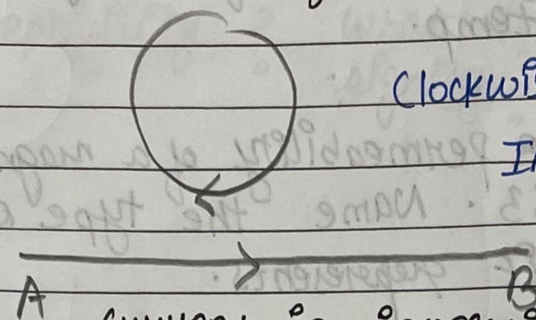
Chap-6

Electromagnetic Induction

Q1. What is direction of induced current in ring

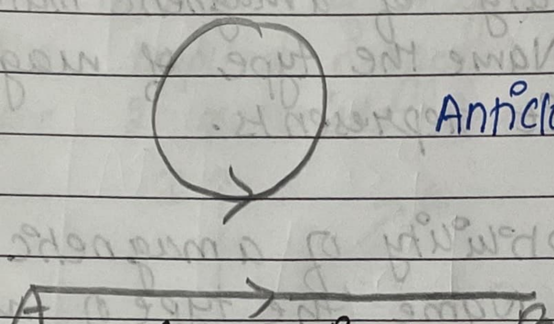


Clockwise

Induced current is in oppo. dirn

current in increasing

Q2.

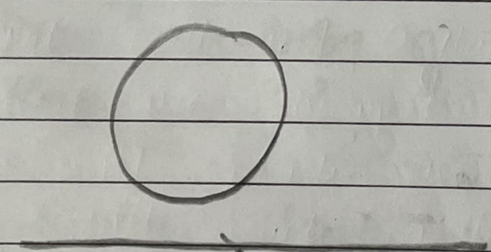


Anticlockwise

Current is decreasing

Induced current ∴ will ↑ current (same dirn)

Q3.

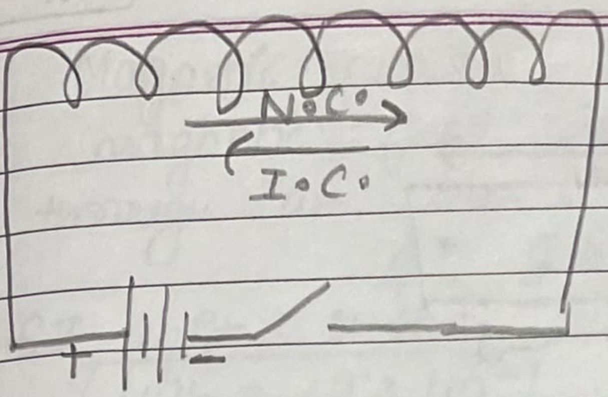


No induced current

A Steady current B

constant ∴ no change

Q4.

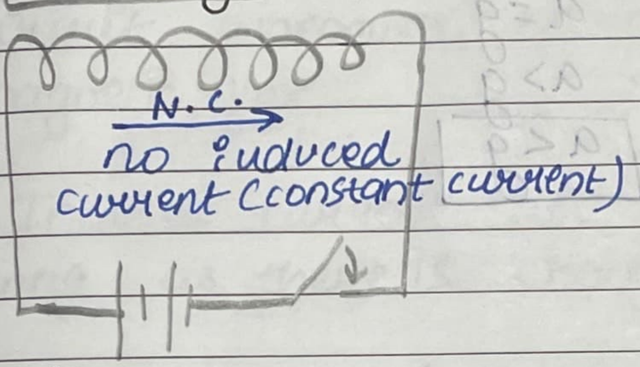


(i) When key is tapped

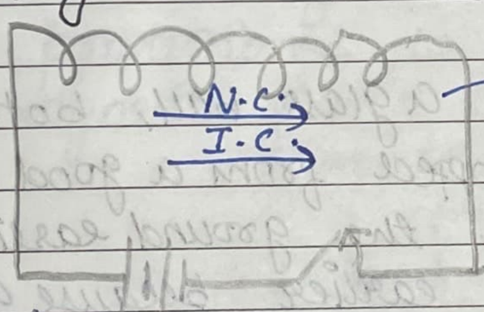
Normal current $\xrightarrow{N.C.}$

Induced current $\xleftarrow{I.C.}$

(ii) constantly tapped



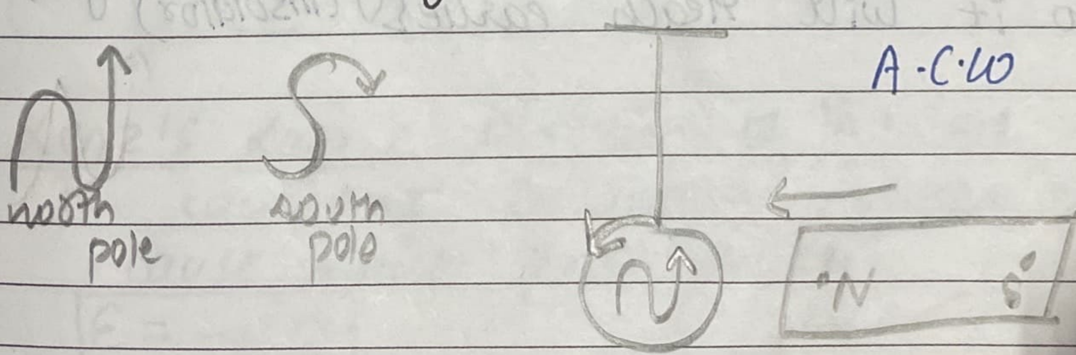
(iii) Key is released



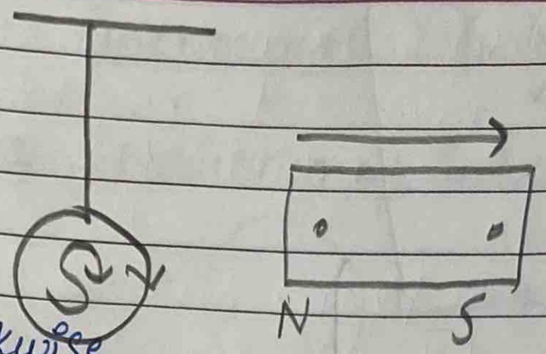
→ current ↓es

I.C → current ↑es (same dir^n)

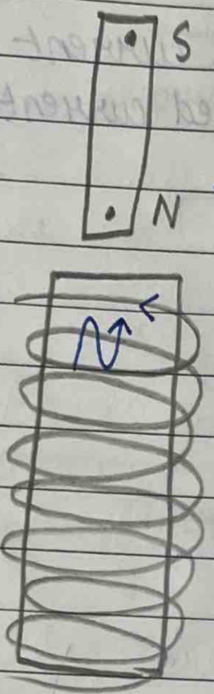
Q5. What is dir^n of induced current in the ring



A.C.W



clockwise



→ it will be in air (will not freefall)

repel.

$a = g$

$a > g$

$\boxed{a < g}$

Q. A metal ball and a glass ball, both identical are dropped from a good height. Which will reach the ground earlier?

Ans. Glass will reach earlier because due to induce current, motion of metal ball (conductor) will be opposed, hence, in metal ball $\boxed{a<g}$ and in glass $\boxed{a=g}$, (insulator) so it will reach earlier.

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Magnetic flux (ϕ) - It is defined as no. of magnetic field lines crossing normally through an area

$$\phi = \oint \vec{B} \cdot \vec{ds}$$

SI unit : weber (Wb).

$\boxed{1 Wb = 1 T \times 1 m^2}$

Magnetic flux - scalar quantity

Faraday's laws of electromagnetic induction (EMI)

(1) An induced current is set up in the circuit whenever there is change in magnetic flux

(2) Induced current will last in the circuit as long as there is change in magnetic flux

(3) Magnitude of induced EMF is directly proportional to rate of change of magnetic flux

$$\varepsilon \propto \frac{d\phi}{dt}$$

$$\varepsilon = k\frac{d\phi}{dt}$$

SI unit $\boxed{K = 1}$

$$\boxed{\varepsilon = -\frac{d\phi}{dt}}$$

Lenz's Law : Direction of induced current is such that it always opposes the cause which produces it

$$\boxed{\varepsilon = -\frac{d\phi}{dt}}$$

Q1. $\phi = 2t^2 + 4t + 6$. Find induced emf and induced current at $t=1$ if $\boxed{R = 10\,\Omega}$

Ans $\mathcal{E} = -\dfrac{d\phi}{dt} = 4t+4$

$\mathcal{E} = 4(1) + 4 = \boxed{8V}$

$I = \dfrac{\mathcal{E}}{R} = \dfrac{8}{10} = \boxed{0.8A}$

Q2. A circular coil of radius R is kept normally in magnetic field $B = B_0 \sin \omega t$. If resistance of coil is R, find induced current.

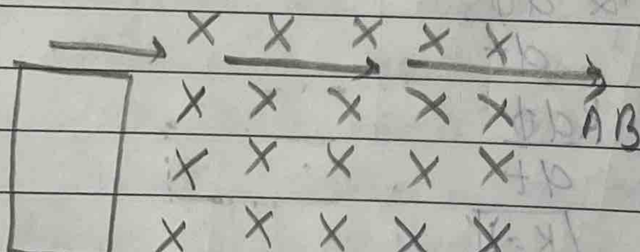
Ans. $\mathcal{E} = -\dfrac{d\phi}{dt}$

$\phi = \oint B \cdot ds = \oint B\,ds = \oint B_0 \sin \omega t\,(\pi r^2)$

$|\mathcal{E}| = B_0 \pi r^2 \cos \omega t \times \omega$

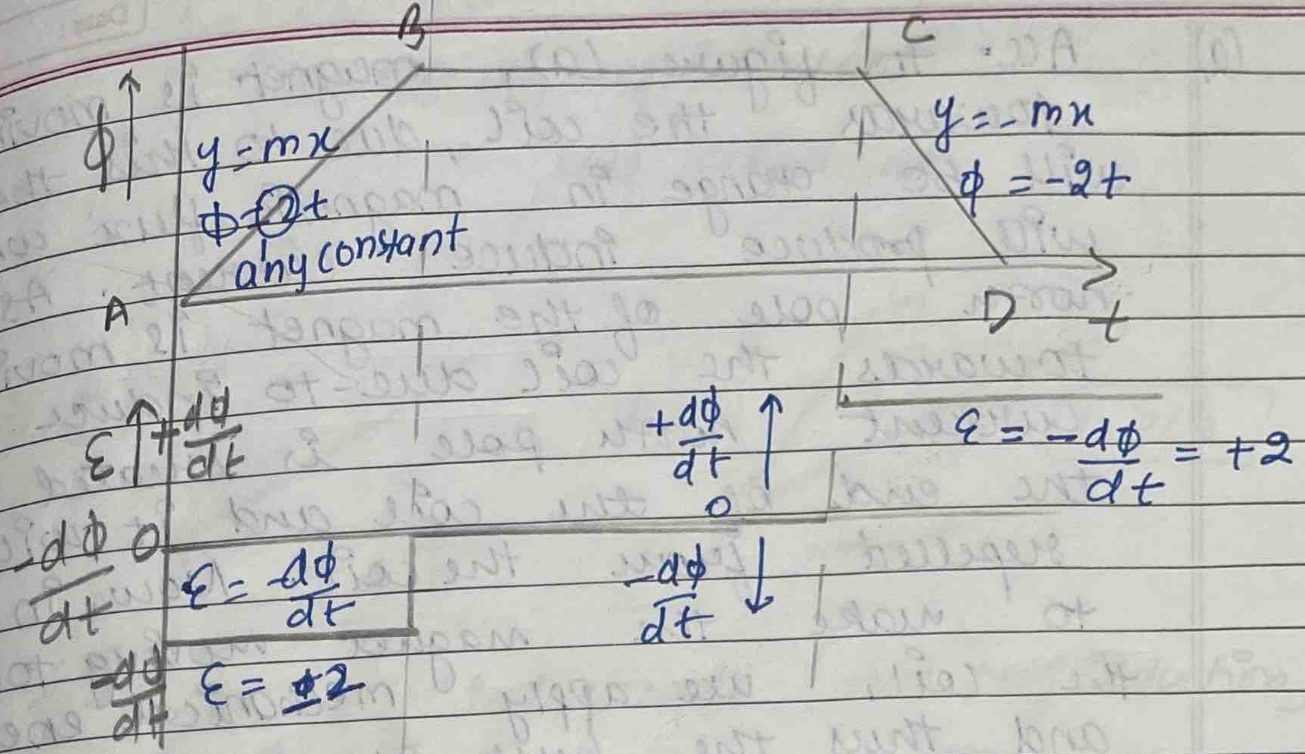
$I = \dfrac{|\mathcal{E}|}{R} = \dfrac{B_0 \pi r^2 \omega \cos \omega t}{R}$

Q3.



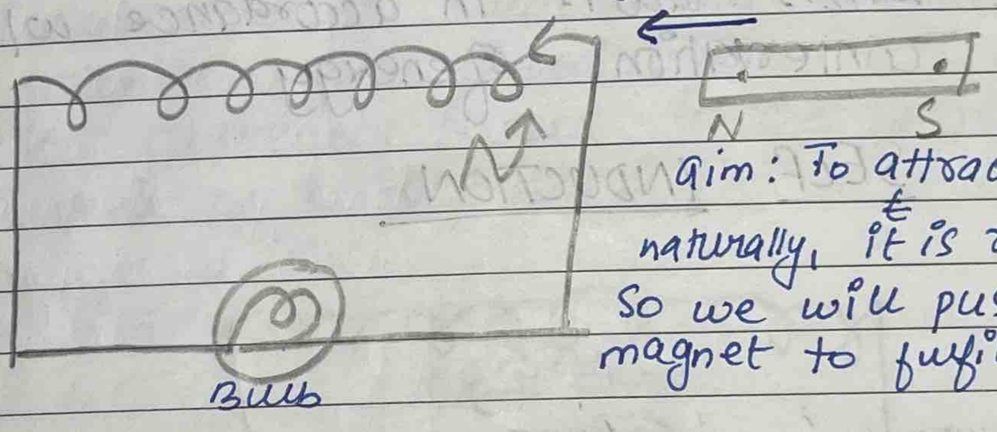
Draw flux v/s time

induced emf v/s time



Lenz's Law is in accordance with law of conservation of energy

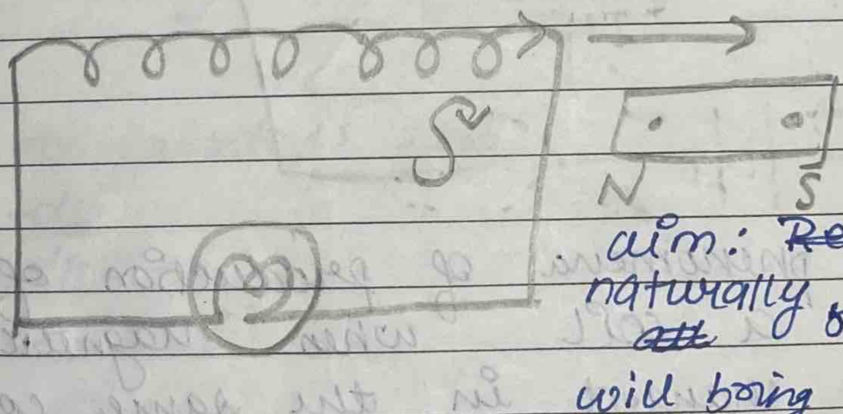
(a)



aim: To attract

naturally, it is repelling so we will push the magnet to fulfill our aim

(b)



aim: ~~Repulsion~~ attraction

naturally it is repelling we will bring south pole towards north

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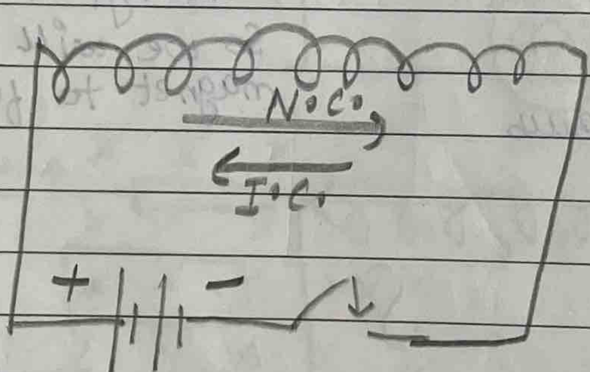
(a) Acc. to figure (a), magnet is moving towards the coil, due to this there will be change in magnetic flux which will produce induce current. As the north pole of the magnet is moving towards the coil due to induce current north pole is created at the end of the coil and it will be repelled from the coil. Now, in order to make the magnet moving towards the coil, we apply mechanical energy and thus the bulb glows as mechanical energy is converted into electrical energy

So, Lenz's law is in accordance w/ law of conservation of energy

SELF INDUCTION



The phenomena of production of I.C. in a coil when magnetic flux is changed in the same coil is called self induction

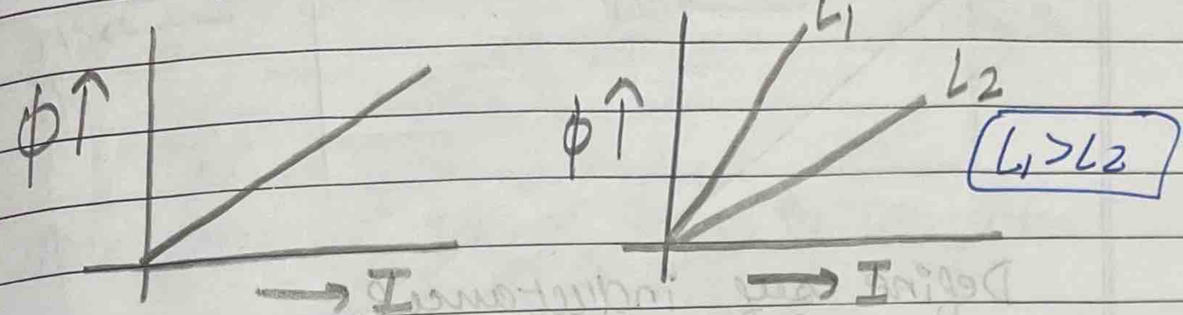
$\phi \propto I$

$\phi = LI$

$L \rightarrow$ constant

$L \rightarrow$ co. eff. of self induction
or
self inductance

$$\phi = LI$$



100%. Define self inductance (not self induction)

$$\phi = LI$$

if $\boxed{I = 1 Amp}$ $\boxed{\phi = L}$

Self inductance is numerically equal to magnetic flux linked with a coil when unit current passes in the same coil.

SI unit of L is Henry (H)

$$\phi = LI$$

$$L = \frac{\phi}{I}$$

$$\boxed{1H = \frac{1Wb}{1A}}$$

Acc. to faraday's law

$$\varepsilon = \frac{-d\phi}{dt}$$

$$\varepsilon = -\frac{d(LI)}{dt}$$

$$\boxed{\varepsilon = -L\frac{dI}{dt}}$$

$$|\varepsilon| = L\frac{dI}{dt}$$

$$y = mx$$

$$y = -mx$$

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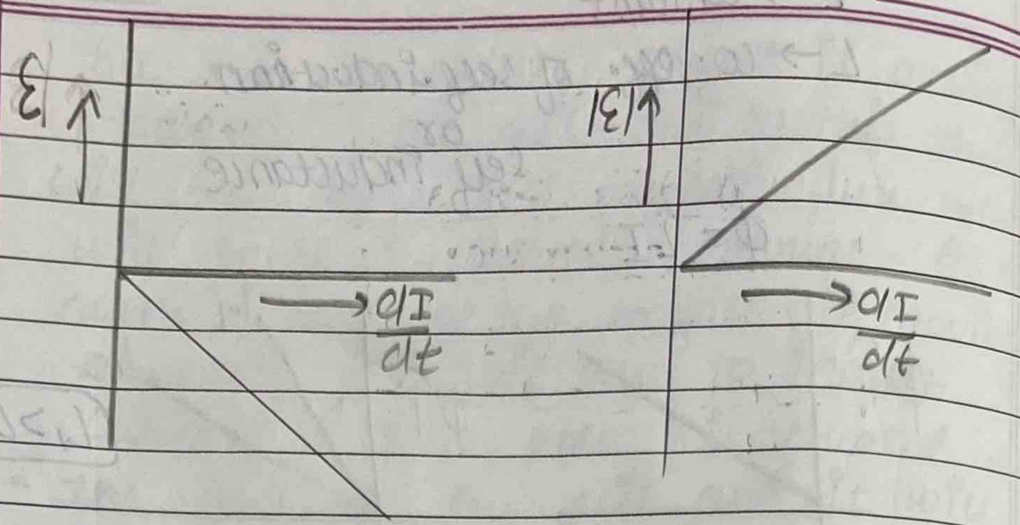

Define self inductance?

$$\varepsilon = -L\frac{dI}{dt}$$

if $\frac{dI}{dt} = \frac{1A}{s}$

$\varepsilon = -L$

$\boxed{|\varepsilon| = L}$

Self inductance is numerically equal to magnitude of induced emf developed in a coil when rate of change of current in the same coil is unity

$$\boxed{\frac{1\,\text{volt} \times 1\,s}{1A} = 1H}$$

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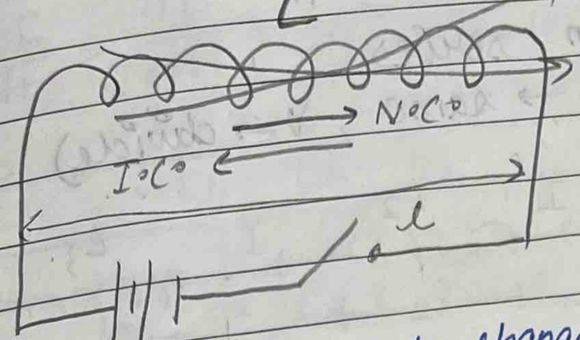
Self inductance of long solenoid



$n = \frac{\text{No. of turns}}{\text{length}}$

Flux produced due to change in current

$\phi = \oint B\cdot ds \times (\text{no. of turns})$

$\phi = \oint B ds \times N$

$\phi = (n \mu_0 I) A n l$

$\phi = n^2 \mu_0 I A l \rightarrow (1)$

using defn of self inductance

$\phi = L I \text{ — } (2)$

Comparing (1) and (2)

$L \cancel{I} = n^2 \mu_0 \cancel{I} A l$

$\boxed{L = n^2 \mu_0 A l}$

Imp.

Q. A solenoid kept in air has self inductance 10 mHenry. If a soft iron rod is kept in it, the self inductance becomes 1000 mH find relative permeability of soft iron. $\longrightarrow \mu_r = \frac{\mu}{\mu_0}$

$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$

Ans.

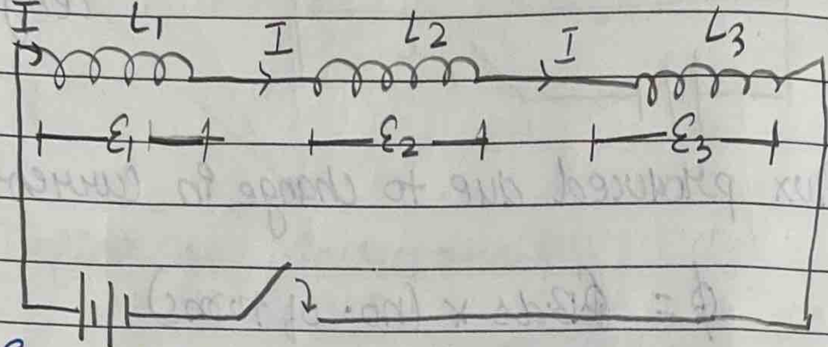
air $L_{air} = n^2 \mu_0 A l$ — (1)

$L_{iron} = n^2 \mu A l \rightarrow (2)$

$\mu_r = \frac{\mu}{\mu_0} = \frac{L_{iron}}{L_{air}} = \frac{1000}{10} = \boxed{100}$ $\boxed{\mu_r = 100}$

Combination of Inductors

(1) Inductors in series
(I → same , V → divide)



$$\mathcal{E} = -L\frac{dI}{dt}$$

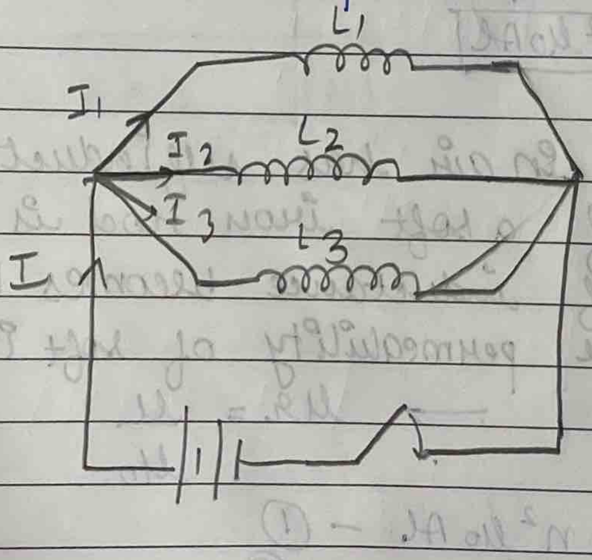
$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3$$

$$-L\frac{dI}{dt} = -L_1\frac{dI}{dt} - L_2\frac{dI}{dt} - L_3\frac{dI}{dt}$$

I → same $\frac{dI}{dt}$ = same

$$\boxed{L = L_1 + L_2 + L_3}$$

(2) Inductors in parallel



(V → same
I → divided)

$$\varepsilon = -L\frac{dI}{dt}$$

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$$-\frac{\varepsilon}{L} = \frac{dI}{dt}$$

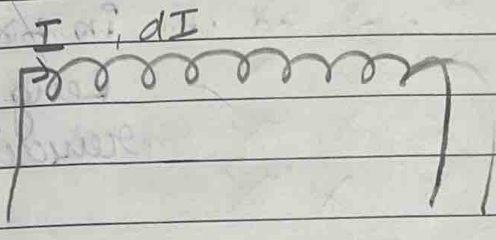
$$I = I_1 + I_2 + I_3$$

$$\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt} + \frac{dI_3}{dt}$$

$$-\frac{\varepsilon}{L} = -\frac{\varepsilon}{L_1} - \frac{\varepsilon}{L_2} - \frac{\varepsilon}{L_3}$$

$$\boxed{\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}}$$

Energy stored in an inductor



$$I = \frac{dq}{dt}$$

$$V = \frac{W}{Q}$$

$$W = QV$$

$$dW = V\,dq$$

$$dW = \varepsilon\,dq$$

work done on the inductor

$$dW = \varepsilon\,dq$$

$$dW = \varepsilon I\,dt$$

$$dW = L\frac{dI}{\cancel{dt}} I\,\cancel{dt}$$

$$dW = L I\,dI$$

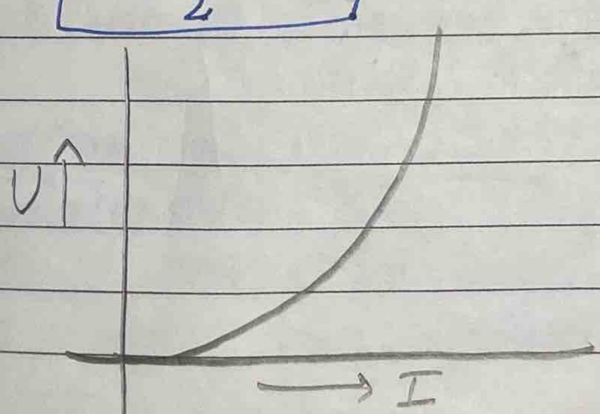
$$W = \int L I'\,dI$$

$$W = \frac{LI^2}{2}$$

stored as

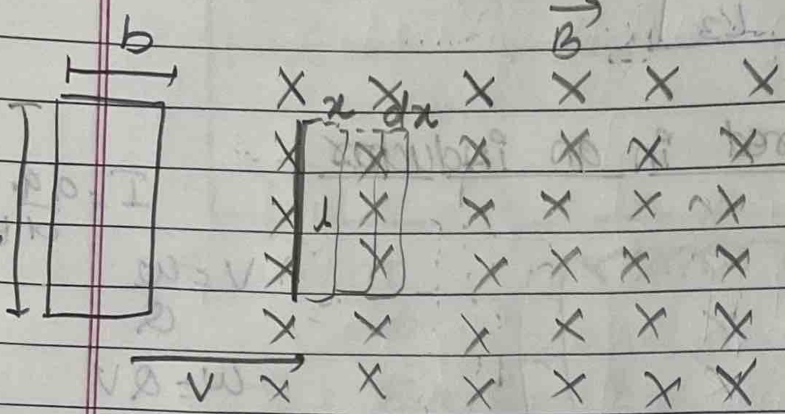
$$\boxed{U = \frac{1}{2}LI^2}$$

$U \propto I^2 \rightarrow$ parabola



MOTIONAL EMF

This is a kind of induced emf which is produced by motion of the conductor in and out of magnetic field



Change in area is due to change in breadth. (x, dx) length (l) remains constant

$$\mathcal{E} = -\frac{d\phi}{dt}$$

$$\mathcal{E} = -\frac{d}{dt}(B \cdot ds)$$

$$\mathcal{E} = \frac{d}{dt}(Bds)$$

$$\mathcal{E} = -\frac{d}{dt}(Blx)$$

$$\mathcal{E} = -Bl\frac{dx}{dt}$$

$$\mathcal{E} = -Blv$$

$$|\mathcal{E}| = Blv$$

$$\boxed{|\mathcal{E}| = Blv}$$

(air↓ mutual induction↑)

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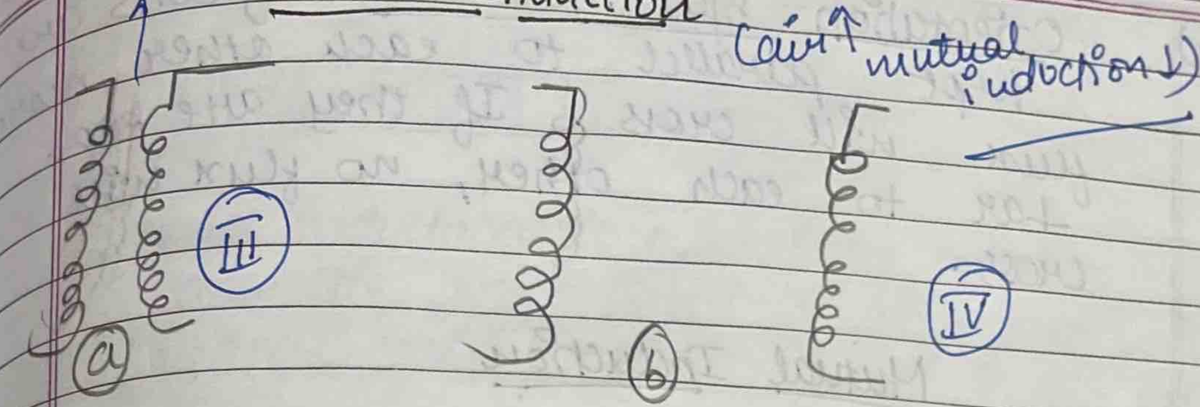
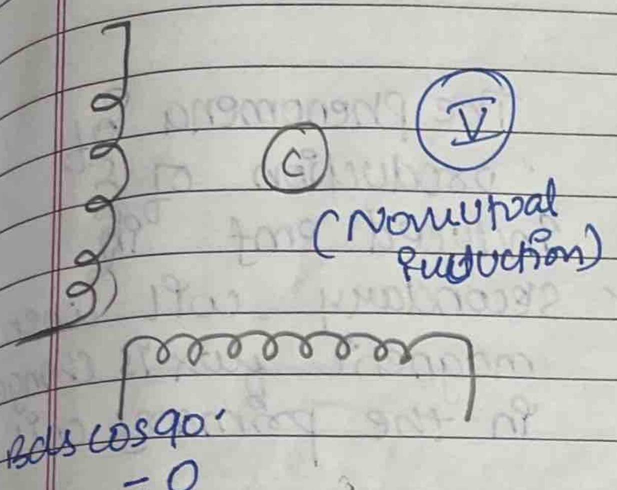
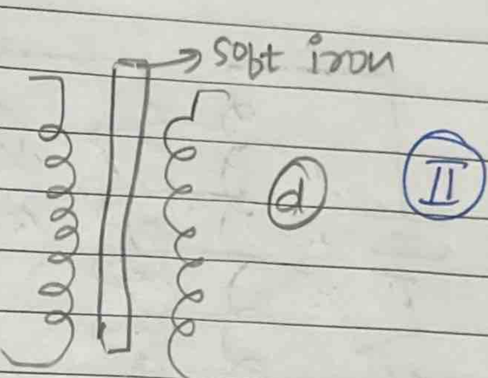
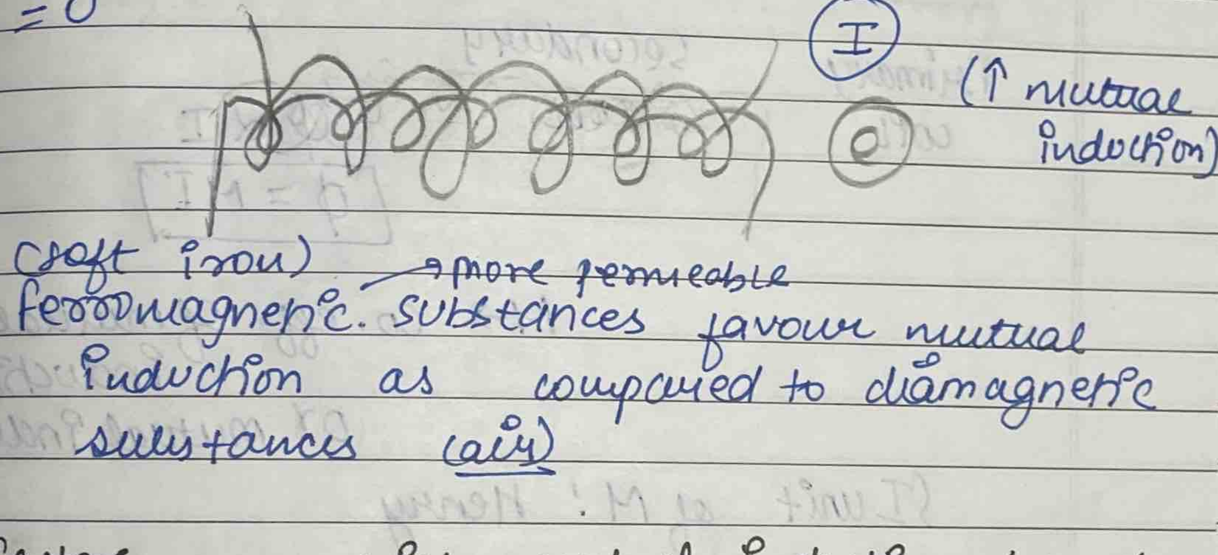
Mutual Induction









→ more permeable

ferromagnetic substances favour mutual induction as compared to diamagnetic substances (air)

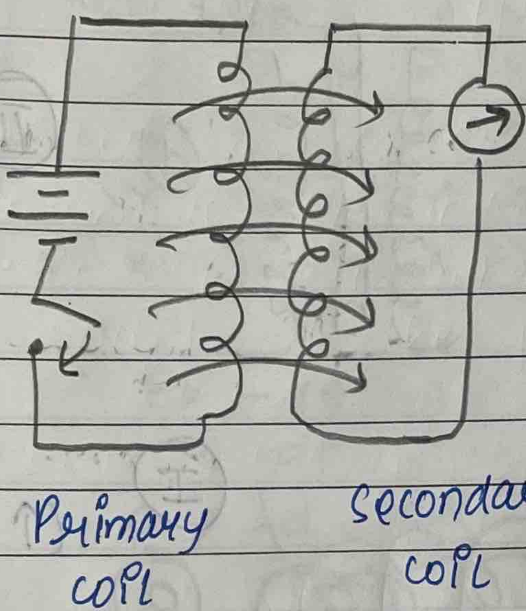
Factors on which mutual induction depends

(1) Distance b/w the coils : more the distance, lesser is the mutual induction and vice-versa

(2) Medium b/w the coils : ferromagnetic material like soft iron, when kept b/w the coils allow flux to cross from one coil to another easily

(3) Orientation b/w the coils: If coils are kept parallel to each other, more flux will cross. If they are kept ⊥ar to each other, no flux will cross.

Mutual Induction



The phenomena of production of induced emf in secondary coil when magnetic flux is changed in the primary coil

$\phi \propto I$

$\boxed{\phi = MI}$

M → Co. eff. of mutual induction or mutual inductance

SI unit of M: Henry

$\boxed{1\,Wb = 1\,H \times 1\,Amp}$

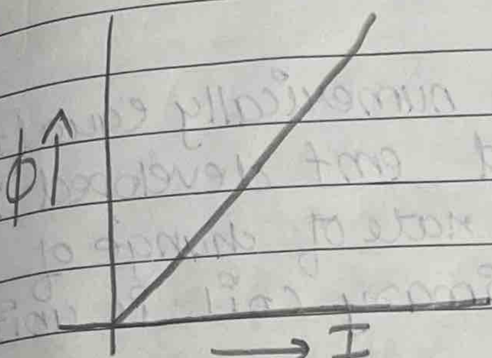
$M = \frac{\phi}{I}$ $\boxed{1\,H = \frac{1\,Wb}{1\,Amp}}$

Define mutual inductance

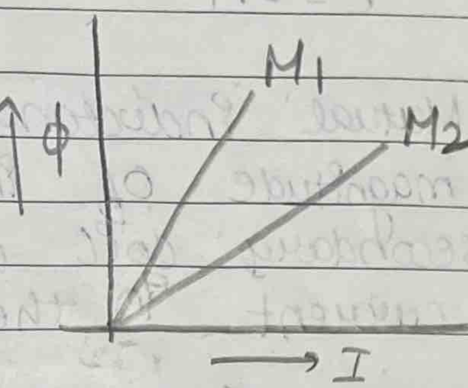
$$\phi = MI$$

if $\boxed{I = 1A}$ $\boxed{\phi = M}$

Mutual inductance is numerically equal to magnetic flux linked w/ secondary coil when unit current passes in the primary coil



Slope = M

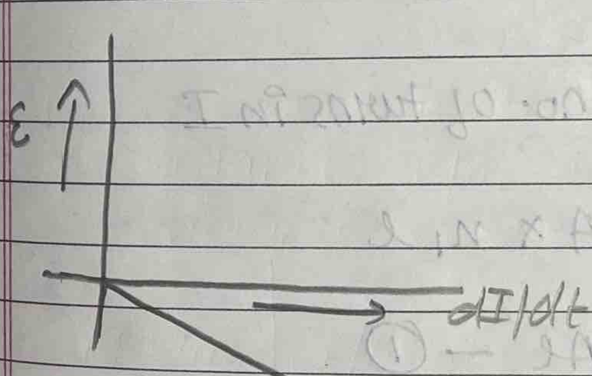


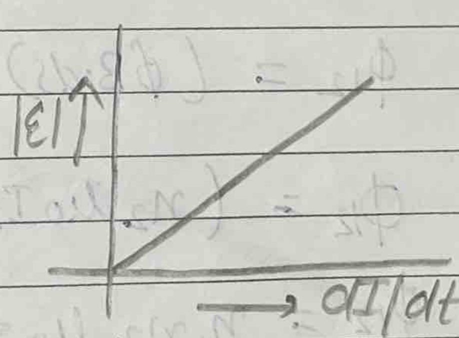
$M_1 > M_2$

$$\varepsilon = -\frac{d\phi}{dt}$$

$$\varepsilon = -\frac{d}{dt}(MI)$$

$$\varepsilon = -M\frac{dI}{dt}$$





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$$\mathcal{E} = -M\frac{dI}{dt}$$

if $\frac{dI}{dt} = 1\,A/s$

$\mathcal{E} = -M$ $\boxed{|\mathcal{E}| = M}$

Define mutual inductance

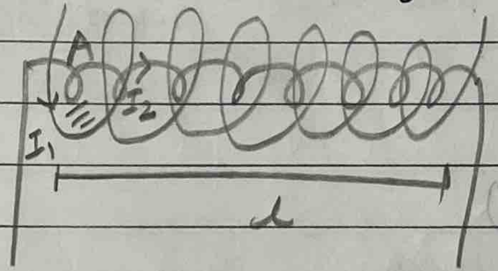
$$\mathcal{E} = -M\frac{dI}{dt} \quad \text{if } \frac{dI}{dt} = 1\,A/s$$

$\mathcal{E} = -M$ $\boxed{|\mathcal{E}| = M}$

Mutual inductance is numerically equal to magnitude of induced emf developed in secondary coil when rate of change of current in the primary coil is unity.

Mutual Inductance of two long solenoid

$n_2 = \frac{N_2}{l}$



$n_1 = \frac{N_1}{l}$

$\phi_{12} \rightarrow$ flux produced in I due to current in II

$$\phi_{12} = (\oint B \cdot ds) \times \text{no. of turns in I}$$

$$\phi_{12} = (n_2 \mu_0 I_2) A \times n_1 l$$

$$\phi_{12} = n_1 n_2 \mu_0 I_2 A l \quad -(1)$$

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By defⁿ. of Mutual Induction
$\phi = M_{12} I_2 \longrightarrow (2)$
Comparing (1) and (2)

$$n_1 n_2 \mu_0 I_2 Al = M_{12} I_2$$
$$\boxed{M_{12} = n_1 n_2 \mu_0 Al}$$

Similarly, $\boxed{M_{21} = n_1 n_2 \mu_0 Al}$

$$\boxed{M_{12} = M_{21}}$$

eg 6.1. (iv) no. of turns ↑
(a) What would you do to obtain a large deflection of the galvanometer?
(b) How would you demonstrate the presence of an induced current in the absence of a galvanometer?

(a) (i) Use a rod of soft iron inside the coil
(ii) connect the coil to a powerful battery
(iii) Move the arrangement rapidly towards the test coil (vel. ↑)
(iv) ↑ no. of turns

(b) Use LED (Light emitting diode)
or
Use a small bulb to obtain induced current
If it lights up, it has induced current

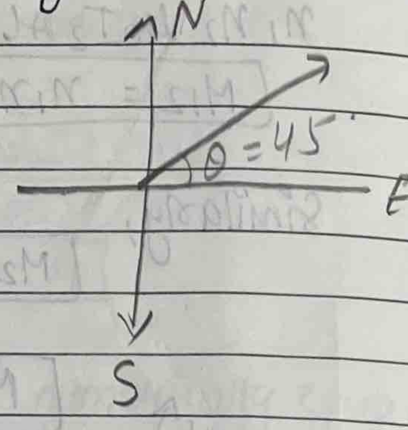
eg 6.2. A square loop of side 10cm and resistance $0.5\,\Omega$ is placed vertically in the east-west plane. A uniform magnetic field of $0.10\,T$ is set up across the plane in the north-east dir^n. The m.f. is decreased to zero in 0.70 sec at a steady state. Determine the magnitudes of induced emf and current during this time-interval.

Sol^n.

$l = 10cm = \frac{10}{100} m$

$R = 0.5 \Omega$

$B_1 = 0.1 T$



$B_2 = 0 T \quad t = 0.7 s$

$\mathcal{E} = -\frac{d\phi}{dt} \qquad \mathcal{E} = -\frac{(\phi_2 - \phi_1)}{t}$

$\phi_2 = B \cdot ds = B ds \cos\theta = B ds \cos 45$.

$A = \frac{10 \times 10}{100 \times 100} = 10^{-2} m^2$

$\vec{B} \cdot \vec{ds}$

$\boxed{\phi_1 = 0} \quad t = 0.7 sec$

$I = \frac{\mathcal{E}}{R}$

$r = 10cm \qquad A = \pi r^2$

6.3. $N = 500 \qquad R = 2\Omega \qquad \theta_1 = 0° \quad \theta_2 = 180°$

$t = 0.25 sec \qquad B = 3.0 \times 10^{-5} T$

$\phi_1 = B \cdot ds = B ds \cos 0°$

$\phi_2 = B ds \cos 180°$

$\mathcal{E} = \frac{N(\phi_2 - \phi_1)}{t}$

$I = \frac{\mathcal{E}}{R}$

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$A = \pi \times (10\times10^{-2})^2$

$\phi_B = 3\times10^{-5} \times (\pi\times10^{-2}) \times \cos 0°$
$= 3\pi \times 10^{-7}\ wb$

$\phi_B = 3\times10^{-5} \times (\pi\times10^{-2}) \times \cos 180°$
$= -3\pi\times 10^{-7}\ wb$

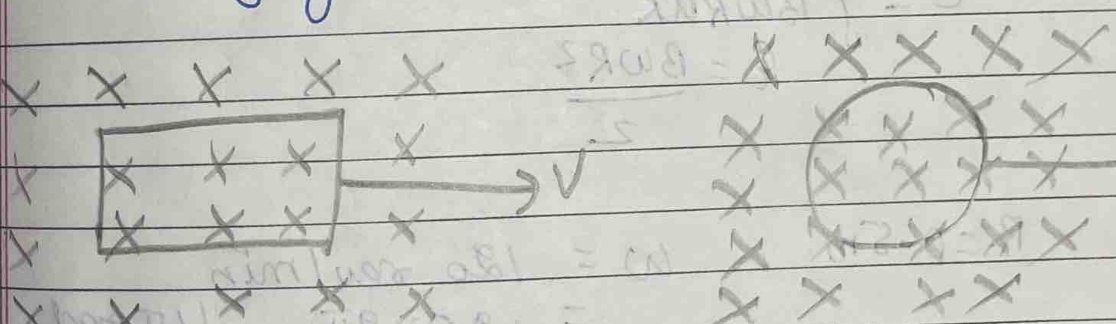
$\varepsilon = \frac{500 \times (6\pi\times10^{-7})}{0.25}$
$= 3.8\times10^{-3}\ V$

$I = \frac{\varepsilon}{R} = 1.9\times10^{-3}\ A$

6.5 (a) No, induced emf cannot be produced without motion of the conductor however strong magnetic field may be

(b) No induced current can be produced by changing electric flux

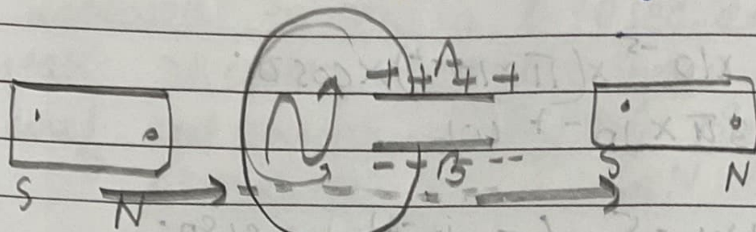
Imp' (c)



The induced emf is expected to be constant only in the case of the rectangular loop. In the case of circular loop, the rate of change of area of the loop during its passage out of the field region is not constant,

North ↳ upper plate = +ve
lower " = -ve

hence induced emf will vary accordingly.

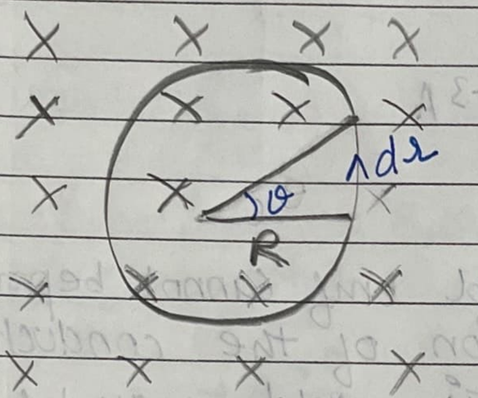


6.6. $l = r = 1m$ $\omega = 50$ rev/sec $= 50 \times 2\pi \frac{rad}{sec}$

$= 100\pi$ rad/s

Metallic rod

$\boxed{R = L}$



$\boxed{\mathcal{E} = \frac{B\omega R^2}{2}}$

$\mathcal{E} = BRv$

$d\mathcal{E} = Bv\,dR$

$d\mathcal{E} = B(\omega R)\,dR$

$\mathcal{E} = \int B\omega R\,dR$

$\mathcal{E} = \frac{B\omega R^2}{2}$

6.7 $R = 0.5m$ $\omega = 120$ rev/min

$= \frac{120 \times 2\pi}{60} = 4\pi \frac{rad}{sec}$

$B = 0.4 \times 10^{-4}\,T$

$\mathcal{E} = \frac{1}{2} B\omega R^2$

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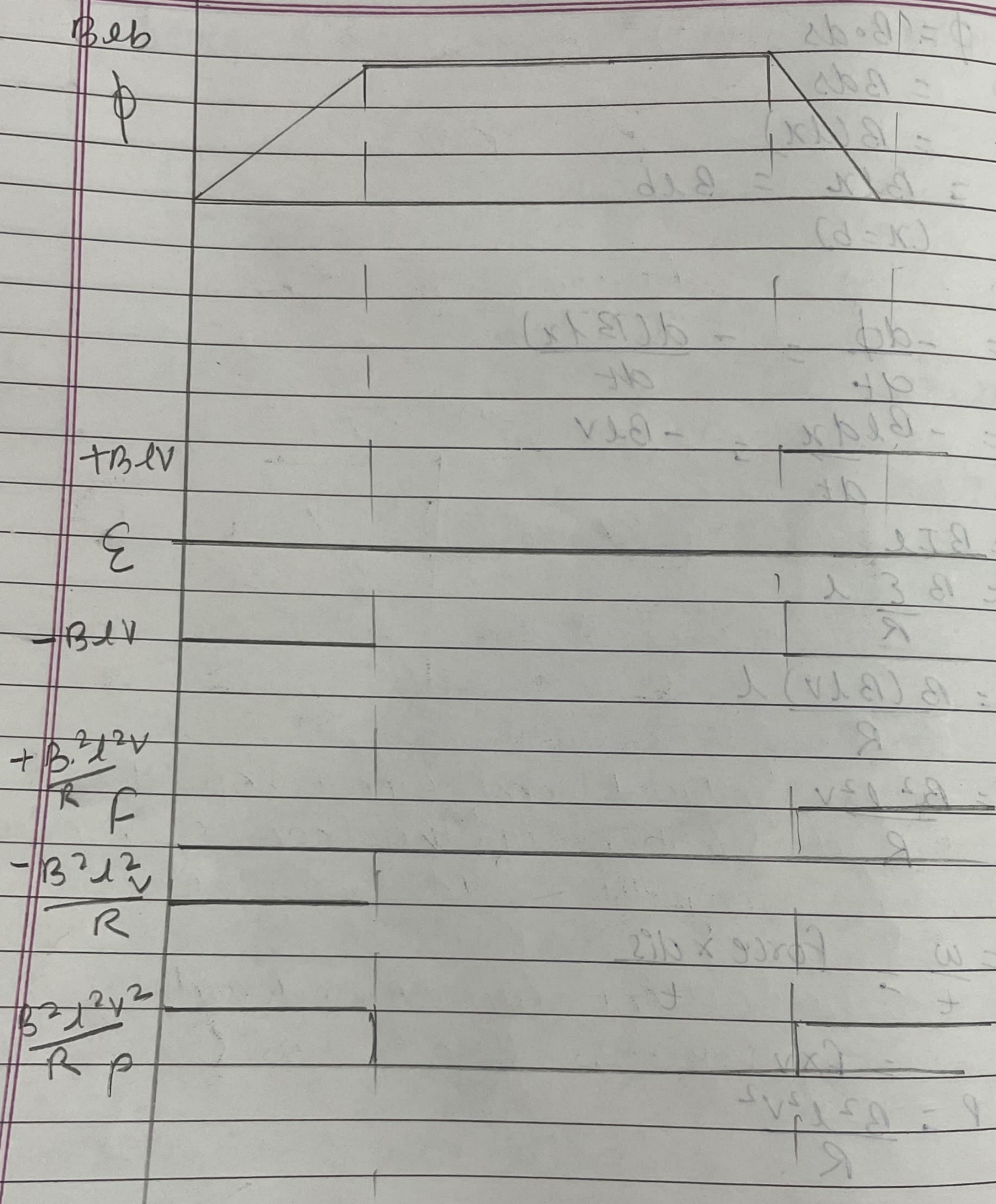
eg 6.8

$$\phi = B \cdot ds$$
$$= Bds$$
$$= B(lx)$$
$$\phi = Blx = Blb$$
$$(x = b)$$

$$\mathcal{E} = \frac{-d\phi}{dt} = -\frac{d(Blx)}{dt}$$
$$\mathcal{E} = -Bl\frac{dx}{dt} = -Blv$$

(iii) $F = BIl$
$$F = B\frac{\mathcal{E}}{R}l$$
$$F = \frac{B(Blv)l}{R}$$
$$F = \frac{B^2 l^2 v}{R}$$

(iv) $P = \frac{w}{t} = \frac{\text{force} \times \text{dis}}{t}$
$$= F \times v$$
$$P = \frac{B^2 l^2 v^2}{R}$$


Blb
ϕ
+Blv
ε
-Blv
+B²l²v/R
F
-B²l²v/R
B²l²v²/R
P