CHAPTER 2

INVERSE TRIGONOMETRIC FUNCTIONS

POINTS TO REMEMBER:

1. (i) $\sin^{-1}x = \theta \Leftrightarrow x = \sin\theta$.

   (ii) $\cos^{-1}x = \theta \Leftrightarrow x = \cos\theta$.

   (iii) $\tan^{-1}x = \theta \Leftrightarrow x = \tan\theta$.

2. Domain & Range:

Functions	Domain	Range (Principle Values)
$\sin^{-1}x$	[-1 , 1]	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1}x$	[-1 , 1]	$[0, \pi]$
$\tan^{-1}x$	$\mathbb{R}$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\cot^{-1}x$	$\mathbb{R}$	$(0, \pi)$
$\sec^{-1}x$	$\mathbb{R} - (-1, 1)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$\text{cosec}^{-1}x$	$\mathbb{R} - (-1, 1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

3. (i) $\sin^{-1}(\sin x) = x$, if $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

   (ii) $\cos^{-1}(\cos x) = x$, if $0 \le x \le \pi$

   (iii) $\tan^{-1}(\tan x) = x$, if $-\frac{\pi}{2} < x < \frac{\pi}{2}$

4. (i) $\sin^{-1} x = \operatorname{cosec}^{-1}\left(\frac{1}{x}\right), -1 \le x \le 1$

(ii) $\cos^{-1} x = \sec^{-1}\left(\frac{1}{x}\right), -1 \le x \le 1$

(iii) $\tan^{-1} x = \cot^{-1}\left(\frac{1}{x}\right), x > 0$

5. (i) $\sin^{-1}(-x) = -\sin^{-1} x, -1 \le x \le 1$

(ii) $\cos^{-1}(-x) = \pi - \cos^{-1} x, -1 \le x \le 1$

(iii) $\tan^{-1}(-x) = -\tan^{-1} x, x \in R$

(iv) $\cot^{-1}(-x) = \pi - \cot^{-1} x, x \in R$

(v) $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x, |x| \ge 1$

(vi) $\sec^{-1}(-x) = \pi - \sec^{-1} x, |x| \ge 1$

6. (i) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}, -1 \le x \le 1$

(ii) $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}, x \in R$

(iii) $\sec^{-1} x + \operatorname{cosec}^{-1} x = \frac{\pi}{2}, |x| \ge 1$

7. (i) $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$, when $x > 0, y > 0$ and $xy < 1$

(ii) $\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$, , when $x > 0, y > 0, xy > 1$

(iii) $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$, when $x > 0, y > 0$ and $xy > -1$

8.

(i) $\sin^{-1} x + \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right), -1 \le x, y \le 1, x^2 + y^2 \le 1$

(ii) $\sin^{-1} x - \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right), -1 \le x, y \le 1, x^2 + y^2 \le 1$

(iii) $\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left(xy - \sqrt{1-y^2}.\sqrt{1-x^2}\right), -1 \le x, y \le 1, x+y \ge 0$

(iv) $\cos^{-1}x - \cos^{-1}y = \cos^{-1}\left(xy + \sqrt{1-y^2}.\sqrt{1-x^2}\right), -1 \le x, y \le 1, x \le y$

9. (i) $\sin^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right), -\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$

(ii) $2\cos^{-1}x = \cos^{-1}(2x^2 - 1), 0 \le x \le 1$

(iii) $2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right), -1 < x < 1$

$2\tan^{-1}x == \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 \le x < \infty$

$2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right), -1 \le x \le 1$

10. (i) $3\sin^{-1}x = \sin^{-1}(3x - 4x^3), -\frac{1}{2} \le x \le \frac{1}{2}$

(ii) $3\cos^{-1}x = \cos^{-1}(4x^3 - 3x), \frac{1}{2} \le x \le 1$

(iii) $3\tan^{-1}x = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

11. (i) For $0 < x < 1$, we have

$$\sin^{-1}x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right) = \cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) = \operatorname{cosec}^{-1}\left(\frac{1}{x}\right)$$

(ii) For $0 < x < 1$, we have

$$\cos^{-1}x - \sin^{-1}\sqrt{1-x^2} - \cot^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) - \operatorname{cosec}^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right) - \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) - \sec^{-1}\left(\frac{1}{x}\right)$$

(iii) For $x > 0$, we have

$$\tan^{-1}x = \sec^{-1}\sqrt{1+x^2} = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) = \operatorname{cosec}^{-1}\left(\frac{\sqrt{1+x^2}}{x}\right) = \cot^{-1}\left(\frac{1}{x}\right)$$

(iv) $\sin^{-1}\left(\frac{a}{\sqrt{a^2+b^2}}\right) = \cos^{-1}\left(\frac{b}{\sqrt{a^2+b^2}}\right) = \tan^{-1}\left(\frac{a}{b}\right)$