CHAPTER 3

MATRICES

POINTS TO REMEMBER:

Matrix: A matrix is an ordered rectangular array of numbers or functions. The numbers or functions are called the elements or the entries of the matrix.

The elements in i[th] row and j[th] column of a matrix is denoted by a_{ij}.

In general, an m × n matrix has the following rectangular array:

$$A=[a_{ij}]_{m\times n}=\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$$

$A=[a_{ij}]_{m\times n}$, $1\le i\le m$, $1\le j\le n$; $i, j \in N$

A matrix having m rows and n columns is called a matrix of order m × n or simply m × n matrix (read as m by n matrix).

A matrix having m rows and n columns is called a matrix of order m × n (read as m by n).

2. Diagonal elements of a square matrix: The elements a_{ij} for which i = j are called the diagonal element of the matrix.

3. Comparable matrices: Two matrices A and B are said to be comparable if they are of the same order.

4. Operations on matrices:

(I) Addition and subtraction of matrices: The sum or difference of two matrices is defined only when they are of same order.

If A = $[a_{ij}]_{m \times n}$ and B = $[b_{ij}]_{m \times n}$ then A+B = $[a_{ij} + b_{ij}]_{m \times n}$ and

A − B = $[a_{ij} - b_{ij}]_{m \times n}$

(II) Scalar multiplication: Let A= $[a_{ij}]_{m \times n}$ and k be any number, then

kA =$[ka_{ij}]_{m \times n}$

(III) Multiplication of matrices: For two matrices A and B, the product AB exist only when number of columns in A = number of rows in B. Otherwise AB does not exist.

Let A = $[a_{ij}]_{m \times n}$ and B = $[b_{jk}]_{n \times p}$ then C = AB = $[c_{ik}]_{m \times p}$ where $c_{ik} = \sum_{j=1}^{n} a_{ij} b_{jk}$

If AB and BA are both defined, then it is not necessary that AB = BA

5. <u>Transpose of a Matrix</u>:

The Matrix obtained by interchanging the rows and columns of a matrix A is called the transpose of A, written as A^T or A′.

For any two matrices A and B of suitable orders, we have

(i) (A′)′ = A (ii) (kA)′ = kA′ (where k is any constant)

(iii) (A ± B)′ = A′ ± B′ (iv) (A B)′ = B′ A′

6. <u>Types of matrices</u>:

(I) Diagonal matrix: A square Matrix A= $[a_{ij}]_{m \times n}$ in which every non diagonal elements is 0 is called a diagonal matrix.

$$D = \begin{bmatrix} a_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & a_{nn} \end{bmatrix} = \text{diag}\,[a_{11}, a_{22}, a_{33}, \ldots a_{nn}]$$

(II) Scalar Matrix: A diagonal Matrix in which all diagonal elements are same is called a scalar matrix.

(III) Unit or Identity Matrix: A scalar Matrix with each diagonal element 1, is called a unit Matrix.

We denote a unit matrix of order n by I_n or I.

(IV) Triangular Matrix: The matrix A= $[a_{ij}]_{n \times n}$ is called:

(i) an upper triangular matrix if $a_{ij} = 0$ when i > j.

(ii) a lower triangular matrix if $a_{ij} = 0$ when i < j.

(V) Symmetric matrix: A square matrix 'A' is said to be symmetric if (A)' = A

(VI) Skew-symmetric Matrix: A square matrix 'A' is said to be skew-symmetric if $(A)' = -A$

For any square matrix A, we have

- $A + A'$ is always symmetric.
- $A - A'$ is always skew - symmetric.