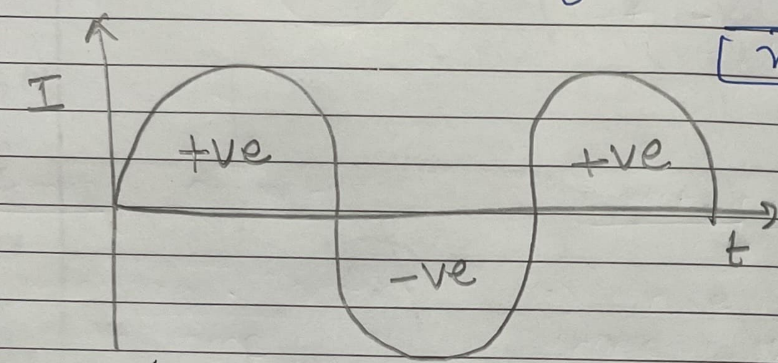
Chap-7

# ALTERNATING CURRENT (AC)

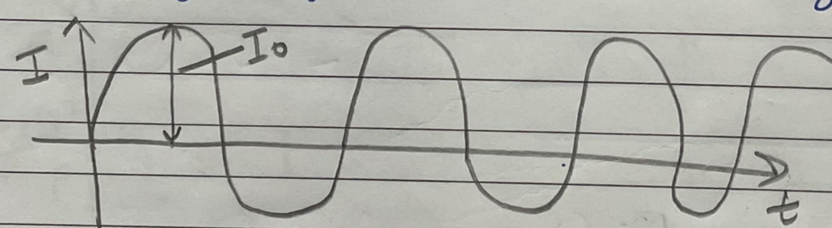
Alternating current

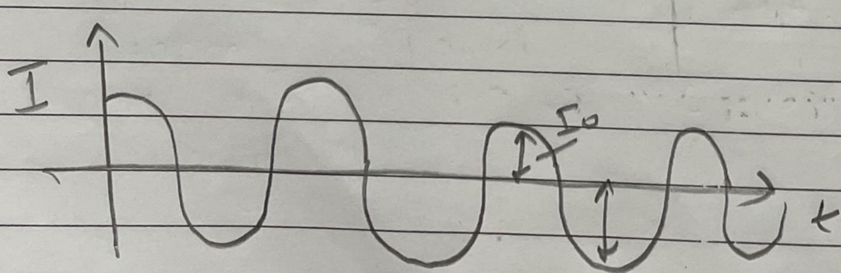
A current whose magnitude changes with time and direction reversed periodically is called alternating current.

$\boxed{\nu = 50 Hz}$




Frequency of A.C. in India is 50Hz.




Sinusoidal graphs

Expression of A.C.:

$$I = I_0 \sin \omega t$$

OR

$$I = I_0 \cos \omega t$$

$I_0$ → peak value [max. amplitude]

$\omega$ → omega

↳ angular frequency [SI unit → $\frac{rad}{s}$]

$$\boxed{\omega = \frac{2\pi}{T}}$$

rpm → rev. per min

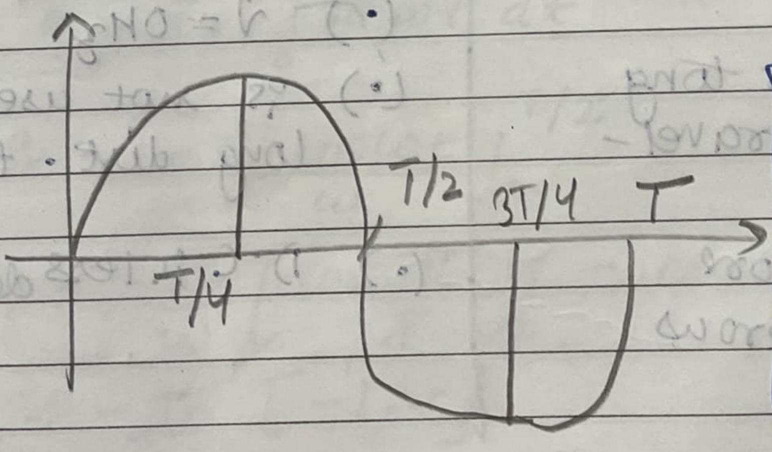
$$\boxed{1\ rpm = \frac{2\pi}{60}\ rev/sec}$$

(sec) T → Time period

↳ Time taken to complete one full oscillation

Freq. ($\nu$) ⇒ SI unit = Hz

↳ $\nu = \frac{1}{T}$ [Reciprocal of time period]




[The graph for alternating current is symmetrical all over.]

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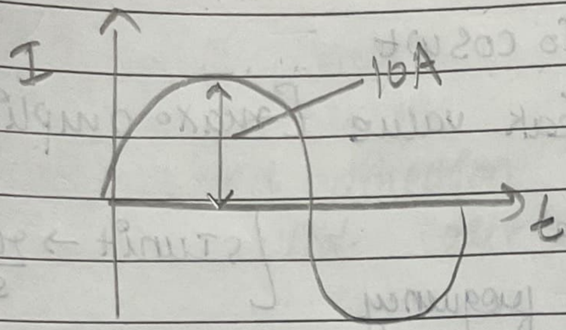
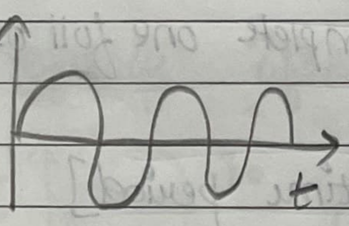
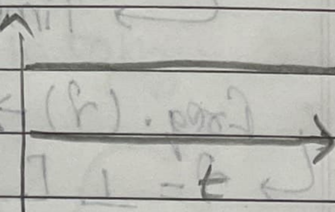
Q1. In what time current will reach to 10A, when $\nu = 60\,Hz$.



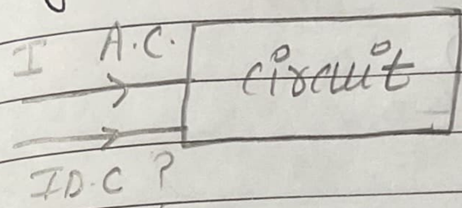

$$t = \frac{T}{4}$$

$$= \frac{1}{4} \times \frac{1}{\nu}$$

$$t = \frac{1}{240}\ sec$$

| Alternating current | Direct current |
|---|---|
| (•) whose mag. changes with time | (•) whose mag. doesn't change with time |
| (•) I  | (•) I  |
| (•) $\nu = 50\,Hz$ | (•) $\nu = 0\,Hz$ |
| (•) is used for long distance travel-portation | (•) is not used for long dist. transportation |
| (•) A.C is more dangerous | (•) D.C is less dangerous. |

(I) Average / Mean value of A.C. If $T = 20\,sec$




$\frac{T}{2}$ (10sec)

$\frac{T}{2}$ (10sec)

It is that value of steady current (D.C.) which when pass through a circuit for half time period of A.C. produces same charge as produced by A.C. in the same circuit for the same time

for A.C. $(I = I_0 \sin \omega t)$

$$I = \frac{dq}{dt}$$

$$dq = I\,dt$$

$$dq = I_0 \sin \omega t\, dt$$

$$q = \int_0^{T/2} I_0 \sin \omega t\, dt$$

$$q = I_0 \int_0^{T/2} \sin \omega t\, dt$$

$$q = -\frac{I_0 \cos \omega t}{\omega}\Big|_0^{T/2}$$

$$q = -\frac{I_0}{\omega}\left[\cos \frac{2\pi}{T}\frac{T}{2} - \cos 0\right]$$

$$q = -\frac{I_0}{\omega}[-1-1]$$

$$q = \frac{2I_0}{\omega}$$

$$q = \frac{\cancel{2} I_0}{\cancel{2}\pi} T$$

$$q = \cancel{2}\,\frac{I_0 T}{\pi} \rightarrow (1)$$

For steady current (D.C.)

$$q = It$$

$$I \rightarrow I_{av} \quad t \rightarrow T/2$$

$$q = I_{av} \times \frac{T}{2} \rightarrow (2)$$

Comparing (1) and (2)

$$\frac{I_{av} T}{2} = \frac{I_0 T}{\pi}$$

$$\boxed{I_{av} = \frac{2}{\pi} I_0}$$

$$\boxed{I_{av} = 0.636\, I_0}$$

Voltage average $\boxed{V_{av} = \frac{2}{\pi} V_0}$ $\boxed{V_{av} = 0.636\, V_0}$

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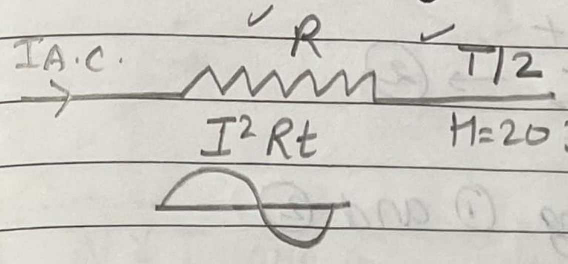
# Root mean square (rms) | virtual | effective value of A.C.




$I_{D.C.} = ?$ It is that value of steady current which when passed through a resistance produces same amt. of heat as produced by A.C. in the same resistance for the same time.

$H = I^2 R t$

for A.C.

$I = I_0 \sin \omega t$

$dH = I_0^2 \sin^2 \omega t \, R \, dt$

$H = \int_0^{T/2} I_0^2 R \sin^2 \omega t \, dt$

$H = I_0^2 R \int_0^{T/2} \sin^2 \omega t \, dt$

$H = I_0^2 R \int_0^{T/2} \left( \frac{1 - \cos 2\omega t}{2} \right) dt$

$H = \frac{I_0^2 R}{2} \left[ t - \frac{\sin 2\omega t}{2\omega} \right]_0^{T/2}$

$H = \frac{I_0^2 R}{2} \left[ \left( \frac{T}{2} - 0 \right) - \frac{1}{2\omega} \left[ \sin 2 \frac{2\pi}{T} \frac{T}{2} - \sin 0 \right] \right]$

$H = \frac{I_0^2 R T}{4} \rightarrow (1)$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos 2\theta = 1 - 2\sin^2\theta$

$2\sin^2\theta = 1 - \cos 2\theta$

$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$

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for steady current

$$H = I_v^2 R t$$

$$H = I_v^2 R \frac{T}{2} \rightarrow (2)$$

Comparing (1) and (2)

$$\frac{I_0^2 R T}{4} = \frac{I_v^2 R T}{2}$$

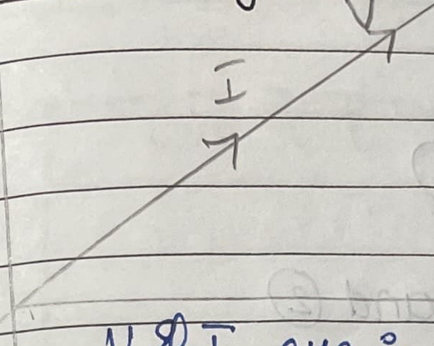
$$I_v^2 = \frac{I_0^2}{2}$$

$$\boxed{I_v = \frac{I_0}{\sqrt{2}}} = \boxed{0.707 I_0}$$

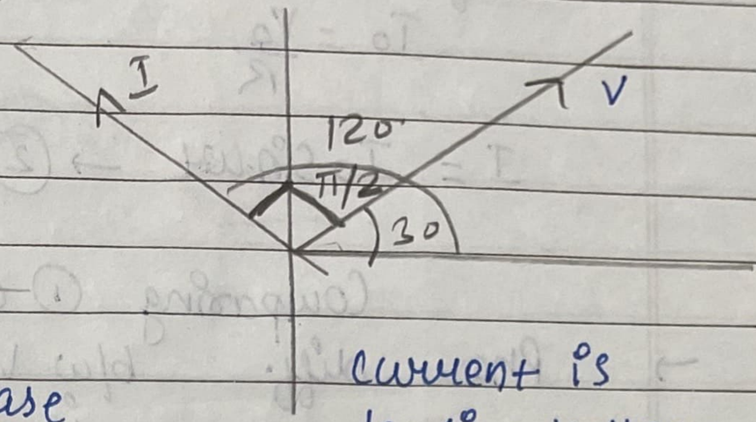
$$\boxed{V_v = \frac{V_0}{\sqrt{2}}} \qquad \boxed{V_v = 0.707 V_0}$$

(1)

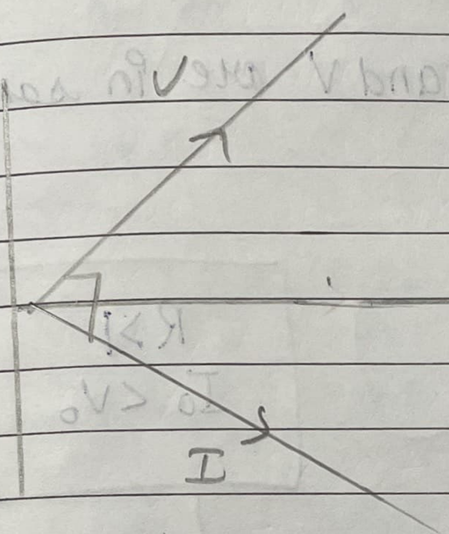
## Phasor Diagrams (♡)




V & I are in same phase

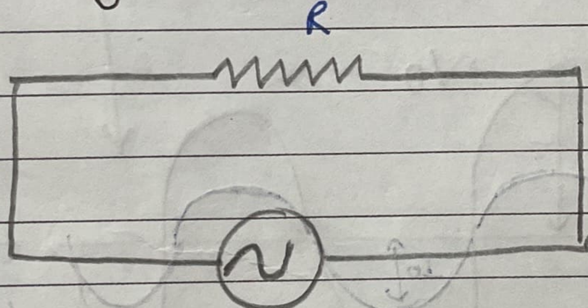



current is leading voltage by π/2



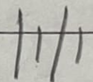

Voltage is leading current by π/2

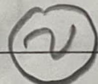
(1) A.C. through resistance




$V = V_0 \sin \omega t$

DC Battery



 AC source

Given

$V = V_0 \sin \omega t$ — (1)

$V = IR$

$I = \frac{V}{R}$

$I = \boxed{\frac{V_0 \sin \omega t}{R}}$

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If

$$I_0 = \frac{V_0}{R}$$

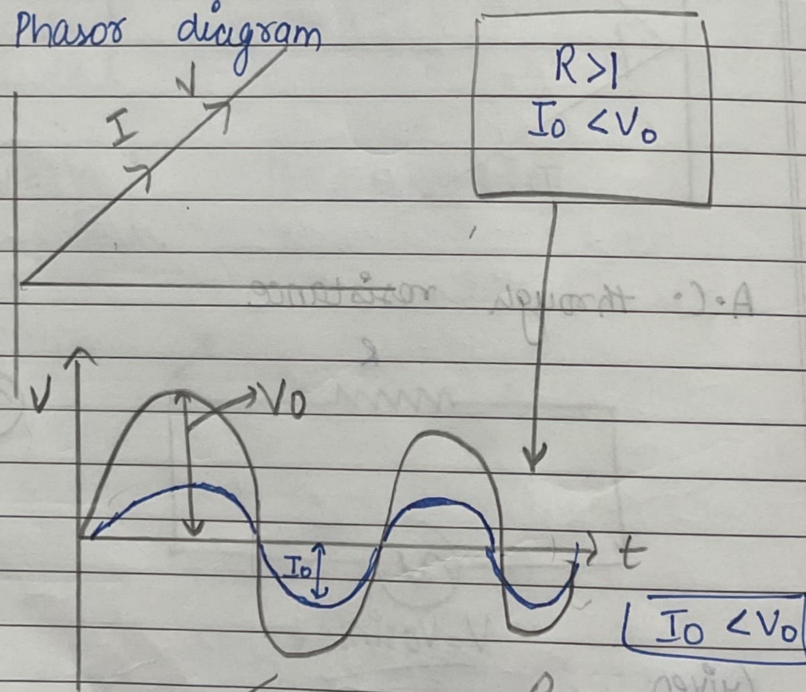
$$I = I_0 \sin \omega t \rightarrow (2)$$

Comparing (1) and (2)

→ Phase diff. b/w V and I for a resistance is 0°.

→ $\boxed{\text{For a resistance}}$ I and V are in same phase

Phasor diagram



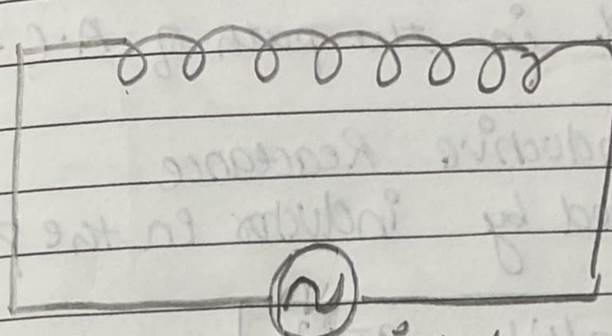

$\boxed{I_0 < V_0}$

R>1 $I = \frac{V}{R}$

(2) A.C. through

Given

(2) A.C. through inductor



$V = V_0 \sin \omega t$

Given

$$V = V_0 \sin \omega t$$

$$V = E = -L\frac{dI}{dt}$$ (V & I both included)

$$V = L\frac{dI}{dt}$$

$$\frac{V\,dt}{L} = dI$$

$$dI = \frac{V_0 \sin \omega t \, dt}{L}$$

$$I = \int \frac{V_0}{L} \sin \omega t \, dt$$

$$I = \frac{V_0}{L} \int \sin \omega t \, dt$$

$$I = \frac{V_0}{L}\left(\frac{-\cos \omega t}{\omega}\right)$$

$$I = -\frac{V_0}{\omega L} \cos \omega t$$

$$I = -I_0 \cos \omega t \rightarrow (2)$$

where $I_0 = \frac{V_0}{\omega L} \rightarrow (3)$

$$\omega L = \frac{2\pi}{T} \frac{V\,dt}{dI} = \frac{Vs}{s\,I} = \frac{\text{volt}}{A}$$

$\boxed{\omega L = \Omega}$

$\omega L = X_L$

$X \rightarrow$ reactance ($\Omega$)

Res. offered in the path of A.C.

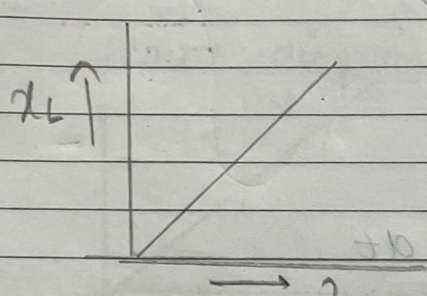
$X_L (\Omega) \rightarrow$ Inductive Reactance

↳ Res. offered by inductor in the path of A.C.

$$\boxed{X_L = \omega L = 2\pi \nu L}$$

$X_L = \omega L$

$X_L = 2\pi \nu L$




$$I_0 = \frac{V_0}{\omega L} = \frac{V_0}{X_L}$$

$$\boxed{V_0 = I_0 X_L}$$

$\boxed{V_\nu = I_\nu X_L}$ Ohm's Law for inductor

$$I_\nu = \frac{V_\nu}{X_L} = \frac{V_\nu}{2\pi \nu L}$$

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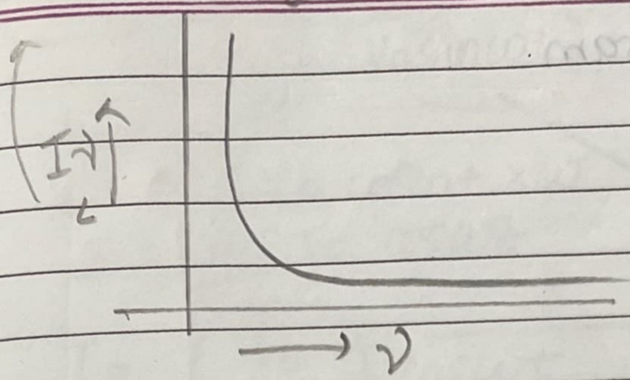
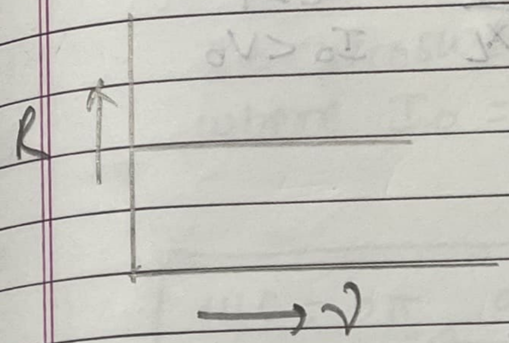
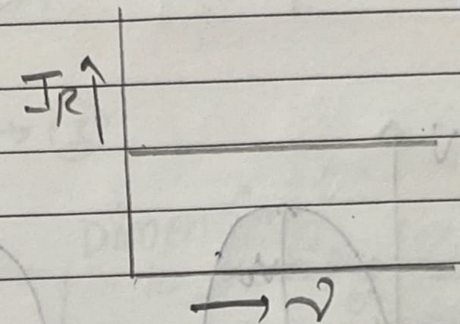
$I_\nu$ : current of L (inductor)

(Current of inductor) It also depends on frequency



Contrary (Resistance is indepen- of frequency)





$$V = V_0 \sin(\omega t)$$

$$I = -I_0 \cos\omega t \rightarrow (4)$$

$$I = -I_0 \sin(90 - \omega t)$$

$\omega t$ is taken as a std. reference

Anything will be subtracted from ($\omega t$)

$$I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right) \rightarrow (5)$$

We know

$$V = V_0 \sin\omega t \rightarrow (6)$$

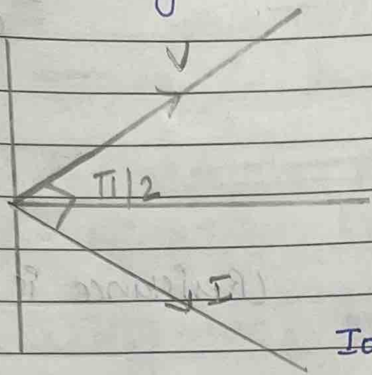
comparing (5) and (6)

→ phase diff. b/w V & I is $\pi/2$

→ Voltage leads current by $\pi/2$

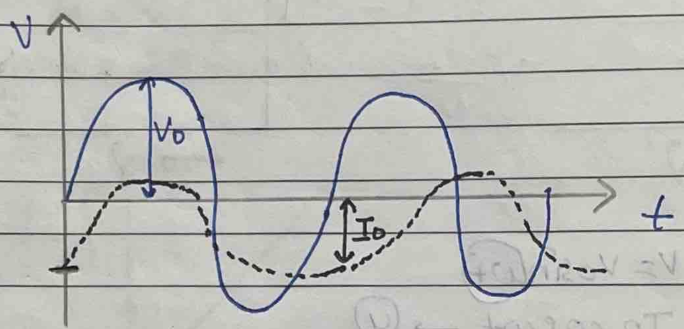
→ current lags behind voltage by $\pi/2$.

## Phasor Diagram

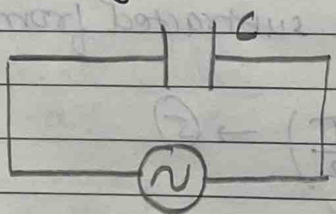



$I_0 = \frac{V_0}{X_L}$ $X_L > 1$

$I_0 < V_0$




(3) A.C. through capacitor




Given $V = V_0 \sin \omega t \rightarrow (1)$

using $\boxed{q = CV}$

Diff. wrt time

$$\frac{dq}{dt} = C\frac{dV}{dt}$$

$$I = C\frac{dV}{dt}$$

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$$I = C\frac{d}{dt}(V_0 \sin\omega t)$$

$$I = CV_0 \cos\omega t \times \omega$$

$$I = \omega C V_0 \cos\omega t$$

$$I = \left(\frac{V_0}{\left(\frac{1}{\omega C}\right)}\right)\cos\omega t$$

$$I = I_0 \cos\omega t \rightarrow (2)$$

where $I_0 = \dfrac{V_0}{1/\omega C} \longrightarrow (3)$

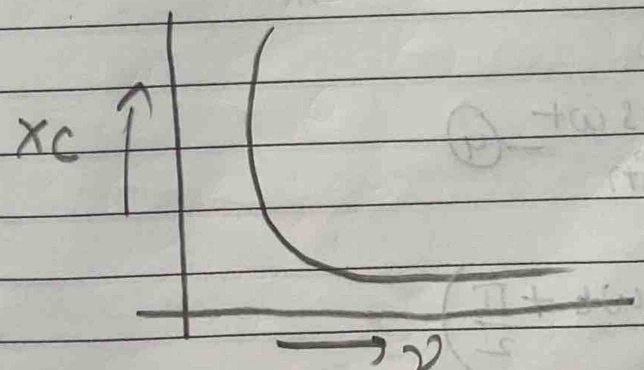
Dimension to verify $\frac{1}{\omega C} = \Omega$

$$\boxed{\begin{aligned}\omega C &= \frac{2\pi}{T} \times \frac{q}{V} = \frac{I}{V}\\ \frac{1}{\omega C} &= \frac{V}{I} = R(\Omega)\end{aligned}}$$

$$\frac{1}{\omega C} = X_C$$

$X_C \rightarrow$ capacitive reactance $(\Omega)$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$$



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using

$$I_0 = \frac{V_0}{\frac{1}{\omega C}}$$

$$I_0 = \frac{V_0}{X_C}$$

$$V_0 = I_0 X_C$$

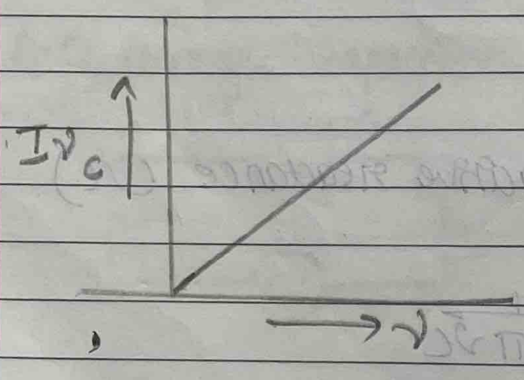
$\boxed{V_\nu = I_\nu X_C}$ Ohm's law for a capacitor

$$I_\nu = \frac{V_\nu}{X_C}$$

$$I_\nu = \frac{V_\nu}{\frac{1}{\omega C}}$$

$$I_\nu = V_\nu \omega C$$

$$I_\nu \propto \nu$$




eq$^n$ 2

$$I = I_0 \cos \omega t \quad \text{--- } 4$$

~~I = I0~~ (sin

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

using
$V = V_0 \sin \omega t \rightarrow (5)$
Comparing (4) and (5)

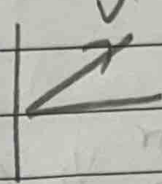
[for a capacitor]

Phase diff. b/w V and I is $\pi/2$

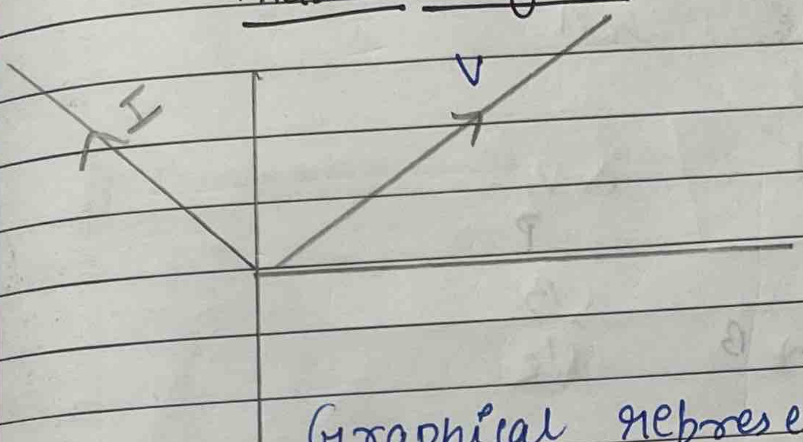
→ current leads vol by $\pi/2$

→ voltage lags behind I by $\pi/2$

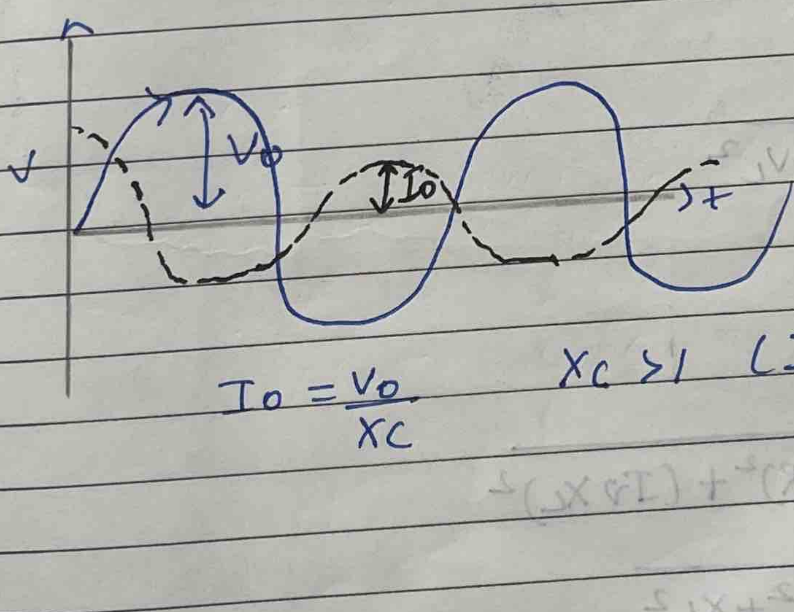
V always like this




Phasor Diagram




Graphical representation

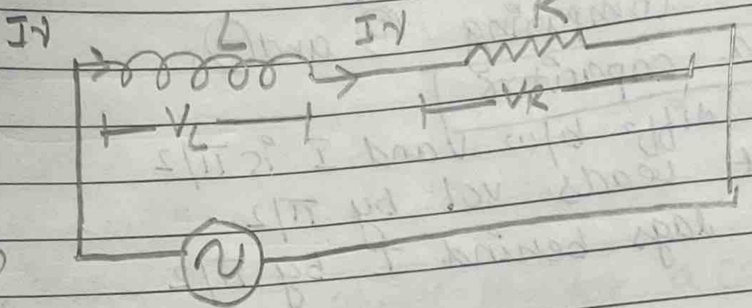


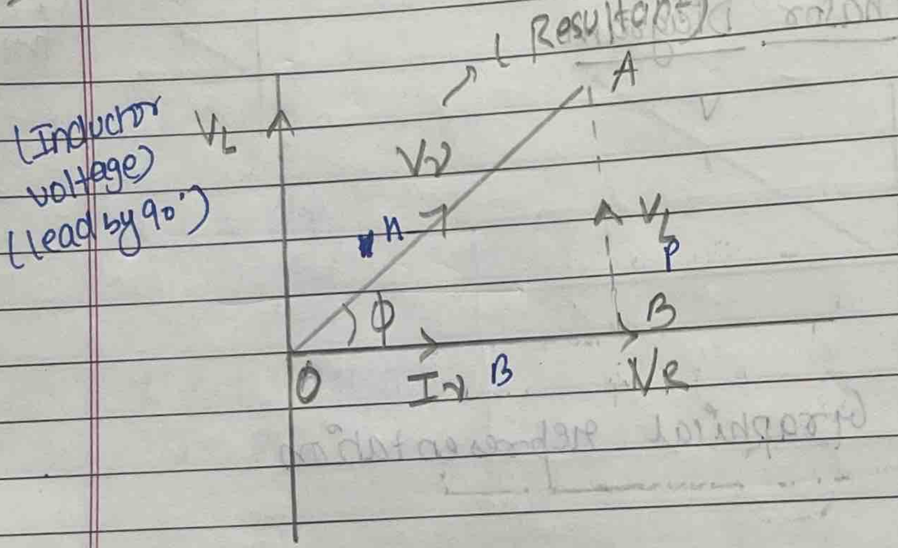

$I_0 = \frac{V_0}{X_C}$ $\quad X_C > 1 \; (I_0 < V_0)$

## (4) A.C. through LR circuit

V → divide

series
↓
current
(same)
$I_v$




(5)

In $\Delta ABO$

$$V_v = \sqrt{V_R^2 + V_L^2}$$

$$V_R = I_v R$$

$$V_L = I_v X_L$$

$$V_v = \sqrt{(I_v R)^2 + (I_v X_L)^2}$$

$$V_v = I_v \sqrt{R^2 + X_L^2}$$

$$V_v = I_0 Z$$

$Z \rightarrow$ impedance ($\Omega$)

for LR circuit

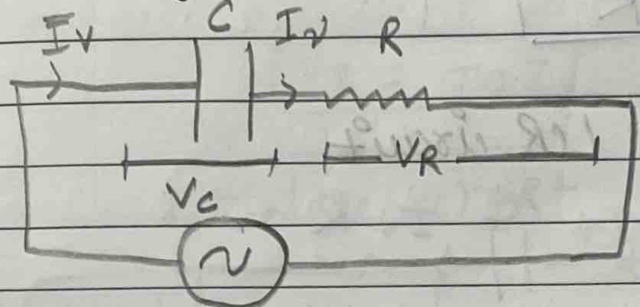
$$Z = \sqrt{X_L^2 + R^2}$$

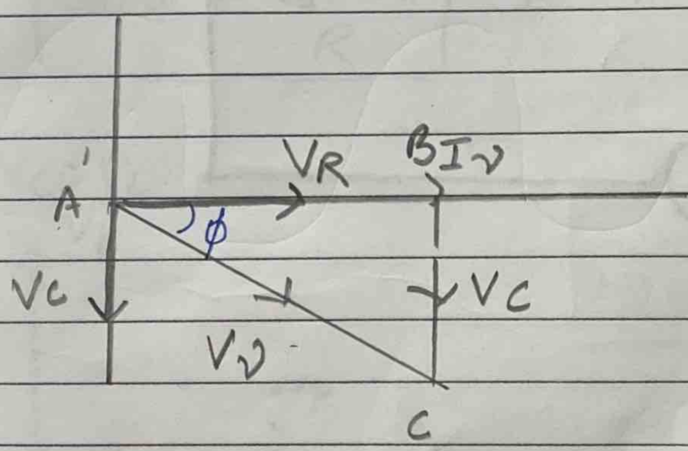
→ Resistance offered by more than one component in the path of A.C.

$$\tan\phi = \frac{V_L}{V_R} = \frac{I_0 X_L}{I_0 R}$$

$$\boxed{\tan\phi = \frac{X_L}{R}}$$

(5) A.C. through CR circuit




In $\Delta ABC$

$$V_v = \sqrt{V_C^2 + V_R^2}$$

$$V_R = I_v R$$

$$V_C = I_v X_C$$

$$V_\nu = \sqrt{(I_\nu R)^2 + (I_\nu X_C)^2}$$

$$V_\nu = I_\nu \sqrt{R^2 + X_C^2}$$

$$\boxed{V_\nu = I_\nu Z}$$

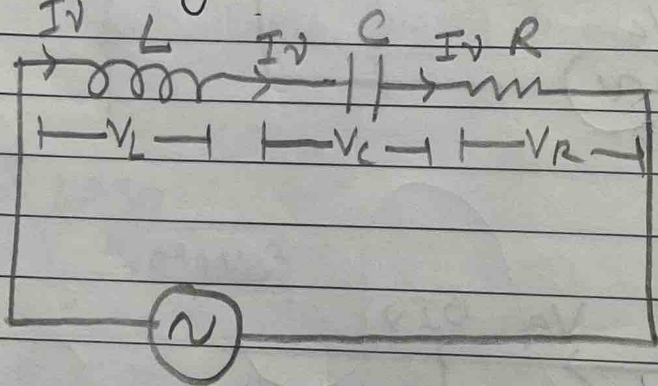
$Z \rightarrow$ Impedence

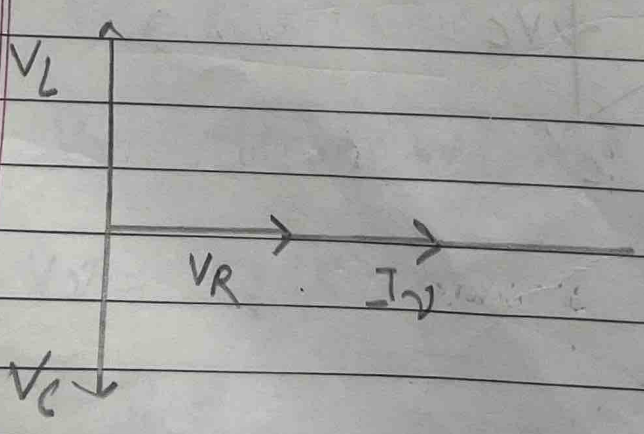
(Ω) $Z = \sqrt{X_C^2 + R^2}$

$$\tan\phi = \frac{V_C}{V_R} = \frac{I_\nu X_C}{I_\nu R}$$

$$\boxed{\tan\phi = \frac{X_C}{R}}$$

(6) A.C. through LCR circuit

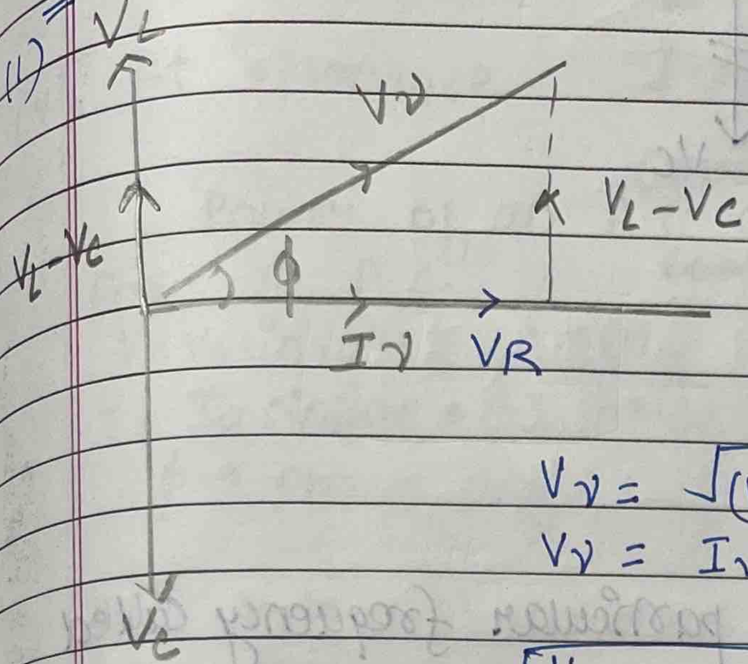


What is phase angle b/w inductive voltage and capacitive voltage in series in LCR ?

Ans $\pi$ (180°)

(i)




$$V_\nu = \sqrt{(V_L - V_C)^2 + V_R^2}$$

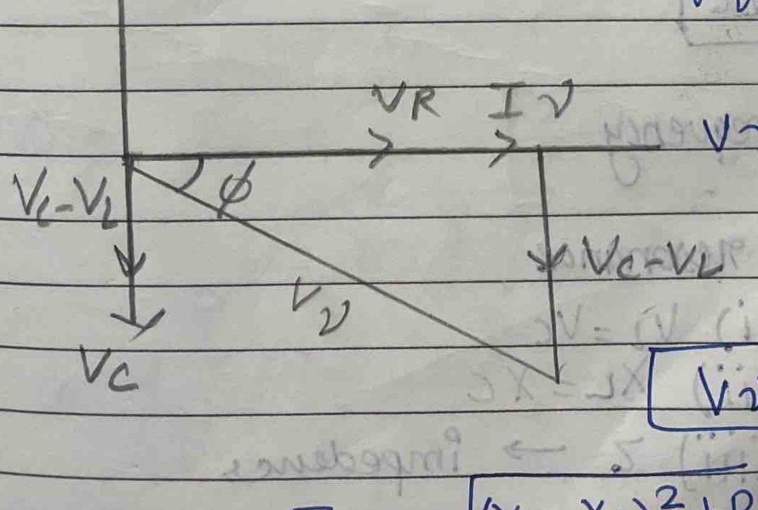
$$V_\nu = I_\nu \underbrace{\sqrt{(X_L - X_C)^2 + R^2}}_{Z}$$

$$\boxed{V_\nu = I_\nu Z}$$

$$Z = \sqrt{(X_L - X_C)^2 + R^2}$$

$$\boxed{\tan\phi = \frac{X_L - X_C}{R}}$$

(ii) $V_C > V_L$




$$V_\nu = \sqrt{(V_C - V_L)^2 + V_R^2}$$

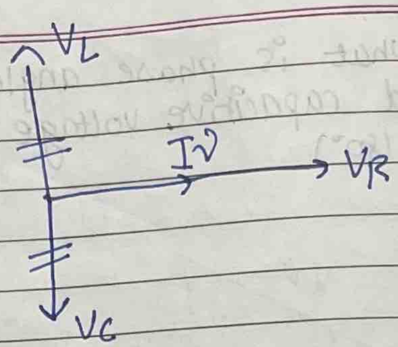
$$V_\nu = I_\nu \underbrace{\sqrt{(X_C - X_L)^2 + R^2}}_{Z \text{ (Impedence)}}$$

$$\boxed{V_\nu = I_\nu Z}$$

$$Z = \sqrt{(X_C - X_L)^2 + R^2}$$

$$\tan\phi = \frac{X_C - X_L}{}$$

(III) case $V_L = V_C$




#

$$V_L = V_C$$
$$I\nu X_L = I\nu X_C$$
$$\boxed{X_L = X_C}$$

$$\boxed{\omega L = \frac{1}{\omega C}}$$

This happens at a particular frequency called resonant frequency

$\omega = \omega_0$
↓
Angular resonant frequency

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\boxed{\nu_0 = \frac{1}{2\pi\sqrt{LC}}}$$
↓
Resonant frequency

At resonance

(i) $V_L = V_C$
(ii) $X_L = X_C$
(iii) $Z \rightarrow$ impedence

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$Z = \sqrt{(X_L - X_C)^2 + R^2}$

(iv) current is max $\boxed{Z = R}$ (Min.)

$I_v = \frac{\varepsilon_v}{Z} = \frac{\varepsilon_v}{R}$

(v) At resonance I and V are in same phase.

Power of an A.C. circuit (over a complete cycle)

For an A.C. ~~from~~

$V = V_0 \sin \omega t \rightarrow (1)$

$I = I_0 \sin(\omega t + \phi) \rightarrow (2)$

$\phi \rightarrow$ phase diff. $\phi \rightarrow$ +ve, -ve, 0

$dw = V dq$

$dw = V I dt$

$dw = V_0 \sin \omega t \, I_0 \sin(\omega t + \phi) dt$

$\sin(A+B) = \sin A \cos B + \sin B \cos A$

$dw = V_0 \sin \omega t \, I_0 [\sin \omega t \cos \phi + \cos \omega t \sin \phi] dt$

$dw = V_0 I_0 [\sin^2 \omega t \cos \phi + \sin \omega t \cos \omega t \sin \phi] dt$

$w = V_0 I_0 \int_0^T [\sin^2 \omega t \cos \phi + \sin \omega t \cos \omega t \sin \phi] dt$

$w = V_0 I_0 \left[ \underbrace{\left[ \int_0^T (\sin^2 \omega t \cos \phi) dt \right]}_{I_1} + \underbrace{\left[ \int_0^T \sin \omega t \cos \omega t \sin \phi \, dt \right]}_{I_2} \right] \rightarrow (3)$

Solving $I_1 = \int_0^T \sin^2 \omega t \cos \phi \, dt$

$= \cos \phi \int_0^T \sin^2 \omega t \, dt$

$= \cos \phi \int_0^T \left( \frac{1 - \cos 2\omega t}{2} \right) dt$

$= \frac{\cos \phi}{2} \left[ t - \frac{\sin 2\omega t}{2\omega} \right]_0^T$

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$$= \frac{\cos\phi}{2}\left[T - \frac{1}{2\omega}\left(\overset{0}{\sin\cancel{2\frac{2\pi}{T}T}} - \underset{0}{\underset{||}{\sin 0}}\right)\right]$$

$$= I_1 = \frac{\cos\phi \, T}{2} \longrightarrow (4)$$

Solving $I_2$

$$\sin\phi \int_0^T \sin\omega t \cos\omega t \, dt$$

Multiply and divide by 2

$$I_2 = \sin\phi \int_0^T \frac{2\sin\omega t\cos\omega t}{2} \, dt$$

$$I_2 = \sin\phi \int_0^T \frac{\sin 2\omega t}{2} \, dt$$

$$I_2 = \frac{\sin\phi}{2}\left[\frac{-\cos 2\omega t}{2\omega}\right]_0^T$$

$$I_2 = -\frac{\sin\phi}{4\omega}\left[\cos 2\frac{2\pi}{T}T - \cos 0\right]$$

$$I_2 = -\frac{\sin\phi}{4\omega}[1-1]$$

$$I_2 = 0$$

Put $I_1$ and $I_2$ in (3)

$$\omega = I_0 V_0\left[\frac{\cos\phi T}{2} + 0\right]$$

~~$\omega = I_0$~~

$$\omega = V_0 I_0 \frac{\cos\phi T}{2}$$

$$\frac{\omega}{T} = \frac{V_0 I_0 \cos\phi}{2}$$

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$$P = \frac{V_0 I_0}{\sqrt{2}\sqrt{2}} \cos\phi$$

$\boxed{P = V_{rms} I_{rms} \cos\phi}$  $\cos\phi \rightarrow$ power factor (P.F.)

Case I : for a resistance

$\phi = 0°$

P.F. $\cos\phi = \cos 0° = 1$

$P = \varepsilon_{rms} I_{rms} \cos\phi$

$\boxed{P = \varepsilon_{rms} I_{rms}}$

Case II : for an inductor

$\phi = \pi/2$

P.F. $\cos\phi = \cos \pi/2 = 0$

$P = \varepsilon_{rms} I_{rms} \cos\phi$

$\boxed{P = 0}$

Ideal inductor Pure inductive circuit consumes zero power.

Case III : For a capacitor

$\phi = \pi/2$

P.F. $= \cos\phi = 0$

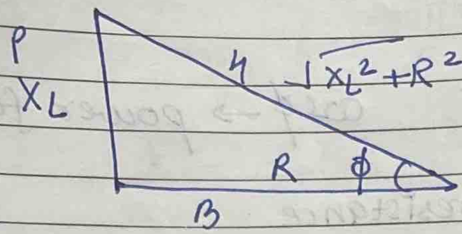
$\boxed{P = 0}$

pure capacitor consumes zero power.

Case IV : LR

$\tan\phi = \frac{X_L}{R}$

~~P~~ impedance triangle




$$\cos\phi = \frac{R}{\sqrt{X_L^2+R^2}} = \boxed{\frac{R}{Z}}$$

$$P = V_r I_r \cos\phi$$

$$P = V_r I_r \frac{R}{Z}$$

$$P = V_r \left[\frac{V_r}{Z}\right] \frac{R}{Z}$$

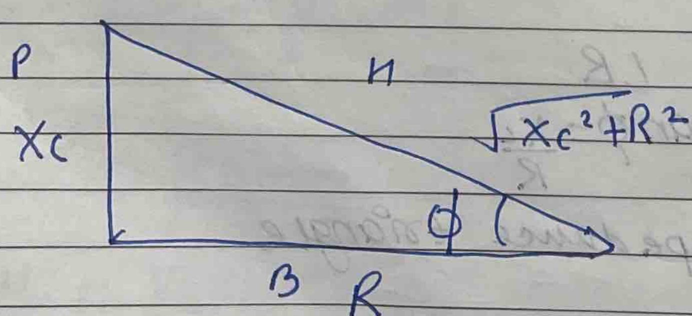
$$P = \frac{V_r^2 R}{Z^2}$$

$$P = \frac{V_r^2 R}{(X_L^2+R^2)}$$

Case-V CR Circuit

$$\tan\phi = \frac{X_C}{R}$$

Impedence $\Delta$



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$$\cos\phi = \frac{R}{\sqrt{X_C^2+R^2}} = \frac{R}{Z}$$

$$P = V_{\nu} I_{\nu} \cos\phi$$
$$P = V_{\nu} I_{\nu} \frac{R}{Z}$$
$$P = V_{\nu}\left[\frac{V_{\nu}}{Z}\right]\frac{R}{Z}$$

$$P = \frac{V_{\nu}^2 R}{Z^2}$$
$$P = \frac{V_{\nu}^2 R}{(X_C^2+R^2)}$$

case VI L-CR

(a) $V_L > V_C$ $\tan\phi = \frac{X_L - X_C}{R}$

$$\cos\phi = \frac{R}{\sqrt{(X_L-X_C)^2+R^2}}$$

$$\cos\phi = \frac{R}{Z}$$
$$P = V_{\nu} I_{\nu} \cos\phi$$
$$= V_{\nu} I_{\nu} \frac{R}{Z}$$
$$= V_{\nu}\left[\frac{V_{\nu}}{Z}\right]\frac{R}{Z}$$
$$= \frac{V_{\nu}^2 R}{Z^2} = \frac{V_{\nu}^2 R}{(X_L-X_C)^2+R^2}$$

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(b) $V_C > V_L$

$$\tan\phi = \frac{X_C - X_L}{R}$$

$$\cos\phi = \frac{R}{\sqrt{(X_C - X_L)^2 + R^2}}$$

$$\cos\phi = \frac{R}{Z}$$

$$P = V_{r} I_{r} \cos\phi$$

$$= V_{r} I_{r} \frac{R}{Z}$$

$$= V_{r}\left[\frac{V_{r}}{Z}\right]\frac{R}{Z}$$

$$= \frac{V_{r}^2 R}{Z^2} = \frac{V_{r}^2 R}{(X_C - X_L)^2 + R^2}$$

(c) LCR $\quad V_L = V_C \quad \tan\phi = 0$

$\phi = 0^\circ \quad \boxed{\cos\phi = 1}$

At resonance p.f. is unity

Wattless current : A current which when flows in a circuit consume zero power is called wattless current. eg. current in inductor or capacitor leads to 0 power consumption

$P = \varepsilon_{r} I_{r} \cos\phi$

$\phi = \pi/2 \quad \boxed{P = 0}$

Q. capacitor blocks D.C. How?

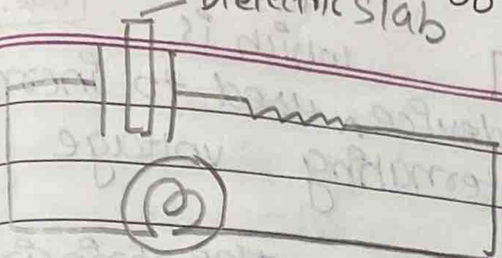
$\rightarrow X_C = \frac{1}{\omega C} \qquad X_C = \frac{1}{2\pi \nu C} \qquad$ D.C. $\boxed{\nu = 0} \quad X_C = \frac{1}{0}$

$X_C = \infty \quad \boxed{I = 0}$

Since it offers $\infty$ resistance, current becomes 0, hence we say capacitor blocks D.C.

What will be the effect on brightness of bulb?

Q1.



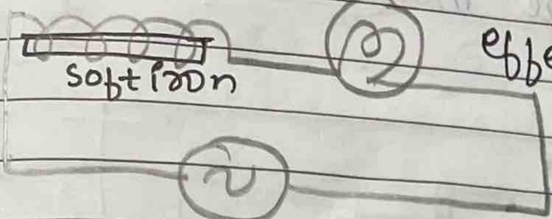

Die-slab $C\uparrow$

$X_C = \frac{1}{\omega C}$  $X_C = \downarrow$

$\boxed{I = \uparrow}$

Brightness $\uparrow$

Q2.




effect on brightness of bulb after inserting soft iron?

Soln. $\mu\uparrow$

$L = n^2 \mu_0 A l$

$L\uparrow$, $\mu\uparrow$

$X_L = \omega L$  $\boxed{X_L \uparrow}$  $I\downarrow$ Brightness $\downarrow$

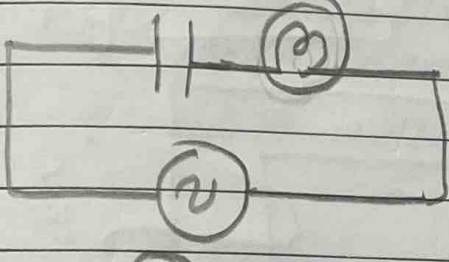
Permeability $\uparrow$, mag. field lines $\uparrow$, $\phi_M\uparrow$, Induced current $\uparrow$

$\downarrow$

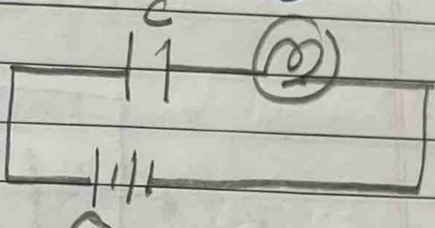
opposes the cause

$\boxed{\text{Brightness} \downarrow} \leftarrow$

Q3.



(a)




(b)

If $\nu$ is $\uparrow$ed, what will effect on brightness in both cases?

Soln (b) $\rightarrow$ $\nu = 0$  C $\rightarrow$ blocks DC current

Bulb will not glow

(a) $\rightarrow$ $\nu\uparrow$  $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$  $\nu\uparrow$, $X_C\downarrow$, $I\uparrow$

Brightness $\uparrow$

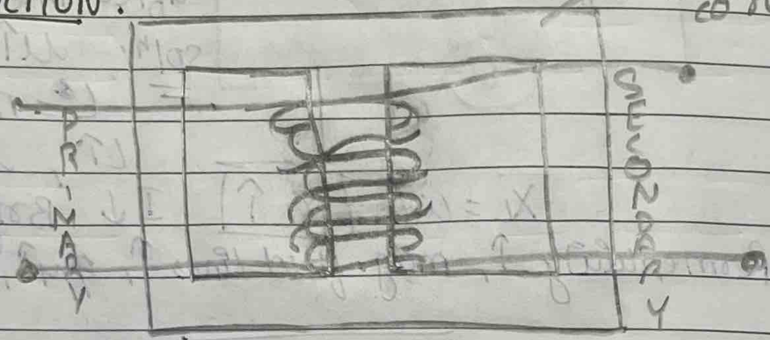
✓ 5 marker

# TRANSFORMER

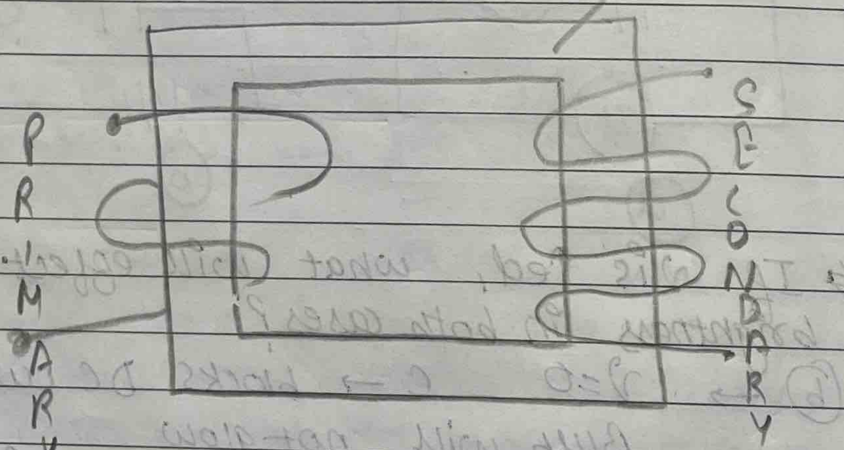
:- **USE:** It is a device which is used to increase or decrease alternating voltage

:- **PRINCIPLE:** It is based on the principle of mutual induction

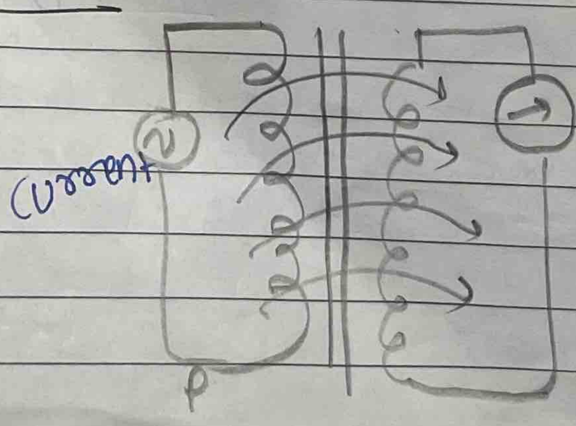
:- **CONSTRUCTION:**




(a)




:- **WORKING:**



change in magnetic flux in primary will produce induced current

$$\varepsilon_P = -N_P \frac{d\phi}{dt} \longrightarrow (1)$$

Due to change in sec$^n$

$$\varepsilon_s = -N_s \frac{d\phi}{dt} \longrightarrow (2)$$

Dividing (1) by (2)

$$\frac{\varepsilon_P}{\varepsilon_S} = \frac{N_P}{N_S}$$

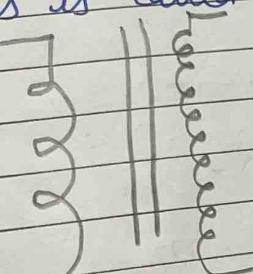
$$\frac{\varepsilon_S}{\varepsilon_P} = \frac{N_S}{N_P} \qquad \boxed{\frac{\varepsilon_S}{\varepsilon_P} = K}$$

$\frac{N_S}{N_P} = K \rightarrow$ Transformation ratio

$K > 1 \qquad N_S > N_P \qquad \varepsilon_S > \varepsilon_P$

output volt > Input volt

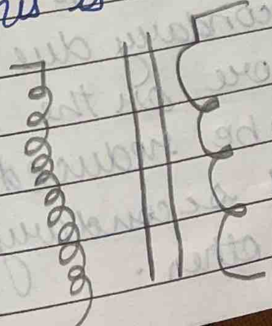
This is called step-up transformer (voltage↑)



$K < 1 \qquad \frac{N_S}{N_P} < 1 \qquad N_S < N_P \qquad \varepsilon_S < \varepsilon_P$

output volt < input volt

This is called step-down transformer (voltage↓)



**Transformer is not a power device.** $VI = K$, $V \propto \frac{1}{I}$

Input power = output power

$$\varepsilon_p I_p = \varepsilon_s I_s = \text{constant}$$

$$\frac{\varepsilon_s}{\varepsilon_p} = \frac{I_p}{I_s} \qquad \boxed{\varepsilon \propto \frac{1}{I}} \quad \text{Non-ohmic}$$

Thus, a step up transformer changes low voltage high current to high voltage low current

A step down transformer converts high voltage low current to low voltage high current

Ideal transformer

Input power = output power

$$(\text{efficiency})\ \eta = \frac{\varepsilon_s I_s}{\varepsilon_p I_p} \qquad \boxed{\eta = 100\%}$$

Losses in transformer

$$\eta \neq 100\%$$

$$\eta = \frac{\varepsilon_s I_s}{\varepsilon_p I_p}$$

In actual transformers, small energy losses do occur due to the following reasons:-

(i) Flux leakage: There is always some flux leakage; that is, not all of the flux due to primary passes through the secondary due to poor design of the core or the air gaps in the core. It can be reduced by winding the primary and secondary coils one over the other.

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(ii) **Resistance of the windings**: The wire used for the windings has some resistance and so, energy is lost due to heat produced in the wire ($I^2R$). In high current, low voltage windings, these are minimised by using thick wire.

(iii) **Eddy currents**: The alternating magnetic flux induces eddy currents in the iron core and causes heating. The effect is reduced by using a laminated core.

(iv) **Hysteresis**: The magnetisation of the core is repeatedly reversed by the alternating magnetic field. The resulting expenditure of energy in the core appears as heat and is kept to a minimum by using a magnetic material which has a low hysteresis loss.

Ques: The primary coil of an ideal step-up transformer has 100 turns and the transformation ratio is also 100. The input voltage and power are 220V and 1100W resp.
Calculate -
(i) no. of turns in the secondary
(ii) the current in the primary
(iii) voltage across the secondary
(iv) the current in the secondary
(v) power in the secondary

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Ques: A transformer of 100% efficiency has 200 turns in the primary and 40,000 turns in the secondary. It is connected to a 220V main supply and the secondary feeds to a 100kΩ resistance. Calculate

(i) the output potential diff. per turn

(ii) total output pot. diff.

(iii) power delivered to the load

Ans) $K = 100 = \frac{N_s}{N_p}$

$N_p = 100$

$\frac{N_s}{N_p} = 100$

(i) $\boxed{N_s = 10,000}$

$E_p = 220V$

$P_p = 1100W$

$P_p = E_p I_p$

$\cancel{100}\ 1100 = 220 \times I_p$

(ii) $I_p = \frac{110\cancel{0}}{22\cancel{0}} = 5A$

$\frac{E_s}{E_p} = \frac{N_s}{N_p}$

$\frac{E_s}{220} = 100$

(iii) $E_s = 22,000V$

$E_p I_p = E_s I_s$

$220 \times 5 = 22000\, I_s$

(IV) = $\boxed{I_s = \frac{1}{20} A}$

(V) $P_p = P_s$

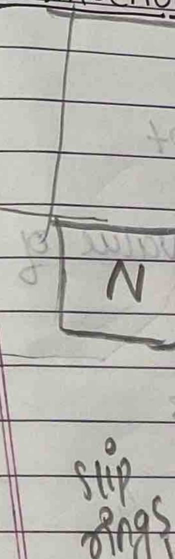
Ans. $\eta = 100\%$

A.C.

USE: It is ... and alt...

PRINCIPLE: ... an in... neric Ind...

CONSTRUCTIO...



(v) $P_P = P_S$

$\boxed{P_S = 1100\,W}$

Ans. $\eta = 100\%$

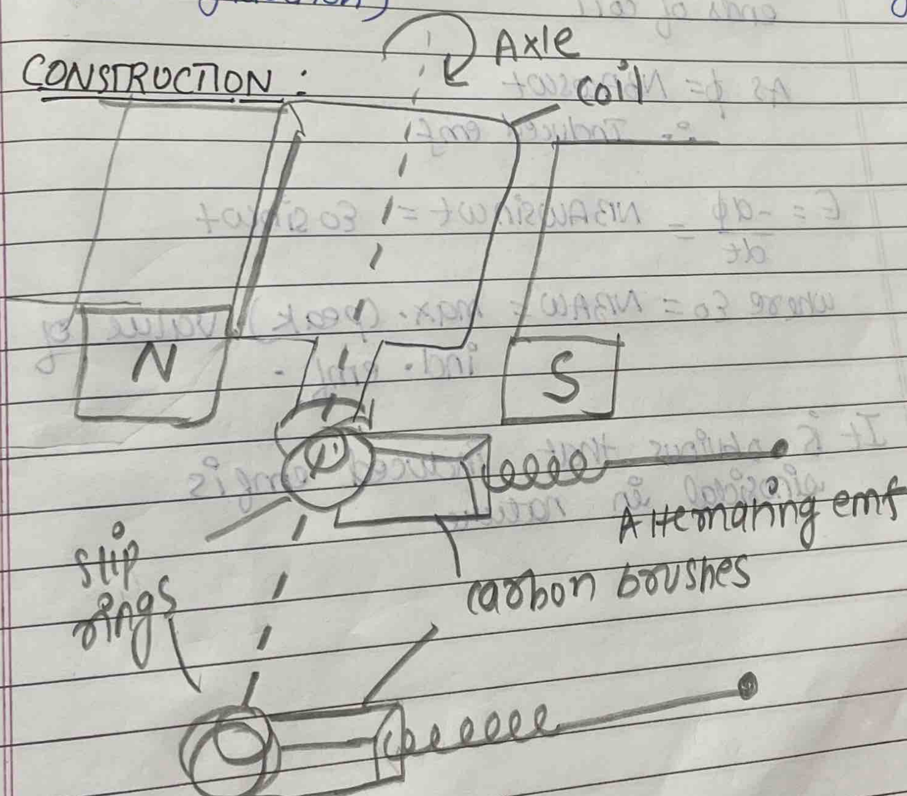
4w

## A.C. Generator

**USE:** It is used to produce alternating current and alternating voltage

**PRINCIPLE:** Whenever magnetic flux is changed, an induce current is produced (Electromagnetic Induction)

**CONSTRUCTION:**



# Working

When an armature coil of N turns and each turn enclosing area A is placed in a uniform mag. field of strength B making an angle $\theta$ from normal to the direction of magnetic field, mag. flux linked with the coil is $\phi = NBA\cos\theta$. As the coil is rotated about its own axis with an ang. speed $\omega$ then value of angle $\theta = \omega t$ and hence, mag. flux changes and an induced emf is developed across the ends of coil

As $\phi = NBA\cos\omega t$

$\therefore$ Induced emf

$$E = \frac{-d\phi}{dt} = NBA\omega\sin\omega t = \varepsilon_0 \sin\omega t$$

where $\varepsilon_0 = NBA\omega$ = max. (peak) value of ind. emf.

It is obvious that induced emf is sinosidal in nature

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# Numericals.

**Q1.** In a LCR circuit $L = 100\,mH$, $C = 10\mu F$, $R = 40\Omega$ is connected to $220V$, $50Hz$ A.C.

find (i) $\nu$ at resonance
(ii) $Z$ at "
(iii) $I$ at "

(i) $\omega_0 L = \frac{1}{\omega_0 C}$

$\nu_0 = \frac{1}{2\pi\sqrt{LC}}$

$\nu_0 = \frac{1}{2\pi\sqrt{100\times10^{-3}\times10\times10^{-6}}}$

$\nu_0 = \frac{1000}{2\pi} = \frac{500}{\pi}\,Hz.$

(ii) $Z = R = 40\Omega$

(iii) $\mathcal{E}_\nu = I_\nu Z$

$220 = I_\nu 40$

$I_\nu = \frac{220}{40} = \frac{11}{2} = 5.5A$

**Q2.** $L = 100mH$, $R = 10\Omega$ and $C$ connected to $200\,V$ AC, $\nu = 50Hz$. find $C$ if phase angle b/w $V$ and $I$ is zero.

Ans. $\boxed{LCR}$ $(\phi = 0°)$ Resonance

$\nu = 50Hz$
$L = 100\times10^{3}H$

$X_L = X_C$

$\omega L = \frac{1}{\omega C}$

$C = \frac{1}{\omega^2 L}$

$C = \frac{1}{(2\pi\nu)^2 L}$

$\pi^2 \approx 10$

$$C = \frac{1}{2\pi \times 50 \times 50 \times 2\pi \times 10^5}$$

$$C = \frac{1}{10 \times 2 \times 2 \times 2500 \times 10^5}$$

$$C = \frac{1}{40 \times 2500 \times 10^5} = \frac{1}{10^{10}} \text{ F}$$

Q3. CR circuit connected to A.C. Voltage across C and R are 120V, 90V resp. if rms current in circuit is 3A. find Z, power factor.

Ans. $\mathcal{E}_C = 120V$ $\quad \mathcal{E}_R = 90V$ $\quad I_v = 3A$ (series → same current)

$\mathcal{E}_C = I_v X_C$

$120 = 3X_C$ $\quad \boxed{X_C = 40\Omega}$

$\mathcal{E}_R = I_v R$ $\quad 90 = 3R$

$\boxed{R = 30\Omega}$

$Z = \sqrt{X_C^2 + R^2}$

$Z = \sqrt{1600 + 900}$ $\quad \boxed{Z = 50\Omega}$

$\cos\phi = \frac{R}{Z}$

$= \frac{30}{50}$

$\boxed{\cos\phi = \frac{3}{5}}$ $\quad$ Phase angle → $\phi$

MCQ.

**Q4.** Power factor of a circuit is 1/2. Find phase angle b/w V and I ?

**Ans.** $\cos\phi = \frac{1}{2}$ $\boxed{\phi = 60^\circ} = \pi/3$

**Q5.** An inductor and a resistor are connected across 12V, 50Hz and a current of 0.5A flows. If phase diff. b/w V and I is $\pi/3$. Find impedence and resistance.

**Ans.** LR $I = 0.5A$ $Z = ?$ $\nu = 50Hz$

$E_\nu = 12V$ $\phi = \pi/3$ $r = ?$

$Z = \sqrt{X_L^2 + R^2}$

$E_\nu = I_\nu Z$

$12 = \frac{1}{2} Z$

$\boxed{Z = 24\Omega}$

$Z = \sqrt{X_L^2 + R^2}$

$\tan\phi = \frac{X_L}{R}$

$\sqrt{3} = \frac{X_L}{R}$

$X_L = \sqrt{3}R$

$24 = \sqrt{4R^2}$

$\boxed{R = 12\Omega}$

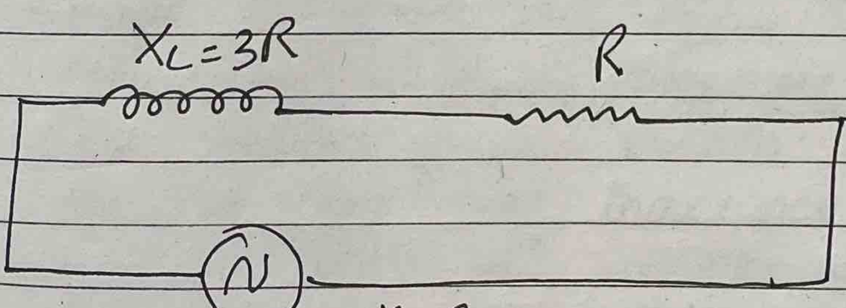
or

$\cos\phi = \frac{R}{Z}$

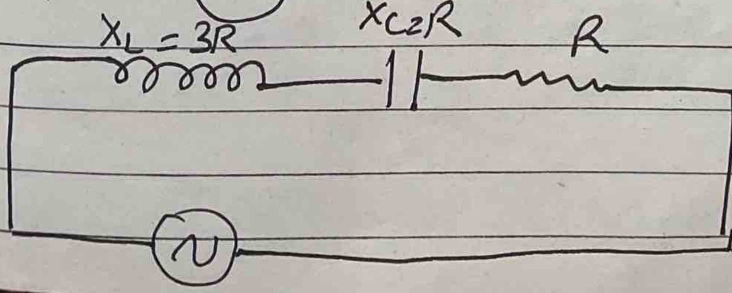
$\frac{\pi}{3} = \frac{R}{24}$

$\frac{24\pi}{3} = R$

**Q6.**




(a) Ratio of power factor in a to b.




(b)

Sol$^n$:

$$Z_1 = \sqrt{(X_L)^2 + R^2}$$

$$Z_1 = \sqrt{(3R)^2 + R^2}$$

$$Z_1 = \sqrt{9R^2 + R^2}$$

$$Z_1 = \sqrt{10R^2} \qquad Z_1 = \sqrt{10}\,R$$

$$Z_2 = \sqrt{(X_L - X_C)^2 + R^2}$$

$$Z_2 = \sqrt{(3R - R)^2 + R^2} \qquad Z_2 = \sqrt{(2R)^2 + R^2}$$

$$Z_2 = \sqrt{4R^2 + R^2}$$

$$Z_2 = \sqrt{5R^2}$$

$$Z_2 = \sqrt{5}\,R$$

$$\cos\phi_1 = \frac{R_1}{Z_1}$$

$$\cos\phi_2 = \frac{R_2}{Z_2}$$

$$\cos\phi_1 = \frac{R}{\sqrt{10}R}$$

$$\cos\phi_2 = \frac{R}{\sqrt{5}R}$$

$$\frac{\cos\phi_1}{\cos\phi_2} = \frac{\sqrt{5}}{\sqrt{10}}$$

$$\boxed{\frac{\cos\phi_1}{\cos\phi_2} = \frac{1}{\sqrt{2}}}$$

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Q. A capacitor of unknown capacitance, a resistor of 100 $\Omega$ and an inductor of self-inductance $L = \frac{4}{\pi^2}$ H are connected in series to an a.c. source of 200V and 50Hz. Calculate the value of capacitance and the current that flows in the circuit when the current is in ~~same~~ phase w/ the voltage.

Resonance

$X_L = X_C$

$\omega L = \frac{1}{\omega C}$

$C = \frac{1}{\omega^2 L}$

$\boxed{C = 25 \mu F}$

$Z = R$

$Z = 100 \Omega$

$E_v = 200V$

$E_v = I_v Z$

$200 = I_v \times 100$ → In this find $I_v$ only.

$I_v = 2A$

$I_v = \frac{I_0}{\sqrt{2}}$ $\boxed{I_0 = 2\sqrt{2}}$

Q. Imp.

When an inductor is connected to a 200V d.c. voltage, a current of 1A flows through it when the same inductor is connected to a 200V, 50Hz ac source only. 0.5A current flows. Explain why? Also calculate the self inductance of inductor.

Whenever inductor is connected w/ d.c voltage, it behave as resistor

$R = \frac{V}{I} = \frac{200}{1} = 200 \Omega$

Same inductor w/ ac:- LR circuit

$R = 200\,\Omega$ $R = 400\,\Omega$

$I = 1\,A$ $I = \frac{1}{2}\,A$

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$\mathcal{E}_v = I_v Z$

$Z = \frac{\mathcal{E}_v}{I_v} = \frac{200}{\frac{1}{2}} = \boxed{400\,\Omega}$

As resistance $\uparrow$ ~~com~~, $I \downarrow$

$Z = \sqrt{X_L^2 + R^2}$

$(400)^2 = X_L^2 + (200)^2$

$X_L^2 = 120000$

$X_L = \sqrt{120000}$

$X_L = \omega L$

$X_L = 2\pi \nu L$