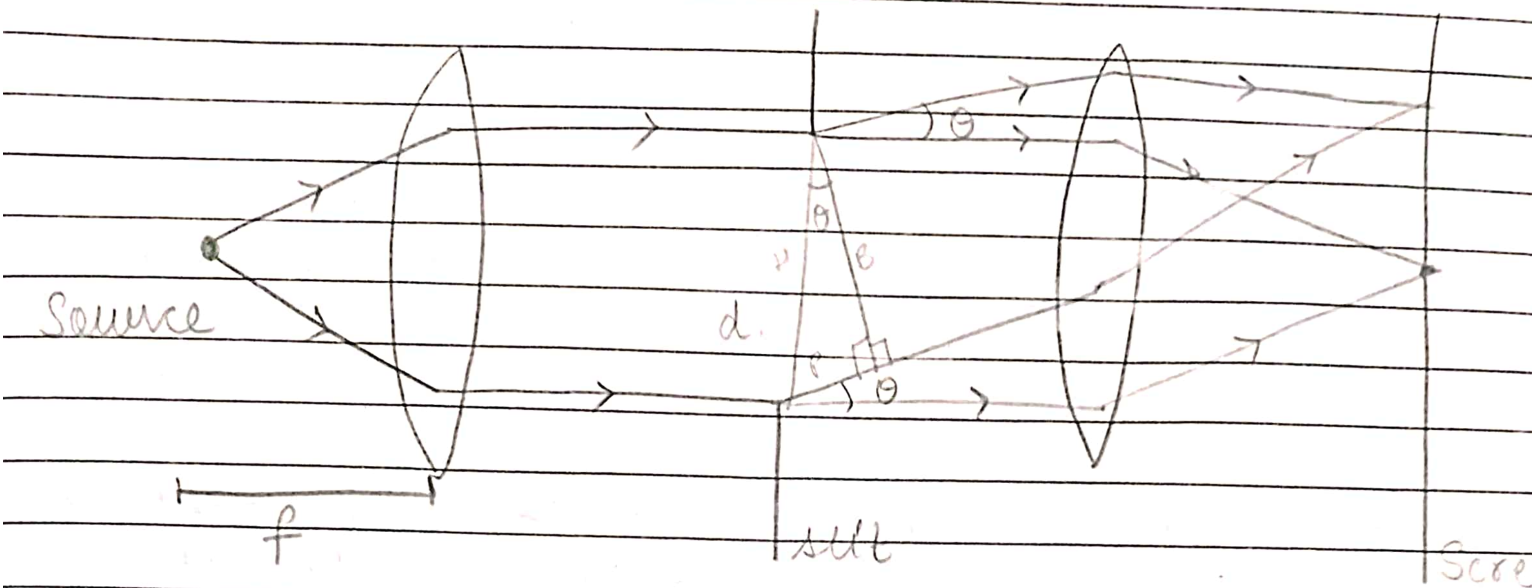
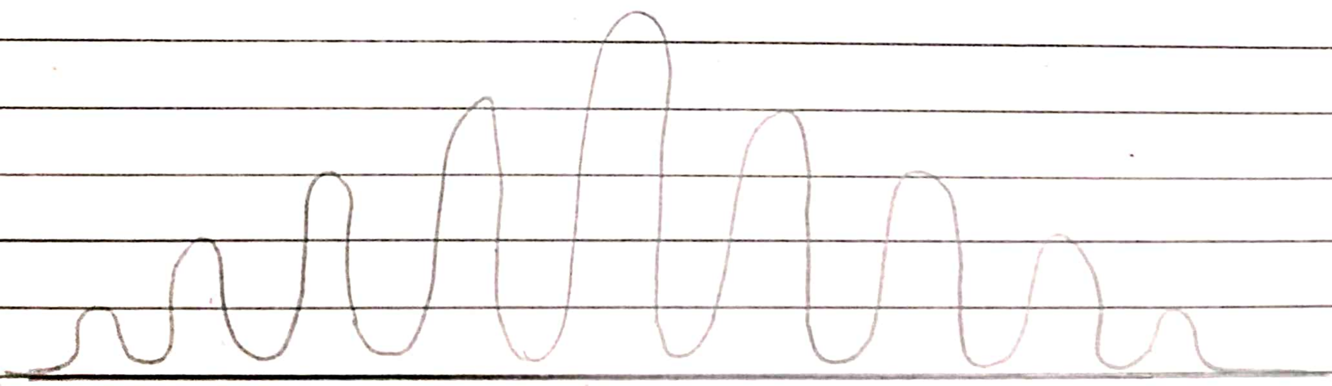
Fraunhoffer (single slit experiment)



Central max:-

$$d \sin\theta = n\lambda \text{ (minima).}$$
$$d \sin\theta = (2n+1)\frac{\lambda}{2} \text{ (Maxima).}$$



Interference	Diffraction.
It is due to the redistribution of light energy on account of principle of superposition of waves.	It is due to bending of light at the edges of the obstacles.
All bright fringes are of same intensity.	The intensity of fringes decreases with increase in distance from centre.
All fringes are of same width.	Width of central maxima is twice the width of secondary maxima.
Minima is formed at $(2n+1)\frac{\lambda}{2}$ from central maxima.	Minima is formed at $n\lambda$ from the central maxima.
Maxima is formed at $n\lambda$ from the central maxima.	Maxima is formed at $(2n+1)\frac{\lambda}{2}$ from the central maxima.