CHAPTER 7

INTEGRALS

POINTS TO REMEMBER:

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + C \ (n \neq -1)$

2. $\int \frac{1}{x} dx = \log|x| + C$

3. $\int e^x dx = e^x + C$

4. $\int a^x dx = \frac{a^x}{\log a} + C$

5. $\int \sin x \, dx = -\cos x + C$

6. $\int \cos x \, dx = \sin x + C$

7. $\int sec^2 x \, dx = \tan x + C$

8. $\int cosec^2 x \, dx = -\cot x + C$

9. $\int \sec x \tan x \, dx = \sec x + C$

10. $\int cosec \, x \cot x \, dx = -cosec \, x + C$

11. $\int \tan x \, dx = \log|\sec x| + C = -\log|\cos x| + C$

12. $\int \cot x \, dx = \log|\sin x| + C = -\log|cosec \, x| + C$

13. $\int \sec x \, dx = \log|\sec x + \tan x| + C = \log tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + C$

14. $\int cosec \, x dx = \log|cosec \, x - \cot x| + C = \log|\tan\frac{x}{2}| + C$

15. $\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + C$

16. $\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + C$

17. $\int \frac{1}{a^2+x^2}dx = \frac{1}{a}tan^{-1}\frac{x}{a} + C$

18. $\int \frac{1}{\sqrt{a^2-x^2}}dx = sin^{-1}\frac{x}{a} + C$

19. $\int \frac{1}{\sqrt{x^2-a^2}}dx = \log|x + \sqrt{x^2 - a^2}| + C$

20. $\int \frac{1}{\sqrt{x^2+a^2}}dx = \log|x + \sqrt{x^2 + a^2}| + C$

21. $\int \sqrt{a^2 - x^2}dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}sin^{-1}\frac{x}{a} + C$

22. $\int \sqrt{x^2 - a^2}dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log|x + \sqrt{x^2 - a^2}| + C$

23. $\int \sqrt{a^2 + x^2}dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\log|x + \sqrt{a^2 + x^2}| + C$

24. Integration by parts: $\int u.v dx = u.\int v dx - \int \left\{\frac{du}{dx}.\int v dx\right\} dx + C$

25. $\int e^x\{f(x) + f'(x)\}dx = e^x f(x) + C$

26. $\int e^{ax}\sin(bx + c)\,dx = \frac{e^{ax}}{a^2+b^2}[a.\sin(bx + c) - b.\cos(bx + c)] + C$

27. $\int \frac{p.\sin x + q.\cos x}{a.\sin x + b.\cos x}\,dx - Ax + B.\log|a.\sin x + b.\cos x| + C$ where,

$A = \frac{ap+bq}{a^2+b^2}$ & $B = \frac{aq-bp}{a^2+b^2}$

28. $\int e^{mx}.\sin(nx)\,dx = \frac{e^{mx}}{m^2+n^2}[m.\sin(nx) - n.\cos(nx)] + C$

29. $\int e^{mx}.\cos(nx)\,dx = \frac{e^{mx}}{m^2+n^2}[m.\sin(nx) + n.\cos(nx)] + C$

30. $\int \frac{1}{a^2\cos^2x+b^2\sin^2x}\,dx = \frac{1}{ab}tan^{-1}\left(\frac{b}{a}\tan x\right) + C$

32. $\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$, where $F(x) = \int f(x)dx$

33. $\int_a^b f(x)dx = \int_a^b f(t)dt$

34. $\int_a^b f(x)dx = -\int_b^a f(x)dx$

35. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

36. $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

37. $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

38. $\int_{-a}^a f(x)dx = \int_0^a [f(x)+f(-x)]\,dx$

39. $\int_0^{2a} f(x)dx = \int_0^a [f(x)+f(2a-x)]\,dx$

40. If f(x) is a periodic function with period 'T', then $\int_0^{nT} f(x)dx = n\int_0^T f(x)dx$

41. Walli's Formula:-

$$\int_0^{\frac{\pi}{2}} sin^n x dx = \int_b^{\frac{\pi}{2}} cos^n x dx = \begin{cases} \frac{(n-1)(n-3)(n-5)\dots 1}{n.(n-2)(n-4)\dots 2}.\frac{\pi}{2} & if\ n\ is\ even \\ \frac{(n-1)(n-3)(n-5)\dots 2}{n.(n-2)(n-4)\dots 1}.1 & if\ n\ is\ odd \end{cases}$$

42. $\int_a^b |f(x)|\,dx$, limit of this integral will split at all those points for which f(x)=0 and a< f(x)< b

43. $\int_0^{\frac{\pi}{2}} \log|\sin x|\,dx = \int_0^{\frac{\pi}{2}} \log|\cos x|\,dx = -\frac{\pi}{2}\log 2$

44. $\int_a^b \frac{f(x)}{f(x)+f(a+b-x)}dx = \frac{1}{2}(b-a)$

45. $\int_a^b \frac{1}{a^2 cos^2 x+b^2 sin^2 x}dx = \frac{\pi}{2ab}$

46. $\int_0^{\frac{\pi}{2}} \frac{a\sin x+b.\cos x}{\sin x+\cos x}dx = \frac{\pi}{4}(a+b)$

47. $\int_0^{\frac{\pi}{2}} \frac{a.\tan x+b.\cot x}{\tan x+\cot x}dx = \frac{\pi}{4}(a+b)$

48. $\int_0^{\frac{\pi}{2}} \frac{a.\cos ec x+b.\sec x}{\cos ec x+\sec x}dx = \frac{\pi}{4}(a+b)$49.

$$\int_0^{\pi} \sin ax.\cos bx\,dx = \begin{cases} \frac{2a}{a^2-b^2} & : if\ a-b\ is\ odd \\ 0 & : if\ a-b\ is\ even \end{cases}$$