CHAPTER 5

CONTINUITY AND DIFFERENTIABILITY

POINTS TO REMEMBER:

Limits:

We say $\lim_{x \to c^-} f(x)$ is the expected value of f at x = c given the values of f near x to the left of c. This value is called the left hand limit of f at c.

We say $\lim_{x \to c^+} f(x)$ is the expected value of f at x = c given the values of f near x to the right of c. This value is called the right hand limit of f(x) at c.

If the right and left hand limits coincide, we call that common value as the limit of f(x) at x = c and denote it by $\lim_{x \to c} f(x)$.

Continuity:

Suppose f is a real function on a subset of the real numbers and let c be a point in the domain of f. Then f is continuous at c if $\lim_{x \to c} f(x) = f(c)$.

More elaborately, if the left hand limit, right hand limit and value of the function at x = c exist and are equal to each other, then f is said to be continuous at x = c.

A real function f is said to be continuous, if it is continuous at every point in the domain of f.

Differentiability:

Suppose f is a real function and 'a' is a point in its domain. The derivative of f at 'a' is defined by

$\lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$, provided this limit exists. Derivative of f (x) at 'a' is denoted by f' (a)

The derivative of f at 'x' is defined by $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$, provided this limit exists.

The process of finding derivative of a function is called differentiation.

Theorem: If a function f is differentiable at a point 'c', then it is also continuous at that point.

Corollary: Every differentiable function is continuous whereas a continuous function may or may not be differentiable.

<u>Basic rules for differentiation:</u>

1. $\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x)$
2. $\frac{d}{dx}(f(x).g(x)) = f(x) \frac{d}{dx} g(x) + g(x) \frac{d}{dx} f(x)$
3. $\frac{d}{dx}[f(g(x))] = \frac{d}{dy}[f(y)] \times \frac{dy}{dx}, where\ y = g(x)$
4. $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{(g(x))^2}, g(x) \neq 0$

<u>Basic formulae for differentiation:</u>

1. $\frac{d}{dx}(x^n) = n\,x^{n-1}$
2. $\frac{d}{dx}(\log_e x) = \frac{1}{x}$
3. $\frac{d}{dx}(\log_a x) = \frac{1}{x \log a}$
4. $\frac{d}{dx}(e^x) = e^x$
5. $\frac{d}{dx}(a^x) = a^x \log a$
6. $\frac{d}{dx}(\sin x) = \cos x$
7. $\frac{d}{dx}(\cos x) = -\sin x$

8. $\frac{d}{dx}(\tan x) = \sec^2 x$

9. $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

10. $\frac{d}{dx}(\sec x) = \sec x \tan x$

11. $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

12. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}, -1 < x < 1$

13. $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}, -1 < x < 1$

14. $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}, |x| > 1$

15. $\frac{d}{dx}(\operatorname{cosec}^{-1} x) = -\frac{1}{|x|\sqrt{x^2-1}}, |x| > 1$

16. $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}, x \in \mathbb{R}$

17. $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}, x \in \mathbb{R}$