RELATIONS AND FUNCTIONS

POINTS TO REMEMBER

1. Cartesian Product of Sets:-

$A \times B = \{(a,b) : a \in A \text{ and } b \in B\}$.

Results on Cartesian Products of Sets:

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

(iii) $A \times (B - C) = (A \times B) - (A \times C)$.

(iv) If A and B have 'n' elements in common, then $A \times B$ and $B \times A$ have n^2 elements in common.

(v) If $n(A) = a$ and $n(B) = b$, then

(a) $n(A \times B) = a \times b$ (b) $n(B \times A) = b \times a$

(c) $n(A \times A) = a^2$ (d) $n(B \times B) = b^2$

2. Relation: Let A and B be any two non-empty sets. Any subset of $A \times B$ is said to be a relation from the set A to the set B.

3. Binary Relation on a set: Every subset of $A \times A$ is called a binary relation on A.

I. (i) Identity Relation: $I_A = \{(a, a) : a \in A\}$.

(ii) Universal Relation: $A \times A$ is called the universal relation on A.

II. A relation R on A is said to be:

(i) Reflexive, if $a\, R\, a\ \forall\, a \in A$

(ii) Symmetric, if a R b ⇒ b R a ∀ a, b ∈ A

(iii) Transitive, if a R b and b R c ⇒ a R c ∀ a, b & c ∈ A

(iv) Anti-symmetric, if a R b and b R a ⇒ a = b.

III. Equivalence Relation: A relation which is Reflexive, Symmetric and Transitive is called an equivalence relation.

(i) The inverse of an equivalence relation is an equivalence relation.

(ii) The intersection of two equivalence relations is an equivalence relation.

(iii) The union of two symmetric relations is symmetric.

(iv) The union of two transitive relations need not be transitive.

(v) If n (A) = m and n (B) = n, then total no. of relations from A to B is 2^{mn}.

(vi) If n (A) = m and n (B) = n, then total no. of relations from B to A is $2^{m.n}$.

(vii) If n (A) = m, then total no. of relations from A to A is 2^{m^2}.

(viii) If n (B) = n, then total no. of relations from B to B is 2^{n^2}.

IV. A relation R on a set A is:

(i) Reflexive A ⇔ $I_A \subseteq R$

(ii) Symmetric ⇔ $R^{-1} = R$

(iii) Transitive ⇔ $RoR \subseteq R$

4. Functions or Mappings:

Let A and B be two non-empty sets. Then, a rule or a correspondence f which associates to each x ∈ A, a unique element f (x) ∈ B, is called a function or a mapping from A to B, and we write, f: A→ B. f (x) is called the image of x and x is called the pre-image of f (x).

A is called the domain of f and { f (x): x ∈ A} ⊆ B is called the range of f, where B is the co-domain of f.

5. Various Types of Functions:

A function f: A→ B is said to be:

(i) Many-One, if two or more than two elements in A have the same image in B.

(ii) One-One, if distinct elements in A have distinct images in B

i.e., f is one-one, if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \forall x_1, x_2 \in A$

(iii) Into, if ∃ at least one element in B which has no pre-image in A.

(iv) Onto, if Range = Co-domain i.e., each element of B has a pre-image in A.

One-one mapping is called Injective; onto mapping is called surjective and a one-one, onto mapping is called bijective.

6. Let f: A→ B and g: B →C, then

gof: A→ C s.t. $(gof)(x) = g(f(x))$

Note I: gof is defined when Range (f) ⊆ Dom (g).

Note II: fog is defined when Range (g) ⊆ Dom (f).

7. Chart to find the domain of various functions:-

	Function	Domain
1.	$\frac{1}{f(x)}$	$D = \{x : f(x) \neq 0\}$
2.	$\frac{f(x)}{g(x)}$	$D = \{x : g(x) \neq 0\}$
3.	$\sqrt{(f(x))}$	$D = \{x : f(x) \geq 0\}$
4.	$\frac{1}{\sqrt{(f(x))}}$	$D = \{x : f(x) > 0\}$
5.	$\log(f(x))$	$D = \{x : f(x) > 0\}$
6.	$\|f(x)\|, [f(x)], e^{f(x)}$	$D = \text{Domain}(f)$

8. (i) Dom(f + g) = Dom(f) $\cap$ Dom(g).

(ii) Dom (f – g) = Dom (f) $\cap$ Dom (g).

(iii) Dom $\left(\frac{f}{g}\right)$ = Dom (f) $\cap$ {Dom (g) – { x : g (x) = 0}}

9. To find the range of f (x), firstly put y = f (x) and find value of 'x' in terms of y only, i.e. x = g (y), then find the domain of g (y), this domain (g) = Range (f).

Notes:

(i) As range of all functions cannot be derived by one method. So, students are advised to be a bit careful while finding range.

(ii) If a line parallel to x - axis cuts the graph of f (x) at more than one point, then the function is not One-One.

(iii) If a line parallel to x – axis cuts the graph of f (x) at atmost one point, then the function is One-One.

(iv) If f '(x) remains unchanged in its sign for every point in the domain of the function, then f (x) is always One - One.