SET THEORY NOTES
NDA I / NDA II 2025 / CUET 2025

- Sets are usually denoted by capital letters *A*, *B*, *C*, etc.
- The members of a set are called its elements and are usually denoted by small letters ,*a*, *b*, *c*, etc.

Representation of Sets

1. **Tabular or Roster method** In this method, a set is described by writing elements, separated by commas, within the braces {}.

   e.g. $N = \{1, 2, 3, 4, \ldots\}$ is a set of natural numbers.

   $A = \{2, 3, 5, 7, 11, 13\}$ is a set of first six prime numbers.

   $W = \{0, 1, 2, 3, 4, \ldots\}$ is a set of whole numbers.

2. **Set-builder or Rule method** In this method, a rule or a formula is written in the braces that defines the sets.

   e.g. $A = \{x : x = 2n + 1, n \geq 1, n \in N\}$, $B = \{x : 6 \leq x \leq 12, x \in N\}$, $C = \{x : x = 2n, n < 8, n \in N\}$

Different Types of Sets

1. **Null set** A set which does not contain any element is called a null set. It is denoted by ϕ. A null set is also called an empty set or a void set. Therefore, $\phi = \{ \}$

   e.g. $A = \{x : x$ is a prime number between 90 and 96$\} = \phi$

2. **Singleton set** A set which contains only one element is called a singleton set.

   e.g. $A = \{0\}$, $B = \{x : x + 10 = 0, x \in Z\}$

3. **Finite set** A set is called a finite set, if it is either void set or its elements can be counted or labelled by natural numbers 1,2,3, ... and the process of counting stops at a certain natural number (say '*n*').

   e.g. $A = \{a, e, i, o, u\}$ = Set of all vowels

   The number of distinct elements of a finite set *A* is called the **cardinal number** of the set *A* and it is denoted by $n(A)$.

4. **Infinite set** A set which has unlimited number of elements is called infinite set.

e.g. *N* = set of all natural numbers = {1, 2, 3, ...}
Z = set of all integers
= {..., − 2, − 1, 0, 1, 2, ...} are infinite sets.

5. **Equivalent sets** Two finite sets *A* and *B* are equivalent, if their cardinal numbers are same, i.e.

$n(A) = n(B)$.

e.g. $A = \{a, b, c, d, e\}$; $B = \{1, 3, 5, 7, 9\}$
Here, $n(A) = n(B)$
So, these sets are equivalent sets.

6. **Equal sets** Two sets *A* and *B* are said to be equal, if every element of *A* is a member of *B* and every element of *B* is a member of *A*. If sets *A* and *B* are equal, we write *A* = *B* and *A* ≠ *B*, when *A* and *B* are not equal.
e.g.
(i) *A* = {1, 3, 4} and *B* = {3, 1, 4} are equal sets.
(ii) *A* = Letters of the word MASS = {M, A, S, S}
B = Letters of the word SAM = {S, A, M}
Here, *A* = *B*
Since, {M, A, S, S} = {M, A, S} = {S, A, M}

Note Equal sets are equivalent but equivalent sets may or may not be equal. e.g. Set $A = \{4, 5, 3, 2\}$ and $B = \{1, 6, 8, 9\}$ are equivalent but are not equal.

7. **Subset** Let A and B be two sets. If every element of A is an element of B, then A is called a subset of B and it is denoted by $A \subseteq B$.
$\therefore A \subseteq B$, if $x \in A \Rightarrow x \in B$
e.g. If $A = \{1, 2, 3\}$
and $B = \{1, 2, 3, 4, 5, 6\}$, then $A \subseteq B$.
The total number of subsets of a finite set containing n elements is $\mathbf{2^n}$.

8. Superset Let A and B be two sets. If B contains all elements of A, then B is called superset of A and it is denoted by $B \supseteq A$.
e.g. If $B = \{a, b, c, d, \ldots, x, y, z\}$ and $A = \{a, e, i, o, u\}$
Then, $B \supseteq A$.
9. Proper subset A set A is said to be a proper subset of set B, if A is a subset of B and A is not equal to B. It is written as $A \subset B$. The total number of proper subsets of a finite set containing n elements **is $(2^n - 1)$.**
e.g. If $A = \{1, 2, 3\}$, then proper subsets of A are ϕ, $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 2\}$, $\{1, 3\}$, $\{2, 3\}$.

10. Universal set All the sets under consideration are likely to be subsets of a set which is called the universal set and is denoted by U.
e.g. Let $A = \{1, 2, 3\}$; $B = \{2, 4, 6, 8\}$; $C = \{1, 3, 5, 7, 9\}$
and U = set of natural numbers.
Here, A, B and C are the subsets of U.
Therefore U is an universal set.

11. Power set Let A be a set, then the set of all the possible subsets of A is called the power set of A and is denoted by $P(A)$.
i.e. $P(A) = \{S \mid S \subseteq A\}$
e.g. Let $A = \{1, 2, 3\}$. Then, subsets of A are
ϕ, $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 2\}$, $\{1, 3\}$, $\{2, 3\}$ and $\{1, 2, 3\}$.
$\therefore P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

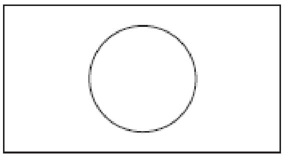
- **Note** Since, the empty set and the set A itself are subsets of A and also the elements of $P(A)$. Thus, the power set of a given set is always **non-empty.**
- If a set A has n elements, then its power set will contain $\mathbf{2^n}$ elements.

Some Important Properties

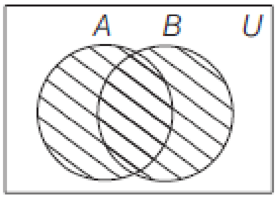
- ***(i)*** $A \subseteq A, \forall A$
- ***(ii)*** $\phi \subseteq A, \forall A$
- ***(iii)*** $A \subseteq U, \forall A$ ***in*** U
- ***(iv)*** $A = B \Leftrightarrow A \subseteq B, B \subseteq A$

Venn Diagram

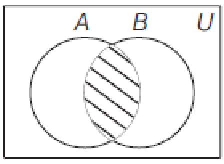
Venn diagram *are used to express relationship among sets.*
In Venn diagram*, the universal set U is represented by a rectangle and its subsets are represented by closed curves (usually circles) within the rectangle.*



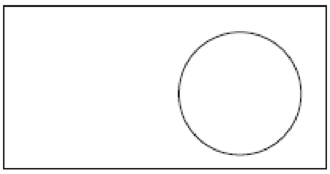
Now, we introduce some operations on sets to construct new sets from the given ones.
1. Union of two sets *Let A and B be two sets. The union of A and B is the set of all those elements which belongs to either A or B or both A and B. The union of A and B is denoted by A U B* ***(read as 'A union B').***
Thus, $A \cup B = \{x : x \in A \text{ or } x \in B\}$
e.g. If $A = \{1, 2, 3, 4\}$ *and* $B = \{3, 4, 5, 6\}$*,*
then $A \cup B = \{1, 2, 3, 4, 5, 6\}$



2. ***Intersection of two sets*** *Let A and B be two sets. The intersection of A and B is the set of all those elements which belong to both A and B. The intersection of A and B is denoted by A∩B (read as 'A intersection B').*
Thus, $A \cap B = \{x : x \in A \text{ and } x \in B\}$
e.g. If $A = \{1, 2, 3, 4\}$ *and* $B = \{3, 4, 5, 6\}$*, then*
$A \cap B = \{3, 4\}$

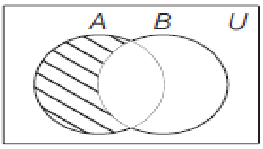


3. ***Disjoint of two sets*** *Two sets A and B are said tobe disjoint, if* $A \cap B = \phi$*,i.e. they don't have any common element. If* $A \cap B \neq \phi$*, then A and B are said to be intersecting or overlapping sets.*
e.g. If $A = \{1, 2, 3\}$ *and* $B = \{4, 5, 6\}$
Then, $A \cap B = \phi$*, so A and B are disjoint sets.*



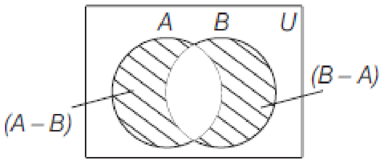
4. ***Difference of two sets*** *Let A and B be two sets. The difference of A and B, written as (A − B) or A\B, is the set of all those elements of A which do not belong to B. Thus,* $(A - B) = \{x : x \in A, x \notin B\}$

The Venn diagram of A − B is as shown in the figure



Similarly, the difference $(B - A)$ *is the set of all those elements of B, which do not belong to A, i.e.* $(B - A) = \{x : x \in B, x \notin A\}$.
e.g. If $A = \{a, b, c, d, e\}$ *and* $B = \{a, c, e, g, i\}$
then, $A - B = \{b, d\}$ *and* $B - A = \{g, i\}$

5. ***Symmetric difference of two sets*** *Let A and B be two sets. The symmetric difference of sets A and B is the set* $(A - B) \cup (B - A)$ *and is denoted by* $A \triangle B$.



Thus, ADB = $(A - B) \cup (B - A) = \{x : x \notin A \cap B\}$
e.g. If $A = \{a, b, c, d, e\}$ *and* $B = \{a, c, e, g, i\}$
Then, $A \triangle B = (A - B) \cup (B - A) = \{b, d\} \cup \{g, i\} = \{b, d, g, i\}$
6. ***Complement of a set*** *The complement of a set A is the set of all those elements which are in universal set but not in A. It is denoted by A¢ or*
A^c *or* $U - A$.
If U is a universal set and A ÌU, then
$A^c = U - A = \{x : x \in U, x \notin A\}$
Clearly, $x \in A^c \iff x \notin A$
e.g. If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ *and* $A = \{1, 3, 5, 7, 9\}$,
then A¢ = $\{2, 4, 6, 8, 10\}$

Laws of Algebra of Sets

1. ***Idempotent laws*** *For any set A, we have*
(i) $A \cup A = A$ (ii) $A \cap A = A$
2. ***Identity laws*** *For any set A, we have*
(i) $A \cup \phi = A$ (ii) $A \cap \phi = \phi$
(iii) $A \cap U = A$ (iv) $A \cup U = U$
3. ***Commutative laws*** *For any two sets A and B, we have*
(i) $A \cup B = B \cup A$ (ii) $A \cap B = B \cap A$
4. ***Associative laws*** *If A, B and C are any three sets, then*
(i) $(A \cup B) \cup C = A \cup (B \cup C)$
(ii) $A \cap (B \cap C) = (A \cap B) \cap C$
5. ***Distributive laws*** *If A, B and C are any three sets, then*
(i) *A ∪ (B Ç C) =* $(A \cup B) \cap (A \cup C)$
(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
6. ***De-Morgan's laws*** *If A, B and C are any three sets, then*
(i) *(A È B)¢ = A¢ ∩ B'*
(ii) $(A \cap B)' = A' \cup B'$
7. (i) $A - (B \cup C) = (A - B) \cap (A - C)$
(ii) $A - (B \cap C) = (A - B) \cup (A - C)$
(iii) $A - B = A \cap B' = B' - A'$
(iv) $A - (A - B) = A \cap B$
(v) $A - B = B - A \iff A = B$
(vi) $A \cup B = A \cap B \iff A = B$
(vii) $A \cup A' = U$
(viii) $A \cap A' = \phi$